5.7 Unit 5 Pretest Review – Cartesian Vectors with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

This review covers all of Unit 5: Cartesian Vectors, and is meant to be used to prepare for the unit test.

Section 1: 5.1 – Vector Notation

1) Express each vector in terms of vectors $\hat{i}$ and $\hat{j}$

a) $[-3, -6]$	b) $[0, -8]$	c) $[-6, 0]$
$-3\hat{i} - 6\hat{j}$	$-8\hat{j}$	$-6\hat{i}$

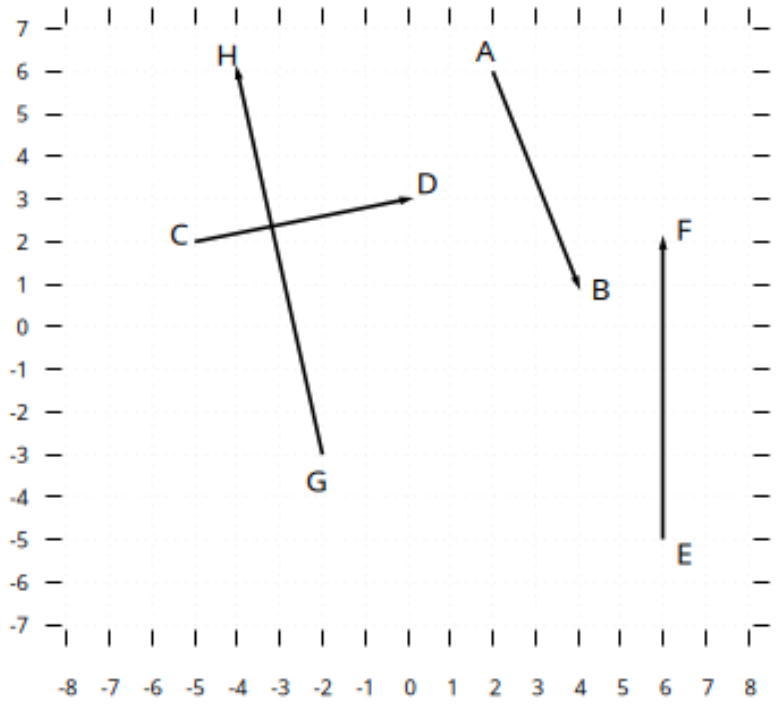
Section 2: 5.1 – Vector Notation

2) Express each vector in the form $[a, b]$

a) $-4\hat{i}$	b) $7\hat{i} - 4\hat{j}$	c) $2\hat{j}$
$[-4, 0]$	$[7, -4]$	$[0, 2]$

Section 3: 5.1 – Magnitude of Vectors from a Grid

3) Write the coordinates of each Cartesian Vector in the form $[a, b]$ and determine its magnitude.



$\overrightarrow{AB} = [4 - 2, 1 - 6] = [2, -5]$

Copyright © 2026 Annie Bernatchez—All rights reserved.

$|\overrightarrow{AB}| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$ units

$\overrightarrow{CD} = [0 - (-5),\ 3 - 2] = [5,\ 1]$

$|\overrightarrow{CD}| = \sqrt{5^2 + 1^2} = \sqrt{26}$ units

$\overrightarrow{EF} = [6 - 6,\ 2 - (-5)] = [0,\ 7]$

$|\overrightarrow{EF}| = \sqrt{0^2 + 7^2} = 7$ units

$\overrightarrow{GH} = [-4 - (-2),\ 6 - (-3)] = [-2,\ 9]$

$|\overrightarrow{GH}| = \sqrt{(-2)^2 + 9^2} = \sqrt{85}$ units

Section 4: 5.1 – Equivalent Vectors

4) You are given the vector $\vec{v} = [5,\ -1]$. An equivalent vector $\overrightarrow{PQ}$ has its initial point at $P(-2,\ -7)$. Determine the coordinates of Q.

