6.6 Intersection of Lines and Planes Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

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1) In each case, determine if the line and the plane are parallel. If so, decide on distinct or coincident.

a) $\ell_1 : \begin{cases} x = 4 + 2t \\ y = -t \\ z = -1 - 4t \end{cases}$ $\pi_1 : 3x + 2y + z - 7 = 0$

$\vec{m} = [2, -1, -4]$, $\vec{n} = [3, 2, 1]$

$\vec{m} \cdot \vec{n} = 2(3) + (-1)(2) + (-4)(1)$

$\vec{m} \cdot \vec{n} = 0$

So $\vec{m}$ and $\vec{n}$ are perpendicular, so ℓ_1 is parallel to the plane.

Check if $(4, 0, -1)$ *is on the plane:*

$LS = 3(4) + 2(0) + (-1) - 7$

$RS = 0$

$LS = 4$

$LS \neq RS$, so ℓ_1 is parallel and distinct from the plane.

b) $\ell_2 : \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}$ $\pi_2 : x - y + 2z = 5$

$\vec{m} = [1, 2, 3]$, $\vec{n} = [1, -1, 2]$

$\vec{m} \cdot \vec{n} = 1(1) + 2(-1) + 3(2)$

$\vec{m} \cdot \vec{n} = 5$

$\vec{m}$ and $\vec{n}$ are not perpendicular, so ℓ_2 is NOT parallel to π_2.

2) In each case, determine if the plane and line intersect. If so, state the solution.

a) $[x, y, z] = [1, 2, 5] + t[1, -1, 2]$ $2x + 6y - z = 5$

$\ell : \begin{cases} x = 1 + t \\ y = 2 - t \\ z = 5 + 2t \end{cases}$

$2(1 + t) + 6(2 - t) - (5 + 2t) = 5$

$2 + 2t + 12 - 6t - 5 - 2t = 5$

$-6t + 9 = 5$

$t = \frac{2}{3}$

$x = 1 + \frac{2}{3} = \frac{5}{3}$

$y = 2 - \frac{2}{3} = \frac{4}{3}$

$z = 5 + 2\left(\frac{2}{3}\right) = \frac{19}{3}$

Point of intersection: $\left(\frac{5}{3}, \frac{4}{3}, \frac{19}{3}\right)$

b) $[x, y, z] = [6, 11, 1] + t[1, 5, 2]$ $x + 3y + 2z - 1 = 0$

$\ell : \begin{cases} x = 6 + t \\ y = 11 + 5t \\ z = 1 + 2t \end{cases}$

$(6 + t) + 3(11 + 5t) + 2(1 + 2t) - 1 = 0$

$6 + t + 33 + 15t + 2 + 4t - 1 = 0$

$20t + 40 = 0$

$t = -2$

$x = 6 + (-2) = 4$

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$y = 11 + 5(-2) = 1$

$z = 1 + 2(-2) = -3$

Point of intersection: $(4, 1, -3)$

c) $[x, y, z] = [9, 8, 3] + t[2, 1, 5]$ $z = 0$

$\ell : \begin{cases} x = 9 + 2t \\ y = 8 + t \\ z = 3 + 5t \end{cases}$

$3 + 5t = 0$

$t = -\frac{3}{5}$

$x = 9 + 2\left(-\frac{3}{5}\right) = \frac{39}{5}$

$y = 8 + \left(-\frac{3}{5}\right) = \frac{37}{5}$

Point of intersection: $\left(\frac{39}{5}, \frac{37}{5}, 0\right)$

d) $[x, y, z] = [4, 2, 6] + t[1, -2, 3]$ $2x + 5y - z - 34 = 0$

$\ell : \begin{cases} x = 4 + t \\ y = 2 - 2t \\ z = 6 + 3t \end{cases}$

$2(4 + t) + 5(2 - 2t) - (6 + 3t) - 34 = 0$

$8 + 2t + 10 - 10t - 6 - 3t - 34 = 0$

$-11t = 22$

$t = -2$

$x = 4 + (-2) = 2$

$y = 2 - 2(-2) = 6$

$z = 6 + 3(-2) = 0$

Point of intersection: $(2, 6, 0)$

e) $[x, y, z] = [3, 2, -1] + t[-2, 1, 3]$ $x + 2y - 3z = 10$

$\ell : \begin{cases} x = 3 - 2t \\ y = 2 + t \\ z = -1 + 3t \end{cases}$

$(3 - 2t) + 2(2 + t) - 3(-1 + 3t) = 10$

$3 - 2t + 4 + 2t + 3 - 9t = 10$

$-9t = 0$

$t = 0$

Point of intersection: $(3, 2, -1)$

f) $[x, y, z] = [4, 2, 6] + t[1, -2, 3]$ $-4x - 5y + 6z = 34$

$\ell : \begin{cases} x = 4 + t \\ y = 2 - 2t \\ z = 6 + 3t \end{cases}$

$-4(4 + t) - 5(2 - 2t) + 6(6 + 3t) = 34$

$-16 - 4t - 10 + 10t + 36 + 18t = 34$

$24t = 24$

$t = 1$

$x = 4 + 1 = 5$

$y = 2 - 2(1) = 0$

$z = 6 + 3(1) = 9$

Point of intersection: $(5, 0, 9)$

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3) Determine if each of the following lines intersects the plane: $\pi : [x, y, z] = [4, -15, -8] + s[1, -3, 1] + t[2, 3, 1]$. If so, how many intersections are there?

