6.5 Intersections of Lines in 2-Space and 3-Space Worksheet

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Solve each linear system in two-space.

a) $3x - 5y = -9,\ 4x + 5y = 23$

b) $\ell_1 : [x, y] = [5, 4] + s[-3, 1],\ \ell_2 : [x, y] = [2, 2] + t[2, -1]$

2) Determine if the parallel lines in each pair are distinct or coincident.

a) $\ell_1 : [x, y, z] = [5, -2, -8] + s[-3, 2, 5]$

$\ell_2 : [x, y, z] = [-4, 0, 2] + t[-3, 2, 5]$

b) $\ell_1 : [x, y, z] = [-3, 0, -6] + s[-3, 0, -6]$

$\ell_2 : [x, y, z] = [9, 1, 18] + t[9, 0, 18]$

3) Triangle ABC is formed from the intersections of the three lines represented by these equations. Find the length of each side of $\triangle ABC$.

$\ell_1 : [x, y] = [1, -3] + t[0, 1]$ $\ell_2 : [x, y] = [2, 4] + s[-1, 6]$ $\ell_3 : [x, y] = [2, 3] + r[1, 7]$

4) Parallelogram ABCD has vertices $A(-1, -4)$, $B(1, -3)$, $C(6, -6)$, and $D(4, -7)$. Find the vector equations of its diagonals and the point of intersection of the diagonals.

5) Determine if the lines in each pair intersect. If so, find the co-ordinates of the point of intersection.

a) $\ell_1 : [x, y, z] = [9, -1, 1] + s[-3, 4, 1]$

$\ell_2 : [x, y, z] = [-3, 11, 5] + t[-3, 4, 1]$

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b) $\ell_1 : [x,\ y,\ z] = [1,\ 4,\ 5] + s[3,\ 0,\ -2]$

$\ell_2 : [x,\ y,\ z] = [9,\ 4,\ -3] + t[3,\ 0,\ -2]$

c) $\ell_1 : [x,\ y,\ z] = [1,\ 0,\ -3] + t[3,\ 5,\ 4]$

$\ell_2 : [x,\ y,\ z] = [0,\ -9,\ -1] + s[-1,\ 2,\ -3]$

d) $\ell_1 : [x,\ y,\ z] = [4,\ 7,\ -1] + s[-2,\ 1,\ 2]$

$\ell_2 : [x,\ y,\ z] = [1,\ 3,\ -1] + t[4,\ -1,\ 2]$

e) $\ell_1 : [x,\ y,\ z] = [-2,\ 0,\ -3] + t[5,\ 1,\ 3]$

$\ell_2 : [x,\ y,\ z] = [5,\ 8,\ -6] + s[-1,\ 2,\ -3]$

6) Determine the distance between the skew lines in each pair.

a) $\ell_1 : [x,\ y,\ z] = [4,\ -3,\ 2] + s[2,\ 7,\ -1]$

$\ell_2 : [x,\ y,\ z] = [-2,\ 5,\ 4] + t[-4,\ 0,\ 3]$

b) $\ell_1 : [x,\ y,\ z] = [-1,\ 6,\ 1] + s[-2,\ 4,\ 3]$

$\ell_2 : [x,\ y,\ z] = [5,\ 1,\ 9] + t[3,\ -2,\ 4]$

Answer Key

1. a) $(2,\ 3)$ b) $(-16,\ 11)$

2. a) parallel and distinct b) parallel and distinct

3. $6.55,\ 14,\ 7.61$

4. $\overrightarrow{AC} = [-1,\ -4] + s[7,\ -2]$, $\overrightarrow{BD} = [1,\ -3] + t[3,\ -4]$; $\left(\frac{5}{2},\ -5\right)$

5. a) No solutions; parallel and distinct b) No solutions; parallel and distinct c) $(-2,\ -5,\ -7)$ d) No solution; skew e) $(8,\ 2,\ 3)$

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6. a) 2.45 b) 0.59

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