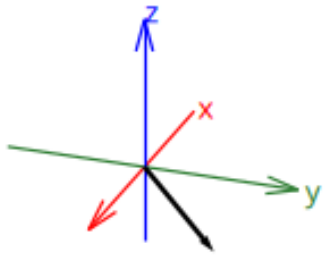
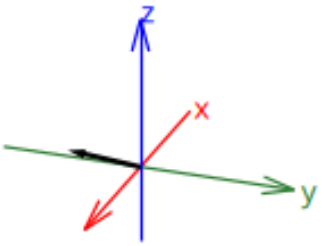
5.4 Vectors in 3-Space Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Draw the position vectors.

a) $[-2, 3, -4]$	b) $[2, -3, 1]$
	

2) Express each vector as the sum of $\vec{i}$, $\vec{j}$ and $\vec{k}$.

a) $[2, -1, 7]$	b) $[-4, -6, 5]$
$= 2\vec{i} - \vec{j} + 7\vec{k}$	$= -4\vec{i} - 6\vec{j} + 5\vec{k}$

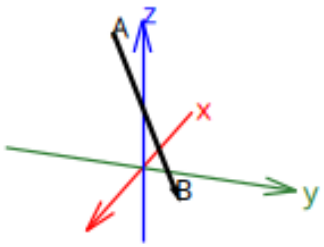
3) Express each vector in the form $[a, b, c]$.

a) $3\vec{i} - 4\vec{j} + 5\vec{k}$	b) $2\vec{i} + 3\vec{k}$
$= [3, -4, 5]$	$= [2, 0, 3]$
c) $-8\vec{i} + 9\vec{j} - 4\vec{k}$	d) $-8\vec{j} - 7\vec{k}$
$= [-8, 9, -4]$	$= [0, -8, -7]$

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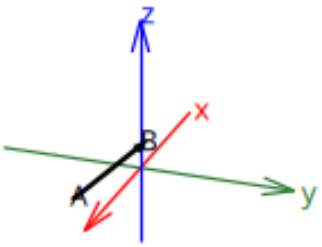
4) Draw vector $\overrightarrow{AB}$ joining each pair of points. Then write the vector in the form $[a, b, c]$.

a) $A(2, -1, 7)$ and $B(0, 2, -1)$



$\overrightarrow{AB} = [-2, 3, -8]$

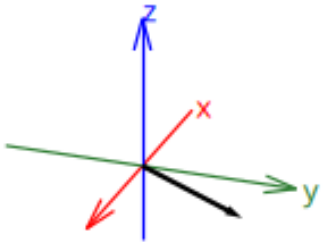
b) $A(0, -4, -2)$ and $B(-3, -1, 0)$



$\overrightarrow{AB} = [-3, 3, 2]$

5) Draw each position vector. Then find its magnitude.

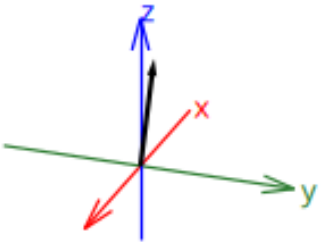
a) $[-1, 5, -2]$



Magnitude $= \sqrt{(-1)^2 + (5)^2 + (-2)^2}$

$= \sqrt{30}$

b) $[-2, 0, 4]$



Magnitude $= \sqrt{(-2)^2 + (0)^2 + (4)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$

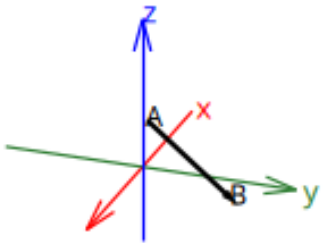
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6) Find a and b such that $\vec{u} = [a, 3, 6]$ and $\vec{v} = [-8, 12, b]$ are collinear.

x :	y :	z :
$a = -8k$ $a = -8\left(\frac{1}{4}\right)$ $a = -2$	$3 = 12k$ $k = \frac{1}{4}$	$6 = bk$ $6 = b\left(\frac{1}{4}\right)$ $b = 24$