$\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2} = [-3(1) - 1(3), 1(2) - 1(1), 1(3) - (-3)(2)]$

$\vec{n} = [-6, 1, 9]$

$\pi : -6x + y + 9z + D = 0$

$-6(4) + (-15) + 9(-8) + D = 0$

$D = 111$

$\pi : -6x + y + 9z + 111 = 0$

a) $[x, y, z] = [5, -9, 3] + k[1, -12, 2]$

$\overrightarrow{m_3} = [1, -12, 2]$

$\overrightarrow{m_3} \cdot \vec{n} = 1(-6) + (-12)(1) + 2(9)$

$\overrightarrow{m_3} \cdot \vec{n} = 0$, so the line is parallel to the plane.

Check if $(5, -9, 3)$ *is on the plane:*

$-6(5) + (-9) + 9(3) + 111$

$= 99 \neq 0$, so $P(5, -9, 3)$ is not on the plane.

The line and plane are parallel and distinct. No intersection.

b) $[x, y, z] = [-2, 9, -21] + k[2, -5, 4]$

$\overrightarrow{m_4} = [2, -5, 4]$

$\overrightarrow{m_4} \cdot \vec{n} = 2(-6) + (-5)(1) + 4(9)$

$\overrightarrow{m_4} \cdot \vec{n} = 19$

The line is not parallel to the plane. There is 1 point of intersection.

c) $[x, y, z] = [3, -2, 1] + k[1, 4, -2]$

$\overrightarrow{m_5} = [1, 4, -2]$

$\overrightarrow{m_5} \cdot \vec{n} = 1(-6) + 4(1) + (-2)(9)$

$\overrightarrow{m_5} \cdot \vec{n} = -20$

The line and plane are not parallel. There is 1 point of intersection.

d) $[x, y, z] = [4, -24, -7] + k[-2, -3, -1]$

$\overrightarrow{m_6} = [-2, -3, -1]$

$\overrightarrow{m_6} \cdot \vec{n} = -2(-6) + (-3)(1) + (-1)(9)$

$\overrightarrow{m_6} \cdot \vec{n} = 0$, so the line and plane are parallel.

Check if $(4, -24, -7)$ *is on the plane:*

$-6(4) + (-24) + 9(-7) + 111$

$= 0$, so the point is on the plane.

The line lies on the plane. Infinite solutions.

e) $[x, y, z] = [2, -3, 0] + k[-1, 3, -1]$

$\overrightarrow{m_7} = [-1, 3, -1]$

$\overrightarrow{m_7} \cdot \vec{n} = -6(-1) + 1(3) + 9(-1)$

$\overrightarrow{m_7} \cdot \vec{n} = 0$, so the line and plane are parallel.

Check if $(2, -3, 0)$ *is on the plane:*

$-6(2) + (-3) + 9(0) + 111$

$= 96 \neq 0$, so the point is not on the plane.

The line and plane are parallel and distinct. No intersection.

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f) $[x, y, z] = [9, 4, 1] + k[-2, 2, 4]$

$\overrightarrow{m_8} = [-2, 2, 4]$

$\overrightarrow{m_8} \cdot \vec{n} = -6(-2) + 1(2) + 9(4)$

$\overrightarrow{m_8} \cdot \vec{n} = 50$

The line and plane are not parallel. There is 1 point of intersection.

4) Find the distance between each point and the given plane.

a) $P(1, 1, -1)$, $x + y - z - 3 = 0$

Point Q on plane:

$0 + 0 - z - 3 = 0$

$z = -3$, so $Q(0, 0, -3)$

$\overrightarrow{PQ} = [-1, -1, -2]$, $\vec{n} = [1, 1, -1]$

$$d = \frac{|\overrightarrow{PQ} \cdot \vec{n}|}{|\vec{n}|} = \frac{|-1(1) + (-1)(1) + (-2)(-1)|}{\sqrt{(1)^2 + (1)^2 + (-1)^2}}$$

$d = \frac{0}{\sqrt{3}} = 0$, so the point is on the plane.

b) $P(7, -3, 2)$, $2x - 3z - 1 = 0$

Point Q on plane:

$2(2) - 3z - 1 = 0$

$z = 1$, so $Q(2, 5, 1)$

$\overrightarrow{PQ} = [-5, 8, -1]$, $\vec{n} = [2, 0, -3]$

$$d = \frac{|-5(2) + 8(0) + (-1)(-3)|}{\sqrt{(2)^2 + (0)^2 + (-3)^2}}$$

$d = \frac{7}{\sqrt{13}}$

$d \cong 1.94$

5) Find the distance between the planes $\pi_1 : 2x + 2y - z - 3 = 0$ and $\pi_2 : 4x + 4y - 2z + 9 = 0$

Point on π_1: $2(0) + 2(0) - z - 3 = 0$

$z = -3$, so $A(0, 0, -3)$

Point on π_2: $4(0) + 4(0) - 2z + 9 = 0$

$z = 4.5$, so $B(0, 0, 4.5)$

$\overrightarrow{AB} = [0, 0, 7.5]$, $\vec{n} = [2, 2, -1]$

$$d = \frac{|0(2) + 0(2) + 7.5(-1)|}{\sqrt{(2)^2 + (2)^2 + (-1)^2}}$$

$d = \frac{7.5}{3} = 2.5$

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