6.8 Unit 6 Pretest Review – Lines and Planes with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

This review covers all of Unit 6: Lines and Planes, and is meant to be used to prepare for the unit test.

Section 1: 6.1 – Vector Equation of a Line in Two-Space

1) Write a vector equation for the line given its direction vector $\vec{m} = [3,\ 1]$ and point on the line $P(2,\ 7)$.

$[x,\ y] = [2,\ 7] + t[3,\ 1]$

Section 2: 6.1 – Vector Equation Through Two Points

2) Write the vector equation of the line through each pair of points.

a) $A(1,\ 7)$ and $B(4,\ 10)$ $\overrightarrow{AB} = [3,\ 3] = 3[1,\ 1]$ $[x,\ y] = [1,\ 7] + t[1,\ 1]$	b) $A(-3,\ 5)$ and $B(-2,\ -8)$ $\overrightarrow{AB} = [1,\ -13]$ $[x,\ y] = [-3,\ 5] + t[1,\ -13]$

Section 3: 6.1 – Point on a Line

3) Determine if the point $P(-1,\ 11)$ is on the line $[x,\ y] = [3,\ 1] + t[-2,\ 5]$

$\ell : \begin{cases} x = 3 - 2t \\ y = 1 + 5t \end{cases}$

$-1 = 3 - 2t \Rightarrow t = 2$

$11 = 1 + 5t \Rightarrow t = 2$

Since both equations give $t = 2$, $P(-1,\ 11)$ is on the line. **Yes**

Section 4: 6.1 – Parametric Equations of a Line

4) Write the parametric equation of each vector equation

a) $[x,\ y] = [10,\ 6] + t[13,\ 1]$ $\ell : \begin{cases} x = 10 + 13t \\ y = 6 + t \end{cases}$	b) $[x,\ y,\ z] = [3,\ 0,\ -1] + t[6,\ -9,\ 1]$ $\ell : \begin{cases} x = 3 + 6t \\ y = -9t \\ z = -1 + t \end{cases}$

Section 5: 6.2 – Vector Equation from Parametric Equations

5) Write the vector equation of each parametric equation

a) $\ell : \begin{cases} x = 1 + 4t \\ y = -6 + t \\ z = 2 - 2t \end{cases}$ $[x,\ y,\ z] = [1,\ -6,\ 2] + t[4,\ 1,\ -2]$	b) $\ell : \begin{cases} x = 7 \\ y = -t \\ z = 0 \end{cases}$ $[x,\ y,\ z] = [7,\ 0,\ 0] + t[0,\ -1,\ 0]$

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Section 6: 6.2 – Line Through Two Points in Three-Space

6) Given the points $A(3,\ 4,\ -5)$ and $B(9,\ -2,\ 7)$, write a vector equation and the parametric equations of the line through A and B.

$\vec{m} = \overrightarrow{AB} = [6,\ -6,\ 12] = 6[1,\ -1,\ 2]$

Vector: $[x,\ y,\ z] = [3,\ 4,\ -5] + t[1,\ -1,\ 2]$

Parametric: $\ell : \begin{cases} x = 3 + t \\ y = 4 - t \\ z = -5 + 2t \end{cases}$

Section 7: 6.1 – Parallel Lines

7) Determine if $\ell_1 : \begin{cases} x = 3 - 2t \\ y = 4 + t \end{cases}$ and $\ell_2 : 2x + 4y - 7 = 0$ are parallel, perpendicular, or neither. Explain.

$\vec{m_1} = [-2,\ 1]$, $\vec{n_2} = [2,\ 4]$

$\vec{m_1} \cdot \vec{n_2} = -2(2) + 1(4)$

$\vec{m_1} \cdot \vec{n_2} = 0$

$\vec{n_2}$ is perpendicular to both ℓ_1's direction and ℓ_2 itself; therefore ℓ_1 and ℓ_2 are **parallel**.