7) Draw the vector $\overrightarrow{AB}$ joining each pair of points. Write the vector in the form $[x, y, z]$. Then determine the exact magnitude of the vector.

a) $A(2, 1, 3)$ and $B(5, 7, 1)$

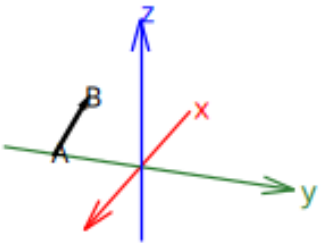


$\overrightarrow{AB} = [3, 6, -2]$

$|\overrightarrow{AB}| = \sqrt{(3)^2 + (6)^2 + (-2)^2}$

$|\overrightarrow{AB}| = 7$

b) $A(3, -4, 1)$ and $B(6, -1, 5)$



$\overrightarrow{AB} = [3, 3, 4]$

$|\overrightarrow{AB}| = \sqrt{(3)^2 + (3)^2 + (4)^2}$

$|\overrightarrow{AB}| = \sqrt{34}$

8) Evaluate each given the vectors $\vec{a} = [-2, 1, 8]$, $\vec{b} = [3, 1, -2]$, and $\vec{c} = [2, -3, 4]$.

a) $3\vec{b}$

$= 3[3, 1, -2]$

$= [9, 3, -6]$

b) $\vec{b} - \vec{c}$

$= [3, 1, -2] - [2, -3, 4]$

$= [1, 4, -6]$

c) $2\vec{a} - 3\vec{c} + 4\vec{b}$

$= 2[-2, 1, 8] - 3[2, -3, 4] + 4[3, 1, -2]$

$= [-4, 2, 16] - [6, -9, 12] + [12, 4, -8]$

$= [-10, 11, 4] + [12, 4, -8]$

$= [2, 15, -4]$

d) $(\vec{a} + \vec{b}) - (\vec{a} + \vec{c})$

$= ([-2, 1, 8] + [3, 1, -2])$

$-([-2, 1, 8] + [2, -3, 4])$

$= [1, 2, 6] - [0, -2, 12]$

$= [1, 4, -6]$

e) $\vec{b} \cdot \vec{c}$

$= 3(2) + 1(-3) + (-2)(4)$

$= 6 - 3 - 8$

$= -5$

f) $\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}$

$= -2(3) + 1(1) + 8(-2) - (-5)$

$= -6 + 1 - 16 + 5$

$= -16$

9) Let $\vec{a} = 3\vec{i} - 2\vec{j} + 4\vec{k}$, $\vec{b} = 7\vec{i} + 4\vec{j} - \vec{k}$ and $\vec{c} = -2\vec{i} + 5\vec{j} + 9\vec{k}$.

a) $(\vec{a} + \vec{b}) \cdot \vec{c}$

$= ([3, -2, 4] + [7, 4, -1]) \cdot [-2, 5, 9]$

$= [10, 2, 3] \cdot [-2, 5, 9]$

$= 10(-2) + 2(5) + 3(9)$

$= 17$

b) $2\vec{a} \cdot (4\vec{b} - 3\vec{c})$

$= 2[3, -2, 4] \cdot (4[7, 4, -1] - 3[-2, 5, 9])$

$= [6, -4, 8] \cdot [34, 1, -31]$

$= 6(34) + (-4)(1) + 8(-31)$

$= -48$

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10) Determine the values of k such that $\vec{u}$ and $\vec{v}$ are orthogonal.

a) $\vec{u} = [2,\ k,\ -1]$ and $\vec{v} = [3,\ -2,\ 7]$ $\vec{u} \cdot \vec{v} = 0$ $2(3) + k(-2) + (-1)(7) = 0$ $6 - 2k - 7 = 0$ $-2k = 1$ $k = -\frac{1}{2}$	b) $\vec{u} = [-3,\ 1,\ k]$ and $\vec{v} = [4,\ -k,\ k]$ $\vec{u} \cdot \vec{v} = 0$ $-3(4) + 1(-k) + k(k) = 0$ $-12 - k + k^2 = 0$ $k^2 - k - 12 = 0$ $(k-4)(k+3) = 0$ $k_1 = 4,\ k_2 = -3$

11) Find a vector orthogonal to each vector.

a) $[2,\ -1,\ 7]$ $[0,\ 7,\ 1]$	b) $[8,\ -3,\ 4]$ $[0,\ 4,\ 3]$

12) Consider the vectors $\vec{u} = [3,\ -5,\ 8]$ and $\vec{v} = [3,\ 1,\ -2]$.

a) Find $\vec{u} \cdot \vec{v}$.

