6.6 Intersection of Lines and Planes Lesson

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Part 1: Intersections of Lines and Planes

Given a line and plane in 3-space, there are three possibilities for the intersection of the line with the plane:

Intersect at 1 Point	Infinite Number of Solutions	No Solutions
		
	The line lies on the plane	The line is parallel and distinct from the plane

Example 1: In each case, determine if the line and the plane intersect. If so, determine the solution.

a) $\pi_1 : 9x + 13y - 2z = 29$

$$\ell_1 : \begin{cases} x = 5 + 2t \\ y = -5 - 5t \\ z = 2 + 3t \end{cases}$$

b) $\pi_2 : x + 3y - 4z = 10$

$$\ell_2 : \begin{cases} x = 4 + 6t \\ y = -7 + 2t \\ z = 1 + 3t \end{cases}$$

c) $\pi_3 : 4x - y + 11z = -1$

$\ell_3 : [x, y, z] = [-2, 4, 1] + t[3, 1, -1]$

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Example 2: Without solving, determine if each line intersects the plane.

a) $\pi_1 : 3x - y + z = -6$

$\ell_1 : [x, y, z] = [2, -5, 3] + s[3, 2, 1]$

b) $\pi_2 : 4x - 2z = 11$

$\ell_2 : [x, y, z] = [1, 0, 1] + s[-2, 1, -4]$

Part 2: Distance from a Point to a Plane

Remember the formula for the magnitude of the projection of $\vec{a}$ on to $\vec{b}$ is $|proj_{\vec{b}}\vec{a}| = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{b}|}$

The shortest distance between a point and a plane is the perpendicular distance. This distance, d, between a point P and a plane is $d = |proj_{\vec{n}}\overrightarrow{PQ}| = \frac{|\overrightarrow{PQ} \cdot \vec{n}|}{|\vec{n}|}$ where Q is ANY point on the plane and $\vec{n}$ is a normal vector to the plane.

Example 3: Find the distance between the plane $4x + 2y + z - 16 = 0$ and the point $P(10, 3, -8)$.

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