$x - (-2) = 5 \Rightarrow x = 3$

$y - (-7) = -1 \Rightarrow y = -8$

$Q(3,\ -8)$

Section 5: 5.1 – Vectors Between Points

5) Given the points $P(-6,\ 1)$, $Q(-2,\ -1)$, and $R(-3,\ 4)$, find

a) $\overrightarrow{QP}$

$\overrightarrow{QP} = [-6 - (-2),\ 1 - (-1)]$

$\overrightarrow{QP} = [-4,\ 2]$

b) $|\overrightarrow{RP}|$

$\overrightarrow{RP} = [-6 - (-3),\ 1 - 4] = [-3,\ -3]$

$|\overrightarrow{RP}| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$ units

c) the perimeter of $\triangle PQR$

$|\overrightarrow{QP}| = \sqrt{(-4)^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$

$\overrightarrow{RQ} = [-2 - (-3),\ -1 - 4] = [1,\ -5]$

$|\overrightarrow{RQ}| = \sqrt{1^2 + (-5)^2} = \sqrt{26}$

Perimeter $= 2\sqrt{5} + \sqrt{26} + 3\sqrt{2}$

Perimeter $\cong 13.8$ units

Section 6: 5.1 – Scalar Multiplication and Vector Addition

6) If $\vec{u} = [4,\ -1]$ and $\vec{v} = [2,\ 7]$, find

a) $8\vec{u}$ $8\vec{u} = [32,\ -8]$	b) $-8\vec{u}$ $-8\vec{u} = [-32,\ 8]$
c) $\vec{u} + \vec{v}$ $\vec{u} + \vec{v} = [6,\ 6]$	d) $\vec{v} - \vec{u}$ $\vec{v} - \vec{u} = [-2,\ 8]$
e) $5\vec{u} - 3\vec{v}$ $5\vec{u} - 3\vec{v} = [20,\ -5] - [6,\ 21]$ $5\vec{u} - 3\vec{v} = [14,\ -26]$	f) $-4\vec{u} + 7\vec{v}$ $-4\vec{u} + 7\vec{v} = [-16,\ 4] + [14,\ 49]$ $-4\vec{u} + 7\vec{v} = [-2,\ 53]$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 7: 5.1 – Collinear Vectors

7) Which vector is collinear with $\vec{a} = [6, -4]$? Give proof.

$\vec{b} = [-9, 6]$, $\vec{c} = [-6, -4]$

Does $\vec{a} = k\vec{b}$? $6 = k(-9) \Rightarrow k = -\frac{2}{3}$; $-4 = k(6) \Rightarrow k = -\frac{2}{3}$

Does $\vec{a} = k\vec{c}$? $6 = k(-6) \Rightarrow k = -1$; $-4 = k(-4) \Rightarrow k = 1$

Since both components give $k = -\frac{2}{3}$ for $\vec{b}$ but inconsistent values of k for $\vec{c}$,

$\vec{b}$ is collinear with $\vec{a}$; $\vec{b} = -\frac{3}{2}\vec{a}$

Section 8: 5.1 – Resolving a Force into Components

8) A person pulls a sleigh, exerting a force of 180 N along a rope that makes an angle of 30° to the horizontal. Write this force in component form as a Cartesian vector.

$\vec{F} = [180\cos 30°, 180\sin 30°]$

$\vec{F} = [90\sqrt{3}, 90]$

Section 9: 5.1 – Resolving a Force into Components

9) A person pushes a lawnmower with a force of 250 N. The handle makes an angle of 35° with the ground. Write this force in component form as a Cartesian vector.

$\vec{F} = [250\cos(-35°), 250\sin(-35°)]$

$\vec{F} \cong [204.8, -143.4]$

Section 10: 5.1 – Resultant Velocity Using Bearings

10) A ship's course is set at a heading of 192°, with a speed of 30 knots. A current is flowing from a bearing of 112°, at 14 knots. Use Cartesian vectors to determine the resultant velocity of the ship.

