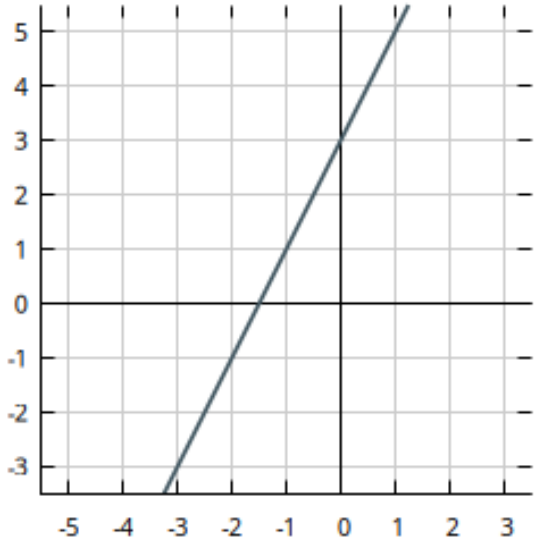
6.5 Intersections of Lines in 2-Space and 3-Space Lesson

This document was composed and formatted by Annie Bernatchez.

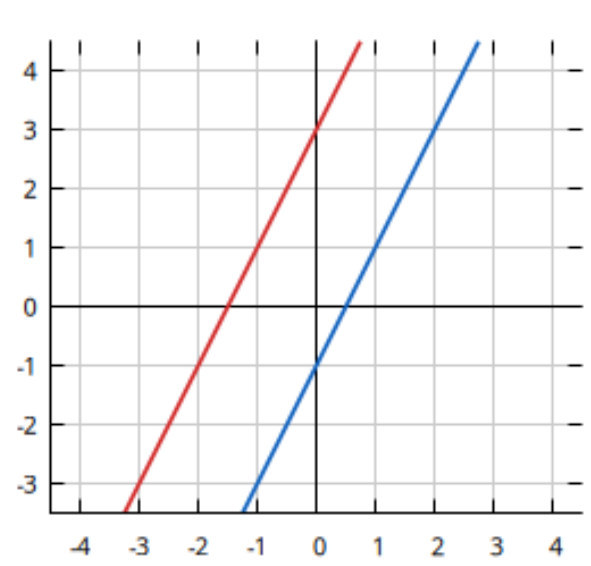
The course material originated from www.jensenmath.ca

Part 1: Intersection of Lines in 2-Space

Possibilities for intersection of lines in 2-Space:

Intersections	Graph	Equations
1 intersection at a point		$y = 2x + 3$ $y = -2x - 1$
Infinite Points of Intersection (parallel and coincident)		$y = 2x + 3$ $y = 2x + 3$

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No points of intersection (parallel and distinct)		$y = 2x + 3$ $y = 2x - 1$

Note: Linear systems in 2-space can be solved using substitution or elimination.

Example 1: Find the solutions of each system if they exist.

a) $\ell_1 : 4x - 6y = -10$

$\ell_2 : 6x - 9y = -15$

b) $\ell_1 : [x, y] = [1, 5] + s[-6, 8]$

$\ell_2 : [x, y] = [2, 1] + t[9, -12]$

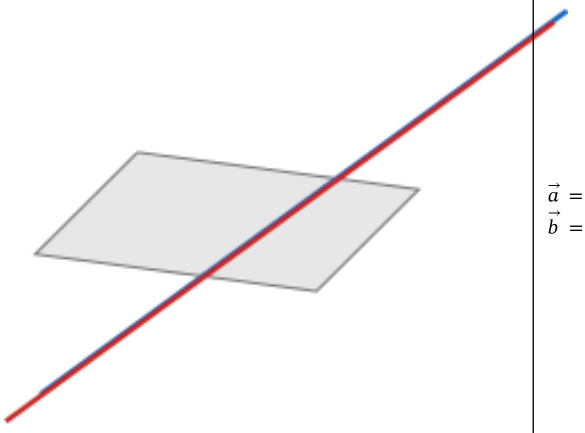
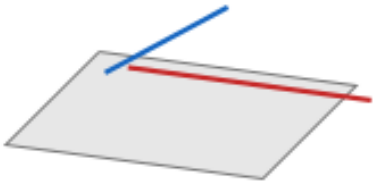
c) $\ell_1 : [x, y] = [3, 4] + s[3, -2]$

