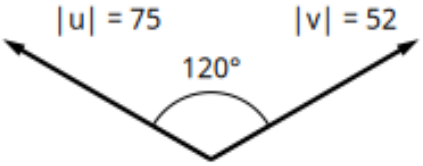
5.5 Cross Product of Vectors Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine $\vec{u}\times\vec{v}$.

a)

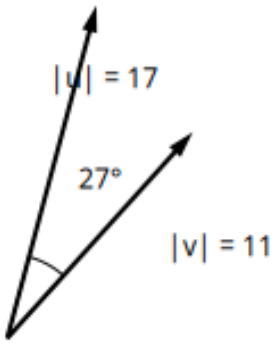


$\vec{u}\times\vec{v} = [(75)(52)\sin(120°)](-\hat{n})$

$\vec{u}\times\vec{v} \cong -3377.5\hat{n}$

$\vec{u}\times\vec{v} \cong 3377.5$ in to the page

b)



$\vec{u}\times\vec{v} = [(17)(11)\sin(27°)](-\hat{n})$

$\vec{u}\times\vec{v} \cong -84.9\hat{n}$

$\vec{u}\times\vec{v} \cong 84.9$ in to the page

c) $\vec{u} = [2, -1, 7]$, $\vec{v} = [2, 1, 3]$

$\vec{u}\times\vec{v} = [-1(3)-7(1),\ 7(2)-2(3),\ 2(1)-(-1)(2)]$

$\vec{u}\times\vec{v} = [-10, 8, 4]$

d) $\vec{u} = [-3, 4, 7]$, $\vec{v} = [4, 3, -5]$

$\vec{u}\times\vec{v} = [4(-5)-7(3),\ 7(4)-(-3)(-5),\ -3(3)-4(4)]$

$\vec{u}\times\vec{v} = [-41, 13, -25]$

e) $\vec{u} = 3\vec{i}+4\vec{j}-\vec{k}$, $\vec{v} = 5\vec{i}+\vec{j}-2\vec{k}$

$\vec{u} = [3, 4, -1]$, $\vec{v} = [5, 1, -2]$

$\vec{u}\times\vec{v} = [4(-2)-(-1)(1),\ -1(5)-3(-2),\ 3(1)-4(5)]$

$\vec{u}\times\vec{v} = [-7, 1, -17]$

f) $\vec{u} = 2\vec{i}-3\vec{j}+7\vec{k}$, $\vec{v} = -\vec{i}+\vec{j}$

$\vec{u} = [2, -3, 7]$, $\vec{v} = [-1, 1, 0]$

$\vec{u}\times\vec{v} = [-3(0)-7(1),\ 7(-1)-2(0),\ 2(1)-(-3)(-1)]$

$\vec{u}\times\vec{v} = [-7, -7, -1]$

2) Find a vector perpendicular to each of the following pairs of vectors. Use the dot product to check your answer.

a) $[5, 0, 1]$ and $[-2, 5, 8]$

$[5, 0, 1]\times[-2, 5, 8] = [0(8)-1(5),\ 1(-2)-5(8),\ 5(5)-0(-2)]$

$= [-5, -42, 25]$

$[-5, -42, 25]\cdot[5, 0, 1] = -25+0+25 = 0$

$[-5, -42, 25]\cdot[-2, 5, 8] = 10-210+200 = 0$

$[-5, -42, 25]$ is orthogonal to both.

b) $[1, 4, -2]$ and $[-4, 9, 0]$

$[1, 4, -2]\times[-4, 9, 0] = [4(0)-(-2)(9),\ -2(-4)-1(0),\ 1(9)-4(-4)]$

$= [18, 8, 25]$

$[18, 8, 25]\cdot[1, 4, -2] = 18+32-50 = 0$

$[18, 8, 25]\cdot[-4, 9, 0] = -72+72+0 = 0$

$[18, 8, 25]$ is orthogonal to both.

3) Find a unit vector perpendicular to $\vec{a} = [6, -2, -3]$ and $\vec{b} = [5, 1, -4]$.

$\vec{a}\times\vec{b} = [-2(-4)-(-3)(1),\ -3(5)-6(-4),\ 6(1)-(-2)(5)]$

$\vec{a}\times\vec{b} = [11, 9, 16]$

$|\vec{a}\times\vec{b}| = \sqrt{11^2+9^2+16^2} = \sqrt{458}$

$$\text{unit vector} = \frac{1}{|\vec{a}\times\vec{b}|}[\vec{a}\times\vec{b}]$$

$$\text{unit vector} = \frac{1}{\sqrt{458}}[11, 9, 16]$$

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4) Given $\vec{a} = [1, -2, -1]$, $\vec{b} = [2, 2, -1]$ and $\vec{c} = [2, -3, -4]$, evaluate each of the following:

a) $\vec{a} \times (\vec{b} \times \vec{c})$

$\vec{b} \times \vec{c} = [2(-4) - (-1)(-3), -1(2) - 2(-4), 2(-3) - 2(2)]$

$\vec{b} \times \vec{c} = [-11, 6, -10]$

$\vec{a} \times (\vec{b} \times \vec{c}) = [-2(-10) - (-1)(6),$

$-1(-11) - 1(-10), 1(6) - (-2)(-11)]$

$\vec{a} \times (\vec{b} \times \vec{c}) = [26, 21, -16]$

b) $(\vec{a} \times \vec{b}) \times \vec{c}$

$\vec{a} \times \vec{b} = [-2(-1) - (-1)(2), -1(2) - 1(-1), 1(2) - (-2)(2)]$

$\vec{a} \times \vec{b} = [4, -1, 6]$

$(\vec{a} \times \vec{b}) \times \vec{c} = [-1(-4) - 6(-3), 6(2) - 4(-4), 4(-3) - (-1)(2)]$

