6.7 Intersection of Planes Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Determine the solution(s) to each system of planes. If there are no consistent solutions, give evidence and or an explanation of why.

a) $\pi_1 : 3x - 2y + z = 4$

$\pi_2 : 6x - 4y + 3z = 7$

Normals are not parallel; they intersect in a line.

$2\times① : \quad 6x - 4y + 2z = 8$

$② : \quad 6x - 4y + 3z = 7$

$-z = 1, \quad z = -1$

Let $x = t$:

$3t - 2y + (-1) = 4$

$3t - 1 - 4 = 2y$

$3t - 5 = 2y$

$y = \frac{3}{2}t - \frac{5}{2}$

$\ell : \begin{cases} x = t \\ y = \frac{3}{2}t - \frac{5}{2} \\ z = -1 \end{cases}$

$[x, y, z] = \left[0, -\frac{5}{2}, -1\right] + t\left[1, \frac{3}{2}, 0\right]$

b) $\pi_1 : 2x - 8y - 6z - 2 = 0$

$\pi_2 : -x + 4y + 3z - 5 = 0$

Normals ARE parallel; check if coincident or distinct.

$① : \quad 2x - 8y - 6z - 2 = 0$

$2\times② : \quad -2x + 8y + 6z - 10 = 0$

$-12 = 0$

No solutions. Parallel and distinct.

c) $\pi_1 : x - 2y + 2.5z - 1 = 0$

$\pi_2 : 3x - 6y + 7.5z - 3 = 0$

Normals are parallel; check if distinct or coincident

$3\times① : \quad 3x - 6y + 7.5z - 3 = 0$

$② : \quad 3x - 6y + 7.5z - 3 = 0$

$0 = 0$

Parallel and coincident. Infinite solutions.

d) $\pi_1 : x + y + 2z = 5$

$\pi_2 : 4x - 3y + z = -8$

$\pi_3 : -5x - 2y + 3z = 7$

No normals are parallel.

$① : \quad x + y + 2z = 5$

$2\times② : \quad 8x - 6y + 2z = -16$

$-7x + 7y = 21$

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$3\times①:\quad 3x+3y+6z=15$

$2\times③:\quad -10x-4y+6z=14$

$13x+7y=1$

$(-7x+7y=21)-(13x+7y=1):$

$-20x=20,\quad x=-1$

$13(-1)+7y=1$

$7y=14,\quad y=2$

$x+y+2z=5:\quad -1+2+2z=5$

$2z=4,\quad z=2$

The three planes intersect at the point $(-1,\ 2,\ 2)$.

e) $\pi_1: x+y+z=-3$

$\pi_2: 2x+2y-3z=4$

$\pi_3: 3x+3y-2z=1$

No normals are parallel.

$2\times①:\quad 2x+2y+2z=-6$

$②:\quad 2x+2y-3z=4$

$5z=-10,\quad z=-2$

$3\times①:\quad 3x+3y+3z=-9$

$③:\quad 3x+3y-2z=1$

$5z=-10,\quad z=-2$

Let $x=t$:

$t+y+(-2)=-3$

$y=-1-t$

$\ell: \begin{cases} x=t \\ y=-1-t \\ z=-2 \end{cases}$

$[x,\ y,\ z]=[0,\ -1,\ -2]+t[1,\ -1,\ 0]$

f) $\pi_1: -x+y+3z=2$

$\pi_2: 2x-2y-6z=-4$

$\pi_3: -3x+3y+9z=6$

$-2\pi_1=\pi_2,\ 3\pi_1=\pi_3$

Three coincident planes. Infinite solutions.

g) $\pi_1: x-4y-13z=4$

$\pi_2: x-2y-3z=2$

$\pi_3: -3x+5y+4z=2$

No parallel normals.

$①:\quad x-4y-13z=4$

$②:\quad x-2y-3z=2$

$-2y-10z=2$

$3\times①:\quad 3x-12y-39z=12$

$③:\quad -3x+5y+4z=2$

$-7y-35z=14$

$7\times(①-②):\quad -14y-70z=14$

$2\times(3①+③):\quad -14y-70z=28$

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$0 = -14$

No solutions. It is an inconsistent system; the planes intersect in pairs.

h) $\pi_1 : 3x - 2y + 5z = 1$

$\pi_2 : 5x + y - 3z = -4$

$\pi_3 : x - 5y + 13z = 6$

No parallel normals.

$5\times① : \quad 15x - 10y + 25z = 5$

$3\times② : \quad 15x + 3y - 9z = -12$

$-13y + 34z = 17$

$① : \quad 3x - 2y + 5z = 1$

$3\times③ : \quad 3x - 15y + 39z = 18$

$13y - 34z = -17$

The two resulting equations are the same (just multiplied by -1), so the planes intersect in a line. Let $y = t$:

$-13t + 34z = 17$

$z = \frac{17}{34} + \frac{13}{34}t$

$③ : \quad x - 5t + 13\left(\frac{17}{34} + \frac{13}{34}t\right) = 6$

$x = -\frac{1}{2} + \frac{1}{34}t$

$\ell : \begin{cases} x = -\frac{1}{2} + \frac{1}{34}t \\ y = t \\ z = \frac{17}{34} + \frac{13}{34}t \end{cases}$

$[x, y, z] = \left[-\frac{1}{2}, 0, \frac{17}{34}\right] + t\left[\frac{1}{34}, 1, \frac{13}{34}\right]$

i) $\pi_1 : -x + y + 3z = 2$

$\pi_2 : -x + y + 3z = 4$

$\pi_3 : x - 3y + 5z = 6$

$\vec{n_1} = \vec{n_2}$ but $\pi_1 \neq k\pi_2$; they are parallel and distinct.

Inconsistent system; two parallel and distinct planes intersected by a third plane.

j) $\pi_1 : x + 4y + 3z = 5$

$\pi_2 : x + 3y + 2z + 4 = 0$

$\pi_3 : x + y - z = -1$

No parallel normals.

$① : \quad x + 4y + 3z = 5$

$② : \quad x + 3y + 2z = -4$

$y + z = 9$

$① : \quad x + 4y + 3z = 5$

$③ : \quad x + y - z = -1$

$3y + 4z = 6$

$3\times(y + z = 9) : \quad 3y + 3z = 27$

$3y + 4z = 6$

$-z = 21, \quad z = -21$

$y + (-21) = 9$

$y = 30$

$x + 4(30) + 3(-21) = 5$

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$x = -52$

They intersect at the point $(-52, 30, -21)$.

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