Section 8: 6.2 – Vector Equations of Lines (Special Conditions)

8) Determine a vector equation of each line.

a) perpendicular to line $4x - 3y = 17$ and through point $P(-2,\ 4)$

$[x,\ y] = [-2,\ 4] + t[4,\ -3]$

b) parallel to the z-axis and through point $P(1,\ 5,\ 10)$

$[x,\ y,\ z] = [1,\ 5,\ 10] + t[0,\ 0,\ 1]$

c) parallel to $[x,\ y,\ z] = [3,\ 3,\ 0] + t[3,\ -5,\ -9]$ with an x-intercept of -10

$[x,\ y,\ z] = [-10,\ 0,\ 0] + t[3,\ -5,\ -9]$

d) with the same x-intercept as $[x,\ y,\ z] = [3,\ 0,\ 0] + t[4,\ -4,\ 1]$ and the same z-intercept as $[x,\ y,\ z] = [6,\ -2,\ -3] + t[3,\ -1,\ -2]$

x-intercept of 1st line $(t = 0) : P_1(3,\ 0,\ 0)$

$\ell_2 : \begin{cases} x = 6 + 3t \\ y = -2 - t \\ z = -3 - 2t \end{cases}$

Set $x = 0 : 0 = 6 + 3t \Rightarrow t = -2$

$z = -3 - 2(-2) = 1$, so $P_2(0,\ 0,\ 1)$

$\vec{m} = \overrightarrow{P_2P_1} = [3,\ 0,\ -1]$

$[x,\ y,\ z] = [3,\ 0,\ 0] + t[3,\ 0,\ -1]$

Section 9: 6.4 – Point on a Plane

9) Does the point $P(1,\ 2,\ 0)$ lie on the plane $4x + 3y - 5z = 10$?

$LS = 4(1) + 3(2) - 5(0) = 10$

$RS = 10$

$LS = RS$, so $P(1,\ 2,\ 0)$ is on the plane. **Yes**

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Section 10: 6.4 – Intercepts of a Plane

10) Find the x-, y-, and z-intercepts of each plane

a) $4x + 2y - 7z + 14 = 0$

x-int: $4x + 14 = 0 \Rightarrow x = -\frac{7}{2}$

y-int: $2y + 14 = 0 \Rightarrow y = -7$

z-int: $-7z + 14 = 0 \Rightarrow z = 2$

x-int: $\left(-\frac{7}{2}, 0, 0\right)$, y-int: $(0, -7, 0)$, z-int: $(0, 0, 2)$

b) $3x + 6z + 18 = 0$

x-int: $3x + 18 = 0 \Rightarrow x = -6$

y-int: no y term; $18 = 0$ is false, so none

z-int: $6z + 18 = 0 \Rightarrow z = -3$

x-int: $(-6, 0, 0)$, y-int: none, z-int: $(0, 0, -3)$

Section 11: 6.3 – Parametric Equation of a Plane

11) Write the parametric equation of the plane $[x, y, z] = [0, -4, 1] + s[1, 10, -1] + t[0, 3, 4]$

$\pi : \begin{cases} x = s \\ y = -4 + 10s + 3t \\ z = 1 - s + 4t \end{cases}$

Section 12: 6.3 – Vector Equation of a Plane

12) Write the vector equation of the plane $\pi : \begin{cases} x = 2 + s + 7t \\ y = 12s - 8t \\ z = 11 + 6s \end{cases}$

$[x, y, z] = [2, 0, 11] + s[1, 12, 6] + t[7, -8, 0]$

Section 13: 6.4 – Point on a Plane (Vector Form)

13) Determine if the point $P(-4, -13, 10)$ is on the plane $[x, y, z] = [6, -7, 10] + s[1, 3, -1] + t[2, -2, 1]$

$\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2} = [3(1) - (-1)(-2), (-1)(2) - 1(1), 1(-2) - 3(2)]$

$\vec{n} = [1, -3, -8]$

$1(6) - 3(-7) - 8(10) + D = 0 \Rightarrow D = 53$

$\pi : x - 3y - 8z + 53 = 0$

$LS = -4 - 3(-13) - 8(10) + 53 = 8$, $RS = 0$

$LS \neq RS$, so $P(-4, -13, 10)$ is **NOT** on the plane.

Section 14: 6.4 – Intercepts of a Plane (Vector Form)

14) Determine the x-, y-, and z-intercepts of $[x, y, z] = [1, 8, 6] + s[1, -12, -12] + t[2, 4, -3]$

$\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2} = [84, -21, 28] = 7[12, -3, 4]$

