6.4 Equations of Planes in Scalar Form Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

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1) Write the scalar equation of each plane given the normal $\vec{n}$ and a point P on the plane.

a) $\vec{n} = [1, -3, -4]$, $P(-2, 5, 7)$

$1(-2) - 3(5) - 4(7) + D = 0$

$D = 45$

$x - 3y - 4z + 45 = 0$

b) $\vec{n} = [0, -3, 5]$, $P(8, -7, 3)$

$0(8) - 3(-7) + 5(3) + D = 0$

$D = -36$

$-3y + 5z - 36 = 0$

2) Write a scalar equation of each plane, given its vector or parametric equation.

a) $[x, y, z] = [7, -3, 4] + s[1, -2, -3] + t[4, -3, 1]$

$\vec{n} = [-2(1) - (-3)(-3), -3(4) - 1(1), 1(-3) - (-2)(4)]$

$\vec{n} = [-11, -13, 5]$

$-11(7) - 13(-3) + 5(4) + D = 0$

$D = 18$

$-11x - 13y + 5z + 18 = 0$

b) $\pi_1 : \begin{cases} x = 4 + t + 2s \\ y = 3 - 2t - 3s \\ z = -1 + 3t + s \end{cases}$

$\overrightarrow{m_1} = [1, -2, 3]$, $\overrightarrow{m_2} = [2, -3, 1]$

$\vec{n} = [-2(1) - 3(-3), 3(2) - 1(1), 1(-3) - (-2)(2)]$

$\vec{n} = [7, 5, 1]$

$7(4) + 5(3) + 1(-1) + D = 0$

$D = -42$

$7x + 5y + z - 42 = 0$

3) For each situation, write the scalar equation of the plane.

a) has normal $\vec{n} = (7, 9, -1)$ and includes the point $(3, -2, 4)$

$7(3) + 9(-2) - 1(4) + D = 0$

$D = 1$

$7x + 9y - z + 1 = 0$

b) contains direction vectors $\vec{a} = [-1, 2, 8]$ and $\vec{b} = [2, -1, 3]$ and includes the point $(2, -7, 8)$

$\vec{n} = \vec{a} \times \vec{b} = [2(3) - 8(-1), 8(2) - (-1)(3), -1(-1) - 2(2)]$

$\vec{n} = [14, 19, -3]$

$14(2) + 19(-7) - 3(8) + D = 0$

$D = 129$

$14x + 19y - 3z + 129 = 0$

c) parallel to the xz-plane and includes the point $(7, 8, -1)$

$\vec{n} = [0, 1, 0]$

$0(7) - 1(8) + 0(-1) + D = 0$

$D = 8$

$y - 8 = 0$

d) contains the points $(3, 8, -1)$, $(-8, 9, -4)$, and $(1, -3, 2)$

$\overrightarrow{m_1} = [-11, 1, -3]$, $\overrightarrow{m_2} = [-2, -11, 3]$

$\vec{n} = [1(3) - (-3)(-11), -3(-2) - (-11)(3), -11(-11) - 1(-2)]$

$\vec{n} = [-30, 39, 123]$

$-30(3) + 39(8) + 123(-1) + D = 0$

$D = -99$

$-30x + 39y + 123z - 99 = 0$

e) contains the line $[x, y, z] = [4, -3, -2] + s[3, -2, 1]$ and parallel to the line defined by the parametric equations $\ell : \begin{cases} x = 5 + 3s \\ y = 1 - s \\ z = 2 + 4s \end{cases}$

$\overrightarrow{m_1} = [3, -2, 1]$, $\overrightarrow{m_2} = [3, -1, 4]$

$\vec{n} = [-2(4) - 1(-1), 1(3) - 3(4), 3(-1) - (-2)(3)]$

$\vec{n} = [-7, -9, 3]$

$-7(4) - 9(-3) + 3(-2) + D = 0$

$D = 7$

$-7x - 9y + 3z + 7 = 0$

f) contains the point $(2, -1, 8)$ and perpendicular to the line $[x, y, z] = [1, -2, -3] + s[5, -4, 7]$

