6.3 Equations of Planes Lesson

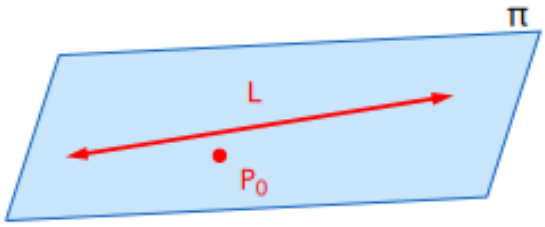
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The course material originated from www.jensenmath.ca

To determine the equation of a plane you need to know two non-collinear vectors parallel to the plane and a point on the plane. We can determine this information if any of the following information is given:

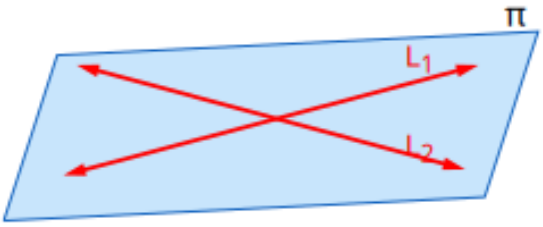
www.geogebra.org/3d/hpnm54gs (bonus interactive visualization)

a.



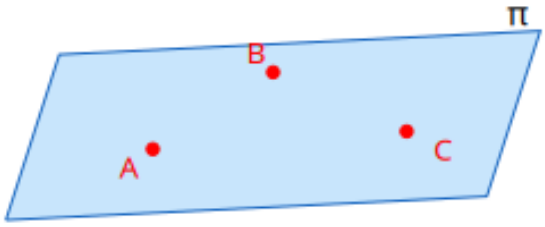
A line and a point not on the line

b.



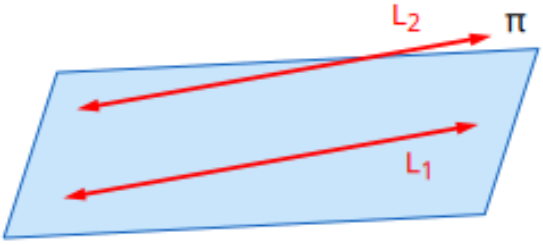
Two intersecting lines

c.



Three noncollinear points

d.



Two parallel and non-coincident lines

The equation of a plane can be written in the following forms:

i) Vector

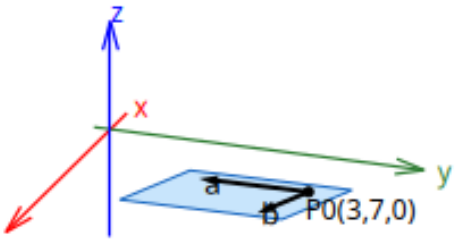
ii) Parametric

iii) Scalar (next lesson)

Vector Equation of a Plane	Parametric Equation of a Plane
$\vec{r} = \vec{r_0} + t\vec{a} + s\vec{b}$ OR $[x, y, z] = [x_0, y_0, z_0] + t[a_1, a_2, a_3] + s[b_1, b_2, b_3]$ $\vec{r} = [x, y, z]$ is a position vector for any point on the plane. $\vec{r_0} = [x_0, y_0, z_0]$ is a position vector for a known point on the plane. $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$ are non-parallel direction vectors parallel to the plane. t and s are scalars	$\begin{cases} x = x_0 + ta_1 + sb_1 \\ y = y_0 + ta_2 + sb_2 \\ z = z_0 + ta_3 + sb_3 \end{cases}$

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Example 1: Consider the plane with direction vectors $\vec{a} = [8, -5, 4]$ and $\vec{b} = [1, -3, -2]$ through $P_0(3, 7, 0)$.
www.geogebra.org/3d/s44t62d7 (bonus interactive visualization)



a) Write the vector and parametric equations of the plane.

b) Determine if the point $Q(-10, 8, -6)$ is on the plane.
If the point is on the plane, then there exists a single set of t and s values that satisfy the equations.

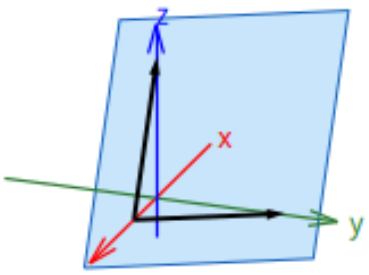
c) Find the coordinates of two other points on the plane.

d) Find the x-intercept of the plane.

Example 2: Find the vector and parametric equations of each plane.
a) The plane with x-intercept = 2, y-intercept = 4, and z-intercept = 5.

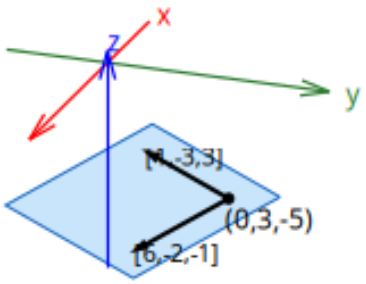
3D visualization: www.geogebra.org/3d/czgh5pk5 (bonus interactive visualization)

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b) The plane containing the line $[x,\ y,\ z] = [0,\ 3,\ -5] + t[6,\ -2,\ -1]$ and parallel to the line $[x,\ y,\ z] = [1,\ 7,\ -4] + s[1,\ -3,\ 3]$.

www.geogebra.org/3d/xskun4uf (bonus interactive visualization)



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