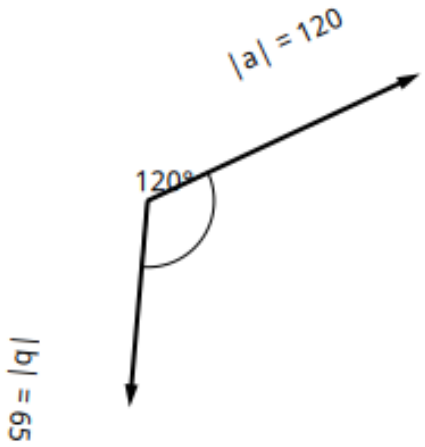
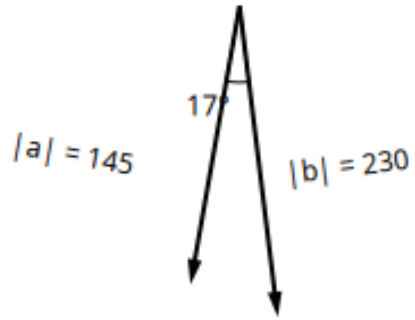
5.2 Dot Product Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Calculate the dot product for each pair.

a)	b)
	
$\vec{a} \cdot \vec{b} = 120(65)\cos(120)$ $\vec{a} \cdot \vec{b} = -3900$	$\vec{a} \cdot \vec{b} = 145(230)\cos(17)$ $\vec{a} \cdot \vec{b} \cong 31\,892.76$

2) Calculate the dot product for each pair of vectors. θ is the angle between the vectors when they are placed tail to tail.

a) $\|\vec{u}\| = 7$, $\|\vec{v}\| = 12$, and $\theta = 47°$ $\vec{u} \cdot \vec{v} = 7(12)\cos(47)$ $\vec{u} \cdot \vec{v} \cong 57.29$	b) $\|\vec{s}\| = 520$, $\|\vec{t}\| = 745$, and $\theta = 135°$ $\vec{s} \cdot \vec{t} = 520(745)\cos(135)$ $\vec{s} \cdot \vec{t} \cong -273\,933.17$

3) Calculate the dot product of each pair of vectors.

a) $\vec{a} = [5, 8]$, $\vec{b} = [-2, 1]$ $\vec{a} \cdot \vec{b} = 5(-2) + 8(1)$ $\vec{a} \cdot \vec{b} = -2$	b) $\vec{c} = [-1, 8]$, $\vec{d} = [3, -3]$ $\vec{c} \cdot \vec{d} = -1(3) + 8(-3)$ $\vec{c} \cdot \vec{d} = -27$
c) $\vec{l} = 2\vec{i} - 3\vec{j}$, $\vec{m} = -9\vec{i} + 4\vec{j}$ $\vec{l} = [2, -3]$, $\vec{m} = [-9, 4]$ $\vec{l} \cdot \vec{m} = 2(-9) + (-3)(4)$ $\vec{l} \cdot \vec{m} = -30$	d) $\vec{u} = -6\vec{i} + 7\vec{j}$, $\vec{v} = 3\vec{i} - 2\vec{j}$ $\vec{u} = [-6, 7]$, $\vec{v} = [3, -2]$ $\vec{u} \cdot \vec{v} = -6(3) + 7(-2)$ $\vec{u} \cdot \vec{v} = -32$

4) Decide whether the following expressions have meaning or not. If not, explain why.

a) $\vec{u} \cdot (\vec{v} \cdot \vec{w})$ NO—can't dot a vector with a scalar	b) $\|\vec{u} \cdot \vec{v}\|$ YES	c) $\vec{u}(\vec{v} \cdot \vec{w})$ YES	d) $\|\vec{u}\|^2$ YES	e) $\vec{v}^2$ NO—can't multiply vectors	f) $(\vec{u} \cdot \vec{v})^2$ YES

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5) Let $\vec{a} = [1, -2]$, $\vec{b} = [2, 5]$, and $\vec{c} = [4, -1]$. Evaluate the following if possible. If not possible, explain why not.