$\ell_2 : [x, y] = [-2, -2] + t[3, 5]$

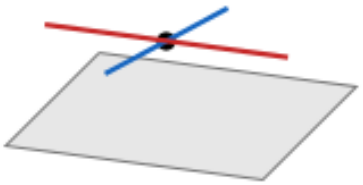
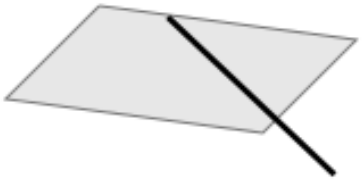
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Part 2: Intersection of Lines in 3-Space

Possibilities for intersection of lines in 3-Space:

Intersections	Graph	Equations
No intersections (parallel and distinct)		$\vec{a} = [-1, 4, 2] + s[-2, 2, 2]$ $\vec{b} = [1, 3, 1] + t[-2, 2, 2]$
No intersections (not parallel but skewed)		$\vec{a} = [-3, -2, 4] + s[1, 1, 1]$ $\vec{b} = [2, 5, 4] + t[-2, 4, -1]$

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1 intersection (not parallel but on same plane)		$\vec{a} = [-3, -2, 4] + s[1, 1, 1]$ $\vec{b} = [-3, -2, 4] + t[-2, 4, -1]$
Infinite points of intersection (parallel and coincident)		$\vec{a} = [-1, 4, -2] + s[-2, 1, -3]$ $\vec{b} = [1, 3, 1] + t[-2, 1, -3]$

Steps for solving the intersection of lines in 3-space:

The first step is always to check if direction vectors are parallel or not by comparing direction vectors and seeing if one is a scalar multiple of the other ($\vec{m_1} = k\vec{m_2}$)

a. If **yes**, there could be infinite solutions OR no solutions. Test to see if a point is on both lines or just solve the system and see if you get no solutions or infinite solutions.

b. If **no**, they could intersect at a point OR they could be skewed and never intersect. Try solving the system and see if you can find parameters that result in the equations being equal. If you can, plug either parameter into an original line and solve for the point of intersection.

Example 2: Determine if the lines intersect. If they do, find the coordinates of the point of intersection.

a) $\ell_1 : [x, y, z] = [5, 11, 2] + s[1, 5, -2]$

$\ell_2 : [x, y, z] = [1, -9, 9] + t[2, 10, -4]$

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b) $\ell_1 : [x,\ y,\ z] = [7,\ 2,\ -6] + s[2,\ 1,\ -3]$

$\ell_2 : [x,\ y,\ z] = [3,\ 9,\ 13] + t[1,\ 5,\ 5]$

c) $\ell_1 : [x,\ y,\ z] = [5,\ -4,\ -2] + s[1,\ 2,\ 3]$

$\ell_2 : [x,\ y,\ z] = [2,\ 0,\ 1] + t[2,\ -1,\ -1]$

Part 3: Distance Between Two Skew Lines

The shortest distance between skew lines is the length of the common perpendicular. It can be calculated using the formula $d = \left|\frac{\overrightarrow{P_1P_2} \cdot \vec{n}}{|\vec{n}|}\right|$, where P_1 and P_2 are any points on each line and $\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2}$ is a normal common to both lines.

Notice this comes from the projection formula learned in the previous unit: $|proj_{\vec{b}}\ \vec{a}| = \left|\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}\right|$

Example 3: Determine the distance between the skew lines.

$\ell_1 : [x,\ y,\ z] = [5,\ -4,\ -2] + s[1,\ 2,\ 3]$

$\ell_2 : [x,\ y,\ z] = [2,\ 0,\ 1] + t[2,\ -1,\ -1]$

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