6.3 Equations of Planes Lesson with solutions

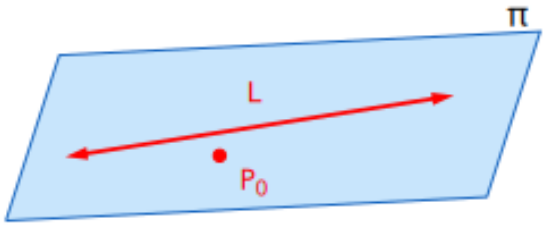
This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

To determine the equation of a plane you need to know two non-collinear vectors parallel to the plane and a point on the plane. We can determine this information if any of the following information is given:

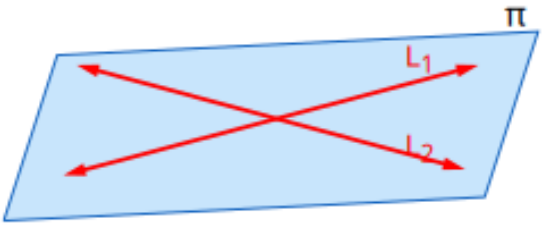
www.geogebra.org/3d/hpnm54gs (bonus interactive visualization)

a.



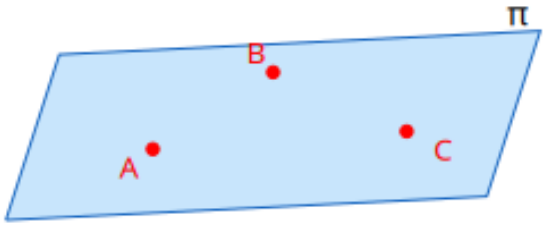
A line and a point not on the line

b.



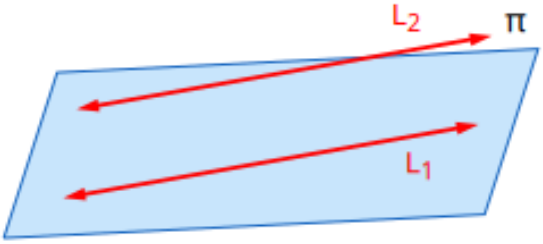
Two intersecting lines

c.



Three noncollinear points

d.



Two parallel and non-coincident lines

The equation of a plane can be written in the following forms:

i) Vector

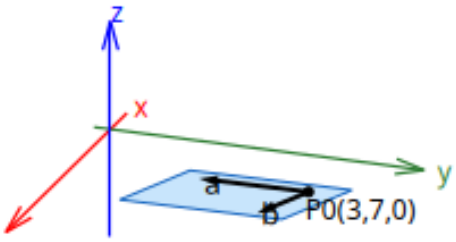
ii) Parametric

iii) Scalar (next lesson)

Vector Equation of a Plane	Parametric Equation of a Plane
$\vec{r} = \vec{r_0} + t\vec{a} + s\vec{b}$ OR $[x, y, z] = [x_0, y_0, z_0] + t[a_1, a_2, a_3] + s[b_1, b_2, b_3]$ $\vec{r} = [x, y, z]$ is a position vector for any point on the plane. $\vec{r_0} = [x_0, y_0, z_0]$ is a position vector for a known point on the plane. $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$ are non-parallel direction vectors parallel to the plane. t and s are scalars	$\begin{cases} x = x_0 + ta_1 + sb_1 \\ y = y_0 + ta_2 + sb_2 \\ z = z_0 + ta_3 + sb_3 \end{cases}$

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Example 1: Consider the plane with direction vectors $\vec{a} = [8, -5, 4]$ and $\vec{b} = [1, -3, -2]$ through $P_0(3, 7, 0)$.
www.geogebra.org/3d/s44t62d7 (bonus interactive visualization)



a) Write the vector and parametric equations of the plane.

$\vec{r_0} = [3, 7, 0]$

Vector Equation: $[x, y, z] = [3, 7, 0] + t[8, -5, 4] + s[1, -3, -2]$

Parametric Equations: $\pi : \begin{cases} x = 3 + 8t + s \\ y = 7 - 5t - 3s \\ z = 4t - 2s \end{cases}$

b) Determine if the point $Q(-10, 8, -6)$ is on the plane.

If the point is on the plane, then there exists a single set of t and s values that satisfy the equations.