$(\vec{a} \times \vec{b}) \times \vec{c} = [22, 28, -10]$

c) $\vec{a} \times \vec{c} - \vec{a} \times \vec{b}$

$\vec{a} \times \vec{c} = [-2(-4) - (-1)(-3), -1(2) - 1(-4), 1(-3) - (-2)(2)]$

$\vec{a} \times \vec{c} = [5, 2, 1]$

$(\vec{a} \times \vec{c}) - (\vec{a} \times \vec{b}) = [5, 2, 1] - [4, -1, 6]$

$\vec{a} \times \vec{c} - \vec{a} \times \vec{b} = [1, 3, -5]$

d) $\vec{b} \times 3\vec{c}$

$3\vec{c} = [6, -9, -12]$

$\vec{b} \times 3\vec{c} = [2(-12) - (-1)(-9), -1(6) - 2(-12), 2(-9) - 2(6)]$

$\vec{b} \times 3\vec{c} = [-33, 18, -30]$

e) $(\vec{a} \times \vec{c}) \cdot \vec{b}$

$(\vec{a} \times \vec{c}) \cdot \vec{b} = [5, 2, 1] \cdot [2, 2, -1]$

$= 5(2) + 2(2) + 1(-1) = 13$

$(\vec{a} \times \vec{c}) \cdot \vec{b} = 13$

f) $(\vec{a} \times \vec{b}) \cdot \vec{c}$

$(\vec{a} \times \vec{b}) \cdot \vec{c} = [4, -1, 6] \cdot [2, -3, -4]$

$= 4(2) + (-1)(-3) + 6(-4) = -13$

$(\vec{a} \times \vec{b}) \cdot \vec{c} = -13$

g) $|\vec{a} \times \vec{b}|$

$|\vec{a} \times \vec{b}| = \sqrt{4^2 + (-1)^2 + 6^2}$

$|\vec{a} \times \vec{b}| = \sqrt{53}$

h) $|\vec{a} \times (\vec{b} - \vec{c})|$

$\vec{b} - \vec{c} = [2, 2, -1] - [2, -3, -4] = [0, 5, 3]$

$\vec{a} \times (\vec{b} - \vec{c}) = [-2(3) - (-1)(5), -1(0) - 1(3), 1(5) - (-2)(0)]$

$\vec{a} \times (\vec{b} - \vec{c}) = [-1, -3, 5]$

$|\vec{a} \times (\vec{b} - \vec{c})| = \sqrt{35}$

5) Use the cross product to determine the angles between the vectors $\vec{a} = [2, 1, -3]$ and $\vec{b} = [5, -4, 3]$. Consider ambiguous case. Use dot product to confirm or use graphing software to inspect.

$\vec{a} \times \vec{b} = [1(3) - (-3)(-4), -3(5) - 2(3), 2(-4) - 1(5)]$

$\vec{a} \times \vec{b} = [-9, -21, -13]$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{691}}{(\sqrt{14})(\sqrt{50})}$$

$\theta_1 \cong 83.5°$

$\theta_2 = 180° - \theta_1 \cong 96.5°$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{2(5) + 1(-4) + (-3)(3)}{(\sqrt{14})(\sqrt{50})} = \frac{-3}{\sqrt{700}}$$

$\theta \cong 96.5°$, verified using the dot product.

$\theta \cong 96.5°$

6) Determine the area of $\triangle PQR$ with vertices of $P(3, -2, 7)$, $Q(2, 2, -3)$, and $R(1, 1, 2)$.

$\overrightarrow{PQ} = [2 - 3, 2 - (-2), -3 - 7] = [-1, 4, -10]$

$\overrightarrow{PR} = [1 - 3, 1 - (-2), 2 - 7] = [-2, 3, -5]$

$\overrightarrow{PQ} \times \overrightarrow{PR} = [4(-5) - (-10)(3), -10(-2) - (-1)(-5), -1(3) - 4(-2)]$

$\overrightarrow{PQ} \times \overrightarrow{PR} = [10, 15, 5]$

Area of $\triangle PQR = \frac{1}{2}|\overrightarrow{PQ} \times \overrightarrow{PR}| = \frac{1}{2}\sqrt{350} = \frac{1}{2}(5)\sqrt{14}$

Area of $\triangle PQR = 2.5\sqrt{14}$ units^2

7) Determine the area of the parallelogram ABCD defined by the vertices $A(2, -1, -1)$, $B(-4, -2, 3)$, $C(2, 3, 2)$, and $D(8, 4, -2)$.

$\overrightarrow{AB} = [-4 - 2, -2 - (-1), 3 - (-1)] = [-6, -1, 4]$

$\overrightarrow{AD} = [8 - 2, 4 - (-1), -2 - (-1)] = [6, 5, -1]$

$\overrightarrow{AB} \times \overrightarrow{AD} = [-1(-1) - 4(5), 4(6) - (-6)(-1), -6(5) - (-1)(6)]$

$\overrightarrow{AB} \times \overrightarrow{AD} = [-19, 18, -24]$

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Area $= |\overrightarrow{AB} \times \overrightarrow{AD}| = \sqrt{1261}$

Area $= \sqrt{1261}$ units^2

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