a) $\vec{a} \cdot (\vec{b} + \vec{c})$ $= [1, -2] \cdot ([2, 5] + [4, -1])$ $= [1, -2] \cdot [6, 4]$ $= 1(6) + (-2)(4)$ $= -2$	b) $(\vec{a} + \vec{b}) \cdot \vec{c}$ $= ([1, -2] + [2, 5]) \cdot [4, -1]$ $= [3, 3] \cdot [4, -1]$ $= 3(4) + 3(-1)$ $= 9$
c) $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{c})$ $= ([1, -2] + [2, 5]) \cdot ([1, -2] + [4, -1])$ $= [3, 3] \cdot [5, -3]$ $= 3(5) + 3(-3)$ $= 6$	d) $(3\vec{a} + 2\vec{b}) \cdot (4\vec{a} - \vec{b})$ $= (3[1, -2] + 2[2, 5]) \cdot (4[1, -2] - [2, 5])$ $= ([3, -6] + [4, 10]) \cdot ([4, -8] - [2, 5])$ $= [7, 4] \cdot [2, -13]$ $= 7(2) + 4(-13)$ $= -38$
e) $\vec{a} \cdot \vec{b} \cdot \vec{c}$ Not possible. $\vec{a} \cdot \vec{b}$ is a scalar, and we can't dot $\vec{c}$ with a scalar.	f) $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ $= [1, -2] \cdot [2, 5] + [1, -2] \cdot [4, -1]$ $= 1(2) + (-2)(5) + 1(4) + (-2)(-1)$ $= -2$
g) $4\vec{b} \cdot (-2\vec{c})$ $= 4[2, 5] \cdot (-2[4, -1])$ $= [8, 20] \cdot [-8, 2]$ $= 8(-8) + 20(2)$ $= -24$	h) $(\vec{a} + \vec{b}) \cdot \vec{c}$ $= ([1, -2] + [2, 5]) \cdot [4, -1]$ $= [3, 3] \cdot [4, -1]$ $= 3(4) + 3(-1)$ $= 9$

6) Determine a value of t so that $\vec{u} = [9, t]$ and $\vec{v} = [-16, t]$ are perpendicular.

If perpendicular, $\vec{u} \cdot \vec{v} = 0$

$[9, t] \cdot [-16, t] = 0$

$9(-16) + t(t) = 0$

$t^2 = 144$

$t = \pm 12$

7) Find a vector that is perpendicular to $\vec{a} = [3, -1]$. Verify that the vectors are perpendicular.

$\vec{a} = [3, -1]$, $\vec{b} = [5, k]$

$\vec{a} \cdot \vec{b} = 0$

$3(5) + (-1)(k) = 0$

$k = 15$

$\vec{b} = [5, 15]$

Shortcut: to find a vector perpendicular to $[3, -1]$, swap the components and change one sign: $[3, -1] \to [-1, 3] \to [1, 3]$.

Check: $[1, 3] \cdot [3, -1] = 3 + (-3) = 0$

8) Which of the following is a right-angled triangle? Identify the right angle in that triangle.

- $\triangle ABC$ for $A(3, 1)$, $B(-2, 3)$, and $C(5, 6)$
- $\triangle STU$ for $S(4, 6)$, $T(-3, 7)$, and $U(-5, -4)$

$\overrightarrow{AB} = [-5, 2]$ $\overrightarrow{BC} = [7, 3]$ $\overrightarrow{AC} = [2, 5]$ $\overrightarrow{AB} \cdot \overrightarrow{BC} = -5(7) + 2(3) = -35 + 6 = -29$ $\overrightarrow{AB} \cdot \overrightarrow{AC} = -5(2) + 2(5) = 0$ So $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are perpendicular. $\angle A = 90°$	$\overrightarrow{ST} = [-7, 1]$ $\overrightarrow{SU} = [-9, -10]$ $\overrightarrow{TU} = [-2, -11]$ $\overrightarrow{ST} \cdot \overrightarrow{SU} = -7(-9) + 1(-10) = 53$ $\overrightarrow{ST} \cdot \overrightarrow{TU} = -7(-2) + 1(-11) = 3$ $\overrightarrow{SU} \cdot \overrightarrow{TU} = -9(-2) + (-10)(-11) = 128$ So $\triangle STU$ is NOT a right triangle.

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