6.1 Equations of Lines in Two-Space Lesson

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

In two-space, a line can be defined many different types of equations:

i) Slope y-intercept form $\rightarrow y = mx + b$

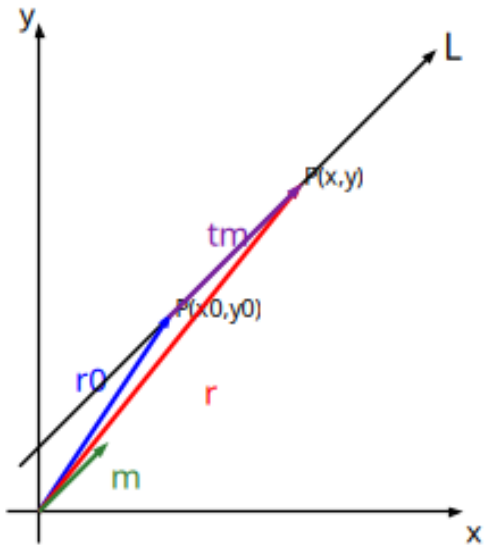
ii) Standard form (scalar equation) $\rightarrow Ax + By + C = 0$

iii) Vector Equation $\rightarrow \vec{r} = \vec{r_0} + t\vec{m}$ or $[x,\ y] = [x_0,\ y_0] + t[m_1,\ m_2]$

iv) Parametric Equation $\rightarrow \begin{cases} x = x_0 + tm_1 \\ y = y_0 + tm_2 \end{cases}$

Part 1: Vector Equation of a Line in Two-Space

In slope y-intercept form and standard form, we have equations that define all points $(x,\ y)$ on the line. A vector equation of a line is an equation that describes resultant vectors that start at the origin and end at a point on the line. In order to create these resultant vectors, we need a position vector that gives us a point on the line, $\vec{r_0} = [x_0,\ y_0]$, and a direction vector parallel to the line, $\vec{m} = [m_1,\ m_2]$. By adding the position and direction vectors together, we get a resultant vector, $\vec{r}$, that has its tip on a point on the line. By multiplying the direction vector by t (to get $t\vec{m}$), the resultant vector $\vec{r}$, can have its tip at any point on the line.



Summary of Vector Equation of a Line:

$\vec{r} = \vec{r_0} + t\vec{m}$ or $[x,\ y] = [x_0,\ y_0] + t[m_1,\ m_2]$

where

- $t \in \mathbb{R}$
- $\vec{r} = [x,\ y]$ is a position vector to any unknown point on the line
- $\vec{r_0} = [x_0,\ y_0]$ is a position vector to any known point on the line
- $\vec{m} = [m_1,\ m_2]$ is a direction vector parallel to the line

Example 1: For a line that goes through the points $A(1,\ 4)$ and $B(3,\ 1)$

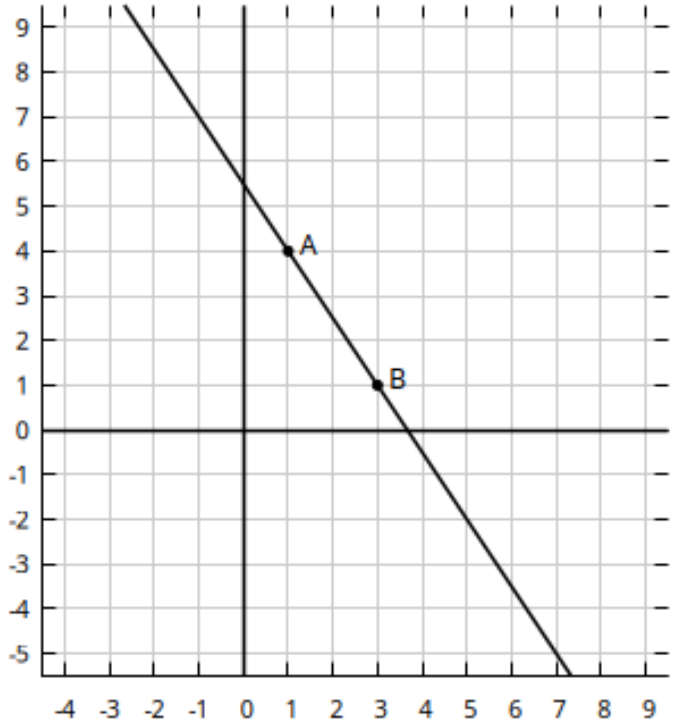
a) Write a vector equation for the line

b) Determine three more position vectors to points on the line. Graph the line.

www.geogebra.org/graphing/j6ntqajv (bonus interactive visualization)

Copyright © 2026 Annie Bernatchez—All rights reserved.

$t = 1$	$t = 2$	$t = -2$



c) Determine if the point $(2, 3)$ is on the line.