$\vec{n} = [5, -4, 7]$

$5(2) - 4(-1) + 7(8) + D = 0$

$D = -70$

$5x - 4y + 7z - 70 = 0$

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g) parallel to the plane $-3x + 2y + 5z + 8 = 0$ and includes the point $(5, -7, 8)$ $\vec{n} = [-3, 2, 5]$ $-3(5) + 2(-7) + 5(8) + D = 0$ $D = -11$ $-3x + 2y + 5z - 11 = 0$	h) contains the lines $[x, y, z] = [4, -1, 0] + s[-2, 1, 3]$ and $[x, y, z] = [-2, 4, 3] + s[-6, 5, 7]$ $\vec{n} = [1(7) - 3(5), 3(-6) - (-2)(7), (-2)(5) - 1(-6)] = -4[2, 1, 1]$ $\vec{n} = [2, 1, 1]$ $2(4) + 1(-1) + 1(0) + D = 0$ $D = -7$ $2x + y + z - 7 = 0$

4) Determine the angle between the planes $x + 2y - 3z - 4 = 0$ and $x + 2y - 1 = 0$. *(Hint: the angle between 2 intersecting planes should be equal to the angle between their normals)*

$\vec{n_1} = [1, 2, -3]$, $\vec{n_2} = [1, 2, 0]$

$\cos\theta = \dfrac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$

$\cos\theta = \dfrac{5}{(\sqrt{14})(\sqrt{5})}$

$\theta \cong 53.3°$

5) Write the vector equation and parametric equations of the plane: $3x - y + 2z - 4 = 0$

x-int: $3x = 4$, $x = \frac{4}{3}$

y-int: $-y = 4$, $y = -4$

z-int: $2z = 4$, $z = 2$

So $X\left(\frac{4}{3}, 0, 0\right)$, $Y(0, -4, 0)$, $Z(0, 0, 2)$ are points on the plane.

$\vec{m_1} = \overrightarrow{XY} = \left[-\frac{4}{3}, -4, 0\right]$, reduces to $[1, 3, 0]$

$\vec{m_2} = \overrightarrow{YZ} = [0, 4, 2]$, reduces to $[0, 2, 1]$

$[x, y, z] = [0, 0, 2] + t[1, 3, 0] + s[0, 2, 1]$

$\pi : \begin{cases} x = t \\ y = 3t + 2s \\ z = 2 + s \end{cases}$

6) Find a vector that is normal and one that is parallel to each plane.

a) $x + 2y + 2z - 5 = 0$ normal: $\vec{n} = [1, 2, 2]$ parallel: $\vec{m} = [-2, 1, 0]$	b) $5x + 2z = 7$ normal: $\vec{n} = [5, 0, 2]$ parallel: $\vec{m} = [-2, 0, 5]$	c) $5y = 8$ normal: $\vec{n} = [0, 5, 0]$ parallel: $\vec{m} = [2, 0, 7]$ (anything in the xz-plane)

7) Consider the plane $-x + 4y + 2z + 6 = 0$

a) Determine a normal vector, $\vec{n}$, to the plane.

$\vec{n} = [-1, 4, 2]$

b) Determine the coordinates of two points, S and T, on the plane.

Let $y = z = 0$: $-x + 6 = 0$, $x = 6$

$S(6, 0, 0)$

Let $x = y = 0$: $2z + 6 = 0$, $z = -3$

$T(0, 0, -3)$

c) Determine $\overrightarrow{ST}$

$\overrightarrow{ST} = [0 - 6, 0 - 0, -3 - 0]$

$\overrightarrow{ST} = [-6, 0, -3]$

d) Show that $\overrightarrow{ST}$ is perpendicular to $\vec{n}$.

$\overrightarrow{ST} \cdot \vec{n} = [-6, 0, -3] \cdot [-1, 4, 2]$

$\overrightarrow{ST} \cdot \vec{n} = 6 + 0 - 6$

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$\overrightarrow{ST} \cdot \vec{n} = 0$, so $\overrightarrow{ST}$ is perpendicular to $\vec{n}$.

8) State the scalar equation of each of the following

a) the xy-plane	b) the xz-plane	c) the yz-plane
$z = 0$	$y = 0$	$x = 0$

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