6.7 Intersection of Planes Worksheet

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Determine the solution(s) to each system of planes. If there are no consistent solutions, give evidence and or an explanation of why.

a) $\pi_1 : 3x - 2y + z = 4$

$\pi_2 : 6x - 4y + 3z = 7$

b) $\pi_1 : 2x - 8y - 6z - 2 = 0$

$\pi_2 : -x + 4y + 3z - 5 = 0$

c) $\pi_1 : x - 2y + 2.5z - 1 = 0$

$\pi_2 : 3x - 6y + 7.5z - 3 = 0$

d) $\pi_1 : x + y + 2z = 5$

$\pi_2 : 4x - 3y + z = -8$

$\pi_3 : -5x - 2y + 3z = 7$

e) $\pi_1 : x + y + z = -3$

$\pi_2 : 2x + 2y - 3z = 4$

$\pi_3 : 3x + 3y - 2z = 1$

f) $\pi_1 : -x + y + 3z = 2$

$\pi_2 : 2x - 2y - 6z = -4$

$\pi_3 : -3x + 3y + 9z = 6$

g) $\pi_1 : x - 4y - 13z = 4$

$\pi_2 : x - 2y - 3z = 2$

$\pi_3 : -3x + 5y + 4z = 2$

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h) $\pi_1 : 3x - 2y + 5z = 1$

$\pi_2 : 5x + y - 3z = -4$

$\pi_3 : x - 5y + 13z = 6$

i) $\pi_1 : -x + y + 3z = 2$

$\pi_2 : -x + y + 3z = 4$

$\pi_3 : x - 3y + 5z = 6$

j) $\pi_1 : x + 4y + 3z = 5$

$\pi_2 : x + 3y + 2z + 4 = 0$

$\pi_3 : x + y - z = -1$

Answer Key

a) $[x, y, z] = \left[0, -\frac{5}{2}, -1\right] + t\left[1, \frac{3}{2}, 0\right]$

b) No solutions; parallel and distinct

c) Infinite solutions, parallel and coincident

d) $(-1, 2, 2)$

e) $[x, y, z] = [-1, 0, -2] + s[-1, 1, 0]$

f) Infinite solutions; three coincident planes

g) Inconsistent system; the planes intersect in pairs

h) $[x, y, z] = \left[-\frac{7}{13}, -\frac{17}{13}, 0\right] + t\left[\frac{1}{13}, \frac{34}{13}, 1\right]$

i) Inconsistent system; two parallel and distinct planes intersected by a third plane

j) $(-52, 30, -21)$

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