Part 2: Parametric Equation of a Line in Two-Space

Remember that the Vector Equation of a line can be written as:

i) $\vec{r} = \vec{r_0} + t\vec{m}$ or $[x, y] = [x_0, y_0] + t[m_1, m_2]$

www.geogebra.org/graphing/f2h5x3wf (bonus interactive visualization)

The vector equation of the line can be separated into two parts, one for each variable.

The parametric equations of a line in two-space are:

$x = x_0 + tm_1$

$y = y_0 + tm_2$

The parametric equation of the line from example 1 would be:

Example 2: Consider the line $\ell_1 : \begin{cases} x = 3 + 2t \\ y = -5 + 4t \end{cases}$

a) Find the coordinates of two points on the line.

For Point 1, let $t = 0$:	For Point 2, let $t = 1$:

Copyright © 2026 Annie Bernatchez—All rights reserved.

b) Write a vector equation of the line.

c) Write the scalar equation of the line.

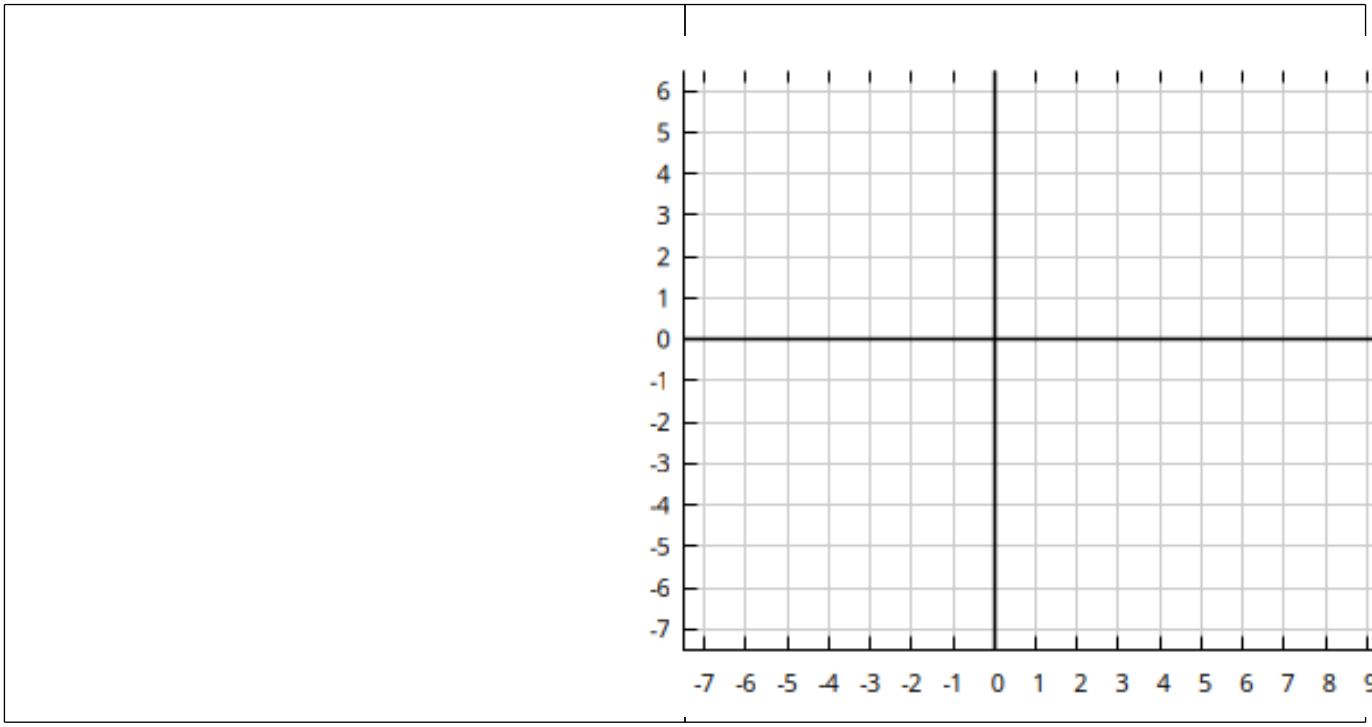
To write the scalar equation, isolate t in both parametric equations, then set them equal to each other and rearrange into standard form.

d) Determine if ℓ_1 is parallel to $\ell_2 : \begin{cases} x = 1 + 3t \\ y = 8 + 12t \end{cases}$

Example 3: Consider the line with scalar equation $4x + 5y + 20 = 0$

www.geogebra.org/graphing/tx4b9esu (bonus interactive visualization)

a) Graph the line



b) Determine a position vector that is perpendicular to the line.

c) How does the position vector from part b) compare to the scalar equation?

d) Write a vector equation of the line

Copyright © 2026 Annie Bernatchez—All rights reserved.

Example 4: Find a scalar equation for the line $[x,\ y] = [0,\ 3] + t[2,\ -1]$

www.geogebra.org/graphing/ykexufu7 (bonus interactive visualization)

	Remember shortcut for finding a perpendicular vector is to swap coordinates and change 1 sign.

Copyright © 2026 Annie Bernatchez—All rights reserved.