$= 3(3) + (-5)(1) + 8(-2)$

$= -12$

b) Calculate the angle between $\vec{u}$ and $\vec{v}$.

$\cos\theta = \frac{-12}{(\sqrt{98})(\sqrt{14})}$

$\theta \cong 108.9°$

13) Determine the projection of $\vec{a}$ on $\vec{b}$.

a) $\vec{a} = [2,\ 1,\ -3]$ and $\vec{b} = [1,\ 7,\ 6]$ $proj_{\vec{b}}\,\vec{a} = \frac{2(1) + 1(7) + (-3)(6)}{(1)^2 + (7)^2 + (6)^2}[1,\ 7,\ 6]$ $= \frac{-9}{86}[1,\ 7,\ 6]$ $= \left[\frac{-9}{86}, \frac{-63}{86}, \frac{-27}{43}\right]$	b) $\vec{a} = [3,\ 4,\ 7]$ and $\vec{b} = [2,\ -1,\ 1]$ $proj_{\vec{b}}\,\vec{a} = \frac{3(2) + 4(-1) + 7(1)}{(2)^2 + (-1)^2 + (1)^2}[2,\ -1,\ 1]$ $= \frac{9}{6}[2,\ -1,\ 1]$ $= \left[3, \frac{-3}{2}, \frac{3}{2}\right]$

14) The initial point of vector $\overrightarrow{CD} = [2,\ -9,\ 1]$ is $C(-3,\ 2,\ 2)$ determine the coordinates of D.

$x - (-3) = 2 \Rightarrow x = -1$

$y - 2 = -9 \Rightarrow y = -7$

$z - 2 = 1 \Rightarrow z = 3$

$D(-1,\ -7,\ 3)$

15) Find 2 unit vectors that are parallel to $\vec{a} = [9,\ -7,\ 2]$.

$|\vec{a}| = \sqrt{(9)^2 + (-7)^2 + (2)^2}$

$|\vec{a}| = \sqrt{134}$

The unit vectors are:

$\frac{1}{\sqrt{134}}[9,\ -7,\ 2] = \left[\frac{9}{\sqrt{134}}, \frac{-7}{\sqrt{134}}, \frac{2}{\sqrt{134}}\right]$

$\frac{-1}{\sqrt{134}}[9,\ -7,\ 2] = \left[\frac{-9}{\sqrt{134}}, \frac{7}{\sqrt{134}}, \frac{-2}{\sqrt{134}}\right]$

16) A triangle has vertices at the points $D = (3,\ -2,\ -3)$, $E(7,\ 0,\ 1)$ and $F(1,\ 2,\ 1)$. What type of triangle is $\triangle DEF$? Explain.

$\overrightarrow{DE} = [4,\ 2,\ 4]$ $|\overrightarrow{DE}| = 6$

$\overrightarrow{DF} = [-2,\ 4,\ 4]$ $|\overrightarrow{DF}| = 6$

$\overrightarrow{EF} = [-6,\ 2,\ 0]$ $|\overrightarrow{EF}| = \sqrt{40} = 2\sqrt{10}$

Isosceles

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$\overrightarrow{DE} \cdot \overrightarrow{DF} = 4(-2) + 2(4) + 4(4) = 16$

$\overrightarrow{DE} \cdot \overrightarrow{EF} = 4(-6) + 2(2) + 4(0) = -20$

$\overrightarrow{DF} \cdot \overrightarrow{EF} = -2(-6) + 4(2) + 4(0) = 20$

No right angles. This is a non-right isosceles triangle because 2 sides of the triangle are the same length but no 2 vectors that make up the sides of the triangle dot to 0, this tells us there are no perpendicular vectors and therefore no right angles.

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