$12(1) - 3(8) + 4(6) + D = 0 \Rightarrow D = -12$

$\pi : 12x - 3y + 4z - 12 = 0$

x-int: $12x - 12 = 0 \Rightarrow x = 1$

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y-int: $-3y-12=0 \Rightarrow y=-4$

z-int: $4z-12=0 \Rightarrow z=3$

x-int: $(1,\ 0,\ 0)$, y-int: $(0,\ -4,\ 0)$, z-int: $(0,\ 0,\ 3)$

Section 15: 6.3 – Equation of a Plane from Points/Intercepts

15) Write a vector and parametric equation of each plane

a) contains the points $A(1,\ 3,\ -2)$, $B(3,\ -9,\ 7)$, and $C(4,\ -4,\ 5)$

$\overrightarrow{m_1}=\overrightarrow{AB}=[2,\ -12,\ 9]$

$\overrightarrow{m_2}=\overrightarrow{BC}=[1,\ 5,\ -2]$

Vector: $[x,\ y,\ z]=[1,\ 3,\ -2]+s[2,\ -12,\ 9]+t[1,\ 5,\ -2]$

Parametric: $\pi:\begin{cases}x=1+2s+t\\y=3-12s+5t\\z=-2+9s-2t\end{cases}$

b) has x-int 8, y-int -3, and z-int 2

$P_1(8,\ 0,\ 0)$, $P_2(0,\ -3,\ 0)$, $P_3(0,\ 0,\ 2)$

$\overrightarrow{m_1}=\overrightarrow{P_2P_1}=[8,\ 3,\ 0]$

$\overrightarrow{m_2}=\overrightarrow{P_2P_3}=[0,\ 3,\ 2]$

Vector: $[x,\ y,\ z]=[8,\ 0,\ 0]+s[8,\ 3,\ 0]+t[0,\ 3,\ 2]$

Parametric: $\pi:\begin{cases}x=8+8s\\y=3s+3t\\z=2t\end{cases}$

Section 16: 6.4 – Scalar Equation of a Plane

16) Determine an equation for each plane

a) parallel to the xz-plane; and through the point $A(3,\ 1,\ 5)$

$y=1$

b) containing the points $A(2,\ 1,\ 1)$, $B(5,\ -3,\ 2)$, and $C(0,\ -1,\ 4)$

$\overrightarrow{m_1}=\overrightarrow{AB}=[3,\ -4,\ 1]$

$\overrightarrow{m_2}=\overrightarrow{CB}=[5,\ -2,\ -2]$

$[x,\ y,\ z]=[2,\ 1,\ 1]+s[3,\ -4,\ 1]+t[5,\ -2,\ -2]$

Section 17: 6.4 – Scalar Equation from Normal and Point

17) Write the scalar equation of a plane given its normal $\vec{n}=[3,\ 7,\ 1]$ and a point $P(3,\ -6,\ 4)$ on the plane.

$3(3)+7(-6)+1(4)+D=0 \Rightarrow D=29$

$3x+7y+z+29=0$

Section 18: 6.4 – Scalar Equation from Vector/Parametric Form

18) Write a scalar equation of each plane, given its vector or parametric equation.

a) $[x,\ y,\ z]=[3,\ 7,\ -5]+s[1,\ 2,\ -1]+t[1,\ -2,\ 3]$

$\vec{n}=\overrightarrow{m_1}\times\overrightarrow{m_2}=[4,\ -4,\ -4]=4[1,\ -1,\ -1]$

$1(3)-1(7)-1(-5)+D=0 \Rightarrow D=-1$

$x-y-z-1=0$

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b) $\pi : \begin{cases} x = 3 - 2s + 2t \\ y = 1 + 3s + t \\ z = 5 - s - 2t \end{cases}$

$\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2} = [-5, -6, -8] = -1[5, 6, 8]$

$5(3) + 6(1) + 8(5) + D = 0 \Rightarrow D = -61$

$5x + 6y + 8z - 61 = 0$

Section 19: 6.3 – Equations of a Plane (Special Conditions)

19) Write a vector and scalar equation of each plane

a) perpendicular to the line $[x, y, z] = [2, 4, -9] + t[3, 5, -3]$ and including the point $P(4, -2, 7)$

$\vec{n} = [3, 5, -3]$; choose $\overrightarrow{m_1} = [0, 3, 5]$, $\overrightarrow{m_2} = [5, -3, 0]$ (both perpendicular to $\vec{n}$)