$(1)\ -10 = 3 + 8t + s$

$(2)\ 8 = 7 - 5t - 3s$

$(3)\ -6 = 4t - 2s$

Solve for s and t using any two equations:

$3\times(1):\ -30 = 9 + 24t + 3s$

$(2):\ \ 8 = 7 - 5t - 3s$

Add: $-22 = 16 + 19t$

$t = -2$

Sub $t = -2$ into (1): $-10 = 3 + 8(-2) + s$

$s = 3$

Now check to see if $t = -2$ and $s = 3$ also satisfy (3):

LS $= -6$

RS $= 4(-2) - 2(3) = -14$

LS $\neq$ RS. Therefore point $Q(-10, 8, -6)$ is NOT on the plane.

c) Find the coordinates of two other points on the plane.

Choose arbitrary values for parameters t and s:

Let $t = 1$ and $s = -1$: $[x, y, z] = [3, 7, 0] + 1[8, -5, 4] - 1[1, -3, -2]$

$[x, y, z] = [10, 5, 6]$

Let $t = 2$ and $s = 1$: $[x, y, z] = [3, 7, 0] + 2[8, -5, 4] + 1[1, -3, -2]$

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$[x,\ y,\ z] = [20,\ -6,\ 6]$

Two points on the plane are $(10,\ 5,\ 6)$ and $(20,\ -6,\ 6)$.

d) Find the x-intercept of the plane.

Set $y = z = 0$ and solve for s and t:

$(1)\ x = 3 + 8t + s$

$(2)\ 0 = 7 - 5t - 3s$

$(3)\ 0 = 4t - 2s$

$2\times(2):\ 0 = 14 - 10t - 6s$

$3\times(3):\ 0 = 12t - 6s$

Subtract: $0 = 14 - 22t$

$t = \frac{7}{11}$

Sub $t = \frac{7}{11}$ into (3): $0 = 4\left(\frac{7}{11}\right) - 2s$

$s = \frac{14}{11}$

$x = 3 + 8\left(\frac{7}{11}\right) + \frac{14}{11} = \frac{33}{11} + \frac{56}{11} + \frac{14}{11}$

$x = \frac{103}{11}$

The x-intercept is $\left(\frac{103}{11},\ 0,\ 0\right)$.

Example 2: Find the vector and parametric equations of each plane.

a) The plane with x-intercept $= 2$, y-intercept $= 4$, and z-intercept $= 5$.

Determine 2 direction vectors:

$\vec{a} = \overrightarrow{XY} = [0,\ 4,\ 0] - [2,\ 0,\ 0]$

$\vec{a} = [-2,\ 4,\ 0]$

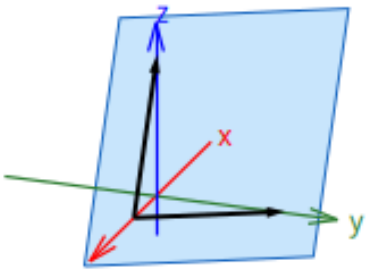
$\vec{b} = \overrightarrow{XZ} = [0,\ 0,\ 5] - [2,\ 0,\ 0]$

$\vec{b} = [-2,\ 0,\ 5]$

Vector Equation: $[x,\ y,\ z] = [2,\ 0,\ 0] + t[-2,\ 4,\ 0] + s[-2,\ 0,\ 5]$

Parametric Equations: $\pi:\ \begin{cases} x = 2 - 2t - 2s \\ y = 4t \\ z = 5s \end{cases}$

3D visualization: www.geogebra.org/3d/czgh5pk5 (bonus interactive visualization)



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b) The plane containing the line $[x, y, z] = [0, 3, -5] + t[6, -2, -1]$ and parallel to the line $[x, y, z] = [1, 7, -4] + s[1, -3, 3]$.

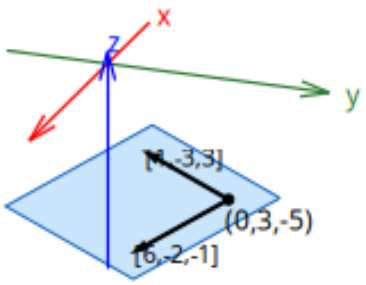
Point $(0, 3, -5)$ is on the plane so a position vector is $[0, 3, -5]$.

Two direction vectors are $[6, -2, -1]$ and $[1, -3, 3]$. Note that these are not scalar multiples of each other and therefore are not parallel.

Vector Equation: $[x, y, z] = [0, 3, -5] + t[6, -2, -1] + s[1, -3, 3]$

Parametric Equations: $\pi : \begin{cases} x = 6t + s \\ y = 3 - 2t - 3s \\ z = -5 - t + 3s \end{cases}$

www.geogebra.org/3d/xskun4uf (bonus interactive visualization)



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