6.2 Equations of Lines in Three-Space Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Write a vector equation for a line given each direction vector $\vec{m}$ and point P_0.

a) $\vec{m} = [2, -3, 5]$, $P_0(8, -11, 2)$ $[x, y, z] = [8, -11, 2] + t[2, -3, 5]$	b) $\vec{m} = [-3, 4, 7]$, $P_0(7, -1, 4)$ $[x, y, z] = [7, -1, 4] + t[-3, 4, 7]$

2) Write the vector equation of the line that passes through each pair of points.

a) $A(4, -3, 1)$, $B(2, -3, 7)$ $\vec{m} = [2-4, -3-(-3), 7-1]$ $\vec{m} = [-2, 0, 6] = -2[1, 0, -3]$ $[x, y, z] = [4, -3, 1] + t[1, 0, -3]$	b) $A(5, 6, -1)$, $B(0, -3, 2)$ $\vec{m} = [0-5, -3-6, 2-(-1)]$ $\vec{m} = [-5, -9, 3]$ $[x, y, z] = [5, 6, -1] + t[-5, -9, 3]$

3) Write the parametric equations for each vector equation.

a) $[x, y, z] = [0, 4, -7] + t[3, -4, 1]$ $\ell : \begin{cases} x = 3t \\ y = 4 - 4t \\ z = -7 + t \end{cases}$	b) $[x, y, z] = [7, 5, -4] + t[-2, 1, 3]$ $\ell : \begin{cases} x = 7 - 2t \\ y = 5 + t \\ z = -4 + 3t \end{cases}$

4) Write a vector equation for each line, given the parametric equations.

a) $\ell : \begin{cases} x = 4 + 3t \\ y = 7 - 2t \\ z = 1 + t \end{cases}$ $[x, y, z] = [4, 7, 1] + t[3, -2, 1]$	b) $\ell : \begin{cases} x = 4t \\ y = 5 - 9t \\ z = -3 \end{cases}$ $[x, y, z] = [0, 5, -3] + t[4, -9, 0]$

5) Write a vector equation and the parametric equations of a line going through the points $A(7, 8, -3)$ and $B(-2, 3, 5)$.

$\vec{m} = [7-(-2), 8-3, -3-5]$

$\vec{m} = [9, 5, -8]$

$[x, y, z] = [7, 8, -3] + t[9, 5, -8]$

$\ell : \begin{cases} x = 7 + 9t \\ y = 8 + 5t \\ z = -3 - 8t \end{cases}$

6) Determine the vector equation of a line through $P_0(-1, -7, 7)$ and perpendicular to $[x, y, z] = [1, -7, 3] + t[4, -1, 2]$.

$\vec{n} = [4, -1, 2]$

Choose $\vec{m}$ such that $\vec{m} \cdot \vec{n} = 0$:

$\vec{m} = [0, 2, 1]$

$[x, y, z] = [-1, -7, 7] + t[0, 2, 1]$

7) Determine which points are on the line $[x, y, z] = [3, 1, -4] + t[2, 0, 5]$.

a) $(-5, 1, -24)$ $-5 = 3 + 2t \Rightarrow t = -4$ $1 = 1$ (always true) $-24 = -4 + 5t \Rightarrow t = -4$ All equations give $t = -4$. The point IS on the line.	b) $(19, 1, 37)$ $19 = 3 + 2t \Rightarrow t = 8$ $1 = 1$ (always true) $37 = -4 + 5t \Rightarrow t = \frac{41}{5}$ The t-values are not equal. The point is NOT on the line.

8) State possible vector, parametric, and symmetric equations for each of the following lines:

a) the line passing through the point $P(-1, 2, 1)$ with direction vector $(3, -2, 1)$

$[x, y, z] = [-1, 2, 1] + t[3, -2, 1]$

Copyright © 2026 Annie Bernatchez—All rights reserved.

$\ell : \begin{cases} x = -1 + 3t \\ y = 2 - 2t \\ z = 1 + t \end{cases}$

$\frac{x+1}{3} = \frac{y-2}{-2} = z - 1$

b) the line passing through the points $A(-1, 1, 0)$ and $B(-1, 2, 1)$

$\vec{m} = [-1 - (-1), 2 - 1, 1 - 0]$

$\vec{m} = [0, 1, 1]$

$[x, y, z] = [-1, 1, 0] + t[0, 1, 1]$

$\ell : \begin{cases} x = -1 \\ y = 1 + t \\ z = t \end{cases}$

$y - 1 = z;\ x = -1$

c) the line passing through the point $Q(1, 2, 4)$ and parallel to the z-axis

$\vec{m} = [0, 0, 1]$

$[x, y, z] = [1, 2, 4] + t[0, 0, 1]$

$\ell : \begin{cases} x = 1 \\ y = 2 \\ z = 4 + t \end{cases}$

No symmetric equation is possible for this line since two of the direction vector coordinates are zero.

9) Determine the parametric equations of the line whose direction vector is perpendicular to the direction vectors of the two lines $\frac{x}{-4} = \frac{y+10}{-7} = \frac{z+2}{3}$ and $\frac{x-5}{3} = \frac{y-5}{2} = \frac{z+5}{4}$ and passes through the point $(2, -5, 0)$.

$\vec{m_1} = [-4, -7, 3]$

$\vec{m_2} = [3, 2, 4]$

$\vec{m_1} \times \vec{m_2} = [(-7)(4) - 3(2), 3(3) - (-4)(4), (-4)(2) - (-7)(3)]$

$\vec{m_1} \times \vec{m_2} = [-34, 25, 13]$

$[x, y, z] = [2, -5, 0] + t[-34, 25, 13]$

$\ell : \begin{cases} x = 2 - 34t \\ y = -5 + 25t \\ z = 13t \end{cases}$

Copyright © 2026 Annie Bernatchez—All rights reserved.