Vector: $[x, y, z] = [4, -2, 7] + s[0, 3, 5] + t[5, -3, 0]$

$3(4) + 5(-2) - 3(7) + D = 0 \Rightarrow D = 19$

Scalar: $3x + 5y - 3z + 19 = 0$

b) parallel to the yz-plane and including the point $P(-1, -2, 5)$

Vector: $[x, y, z] = [-1, -2, 5] + s[0, 1, 0] + t[0, 0, 1]$

$1(-1) + D = 0 \Rightarrow D = 1$

Scalar: $x + 1 = 0$ $(x = -1)$

c) parallel to the plane $3x - 9y + z - 12 = 0$ and including the point $P(-3, 7, 1)$

Vector: $[x, y, z] = [-3, 7, 1] + s[0, 1, 9] + t[1, 0, -3]$

$3(-3) - 9(7) + 1 + D = 0 \Rightarrow D = 71$

Scalar: $3x - 9y + z + 71 = 0$

d) containing the lines $[x, y, z] = [-2, 3, 12] + s[-2, 1, 5]$ and $[x, y, z] = [1, -4, 4] + t[-6, 3, 15]$

Note: the two lines' direction vectors $[-2, 1, 5]$ *and* $[-6, 3, 15] = 3[-2, 1, 5]$ *are parallel, so this plane is defined by the shared direction plus the vector connecting a point on each line.*

$\overrightarrow{m_1} = \overrightarrow{P_1P_2} = [3, -7, -8]$ (connecting the two given points), $\overrightarrow{m_2} = [-2, 1, 5]$ (shared direction)

Vector: $[x, y, z] = [-2, 3, 12] + s[3, -7, -8] + t[-2, 1, 5]$

$\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2} = [-27, 1, -11]$

$-27(1) + 1(-4) - 11(4) + D = 0 \Rightarrow D = 75$

Scalar: $27x - y + 11z - 75 = 0$

Section 20: 6.7 – Angle Between Planes

20) Determine the angle between the planes $\pi_1 : 3x + 2y + 5z = 5$ and $\pi_2 : 4x - 3y - 2z = 2$

$\overrightarrow{n_1} = [3, 2, 5]$, $\overrightarrow{n_2} = [4, -3, -2]$

$$\cos\theta = \frac{\overrightarrow{n_1} \cdot \overrightarrow{n_2}}{|\overrightarrow{n_1}||\overrightarrow{n_2}|} = \frac{3(4) + 2(-3) + 5(-2)}{(\sqrt{38})(\sqrt{29})}$$

$$\cos\theta = \frac{-4}{(\sqrt{38})(\sqrt{29})}$$

$\theta \cong 96.9°$

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Section 21: 6.5 – Intersections of Lines

21) State the intersections of the following systems. Always try to predict the intersection before you spend time solving.

a) $\ell_1 : [x, y] = [-12, -7] + s[8, -5]$, $\ell_2 : [x, y] = [2, -1] + t[3, -2]$

$14 = -3t + 8s$, $6 = 2t - 5s$

Solving: $s = 46$

$x = -12 + 8(46) = 356$, $y = -7 - 5(46) = -237$

Point of intersection: $(356, -237)$

b) $\ell_1 : [x, y, z] = [1, -1, 0] + s[2, -1, 3]$, $\ell_2 : [x, y, z] = [1, 2, 3] + t[3, 2, 4]$

$0 = 3t - 2s$, $-3 = 2t + s$, $-3 = 4t - 3s$

Solving first two: $t = -\frac{6}{7}$, $s = -\frac{9}{7}$

Check third equation: $LS = -3$, $RS = 4t - 3s = \frac{3}{7}$

$LS \neq RS$: no solutions. The lines are **skew**.

c) $\ell_1 : [x, y, z] = [7, 2, -6] + s[2, 1, -3]$, $\ell_2 : [x, y, z] = [3, 9, 13] + t[1, 5, 5]$

$4 = t - 2s$, $-7 = 5t - s$, $-19 = 5t + 3s$

Solving: $s = -3$, $t = -2$

$x = 3 + t = 1$, $y = 9 + 5t = -1$, $z = 13 + 5t = 3$

Point of intersection: $(1, -1, 3)$

Section 22: 6.6 – Intersection of a Line and a Plane

21 (continued)

d) $[x, y, z] = [3, 1, 2] + t[1, -4, -8]$, $\quad 4x + 2y - z - 8 = 0$

$4(3 + t) + 2(1 - 4t) - (2 - 8t) - 8 = 0$

$4t = -4 \Rightarrow t = -1$

Point of intersection: $(2, 5, 10)$

e) $\ell : \begin{cases} x = 2 + t \\ y = 2 + 2t \\ z = 9 + 8t \end{cases}$, $\quad \pi : 2x - 5y + z - 6 = 0$

$2(2 + t) - 5(2 + 2t) + (9 + 8t) - 6 = 0$

$0t = 3$

No solutions. The line and plane are **parallel and distinct**.

f) $\ell : [x, y, z] = [3, -2, 1] + t[14, -5, -3]$, $\quad \pi : x + y + 3z - 4 = 0$

$(3 + 14t) + (-2 - 5t) + 3(1 - 3t) - 4 = 0$

$0t = 0$

Infinite solutions. The line and plane are **parallel and coincident**.

Section 23: 6.7 – Intersection of Planes

21 (continued)

g) $\pi_1 : x + 3y - 4z = 10$, $\quad \pi_2 : 3x + 9y - 12z = -30$

$3\pi_1 : 3x + 9y - 12z = 30$

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Compare to $\pi_2 : 3x + 9y - 12z = -30$

$0 = 60$

No solutions. The planes are **parallel and distinct**.

h) $\pi_1 : 5x - 4y + z = 3, \quad \pi_2 : x + 3y = 9$

$5\pi_2 : 5x + 15y = 45$

Subtract π_1: $19y - z = 42$

Let $y = t$: $x = 9 - 3t, \ z = -42 + 19t$

Line of intersection: $[x, y, z] = [9, 0, -42] + t[-3, 1, 19]$

i) $\pi_1 : x - 5y + 2z - 10 = 0, \pi_2 : x + 7y - 2z + 6 = 0, \pi_3 : 8x + 5y + z - 20 = 0$

$\pi_1 - \pi_2 : -12y + 4z - 16 = 0$

$8\pi_2 - \pi_3 : 51y - 17z + 68 = 0$

Solving these two: consistent (0=0), infinite solutions

Let $y = t$: $z = 4 + 3t, \ x = 2 - t$

Line of intersection: $[x, y, z] = [2, 0, 4] + t[-1, 1, 3]$

j) $\pi_1 : 2x + y + 6z - 7 = 0, \pi_2 : 3x + 4y + 3z + 8 = 0, \pi_3 : x - 2y - 4z - 9 = 0$

$\pi_1 - 2\pi_3 : 5y + 14z + 11 = 0$

$\pi_2 - 3\pi_3 : 10y + 15z + 35 = 0$

Solving: $z = 1, \ y = -5, \ x = 3$

The point of intersection is $(3, -5, 1)$.

k) $\pi_1 : x - y + 2z = 0, \pi_2 : x - y + 2z + 7 = 0, \pi_3 : z - 5 = 0$

$\pi_1 - \pi_2 : 0 = -7$

No solution. Inconsistent system: planes 1 and 2 are **parallel and distinct**.

l) $\pi_1 : x + 3y - z = -10, \pi_2 : 2x + y + z = 8, \pi_3 : x - 2y + 2z = -4$

$2\pi_1 - \pi_2 : 5y - 3z = -28$

$\pi_1 - \pi_3 : 5y - 3z = -6$

Subtracting: $0 = -22$

No solutions. Inconsistent system: **the planes intersect in pairs** (a triangular prism arrangement).

Section 24: 6.6 – Distance from a Point to a Plane

22) Find the distance between the point $P(3, -2, 0)$ and the plane $4x - y + 8z = 2$.

Point on plane ($x = 0, \ z = 0$): $Q(0, -2, 0)$

$\overrightarrow{QP} = [3, 0, 0], \ \vec{n} = [4, -1, 8]$

$$d = \frac{|\overrightarrow{QP} \cdot \vec{n}|}{|\vec{n}|} = \frac{12}{9}$$

$d = \frac{4}{3} \cong 1.33$ units

Section 25: 6.5 – Distance Between Skew Lines

23) Find the distance between these skew lines.

$\ell_1 : [x, y, z] = [1, 0, -1] + s[2, 3, -4]$

$\ell_2 : [x, y, z] = [8, 1, 3] + t[4, -5, 1]$

$P_1(1, 0, -1), \ P_2(8, 1, 3), \ \overrightarrow{P_1P_2} = [7, 1, 4]$

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$\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2} = [-17, \ -18, \ -22]$

$d = \dfrac{|\overrightarrow{P_1P_2} \cdot \vec{n}|}{|\vec{n}|} = \dfrac{225}{\sqrt{1097}}$

$d \cong 6.79$ units

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