$\vec{s} = [30\cos(258°), 30\sin(258°)]$

$\vec{w} = [14\cos(158°), 14\sin(158°)]$

$\vec{r} = \vec{s} + \vec{w} \cong [-19.22, -24.1]$

$|\vec{r}| \cong 30.8$ km/h

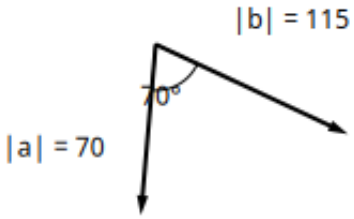
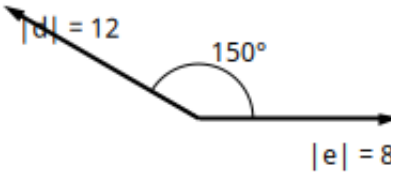
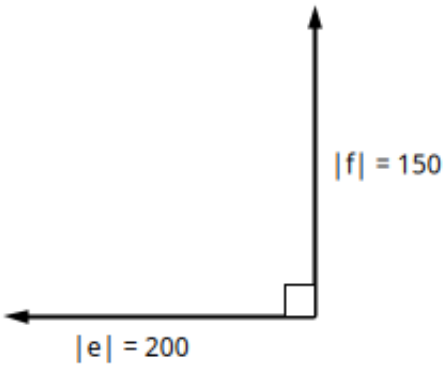
$\tan\theta = \frac{-24.1}{-19.22} \Rightarrow \theta \cong 51.4°$

30.8 km/h on a true bearing of 218.6°

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 11: 5.2 – Dot Product

11) Calculate the dot product of each pair of vectors.

a) $\lvert\vec{a}\rvert = 70$, $\lvert\vec{b}\rvert = 115$, 70° between	b) $\lvert\vec{d}\rvert = 12$, $\lvert\vec{e}\rvert = 8$, 150° between	c) $\lvert\vec{e}\rvert = 200$, $\lvert\vec{f}\rvert = 150$, perpendicular
		
$\vec{a}\cdot\vec{b} = 70(115)\cos(70°)$ $\vec{a}\cdot\vec{b} \cong 2753.3$	$\vec{e}\cdot\vec{d} = 8(12)\cos(150°)$ $\vec{e}\cdot\vec{d} \cong -83.1$	$\vec{e}\cdot\vec{f} = 0$
d) $\vec{u} = [2, 4]$, $\vec{v} = [3, -1]$ $\vec{u}\cdot\vec{v} = 2(3) + 4(-1)$ $\vec{u}\cdot\vec{v} = 2$	e) $\vec{s} = [9, -3]$, $\vec{t} = [3, -3]$ $\vec{s}\cdot\vec{t} = 9(3) + (-3)(-3)$ $\vec{s}\cdot\vec{t} = 36$	f) $\vec{a} = 2\hat{i} + 3\hat{j}$, $\vec{b} = 9\hat{i} - 7\hat{j}$ $\vec{a}\cdot\vec{b} = 2(9) + 3(-7)$ $\vec{a}\cdot\vec{b} = -3$

Section 12: 5.2 – Dot Product Properties

12) Let $\vec{u} = [3, -5]$, $\vec{v} = [-6, 1]$, and $\vec{w} = [4, 7]$. Evaluate each of the following if possible. If it is not possible, explain why not.

a) $\vec{u}\cdot(\vec{v} + \vec{w})$ $= [3, -5]\cdot[-2, 8] = 3(-2) + (-5)(8)$ $\vec{u}\cdot(\vec{v} + \vec{w}) = -46$	b) $(\vec{u} + \vec{v})\cdot(\vec{u} - \vec{v})$ $= [-3, -4]\cdot[9, -6] = -3(9) + (-4)(-6)$ $(\vec{u} + \vec{v})\cdot(\vec{u} - \vec{v}) = -3$
c) $\vec{u} + \vec{v}\cdot\vec{w}$ This is not possible. It is the sum of a vector and a scalar.	d) $-3\vec{v}\cdot(2\vec{w})$ $= [18, -3]\cdot[8, 14] = 18(8) + (-3)(14)$ $-3\vec{v}\cdot(2\vec{w}) = 102$
e) $(\vec{u} + 2\vec{v})\cdot(3\vec{w} - \vec{u})$ $= [-9, -3]\cdot[9, 26] = -9(9) + (-3)(26)$ $(\vec{u} + 2\vec{v})\cdot(3\vec{w} - \vec{u}) = -159$	f) $\vec{v}\cdot\vec{v} + \vec{w}\cdot\vec{w}$ $= [37] + [65]$ $\vec{v}\cdot\vec{v} + \vec{w}\cdot\vec{w} = 102$

Section 13: 5.2 – Right Triangles Using the Dot Product

13) $\triangle ABC$ has points $A(3, 1)$, $B(-2, 3)$, and $C(5, 6)$. Is it a right-angled triangle? If it is, identify the right angle.

$\overrightarrow{AB} = [-5, 2]$, $\overrightarrow{AC} = [2, 5]$

$\overrightarrow{AB}\cdot\overrightarrow{AC} = -5(2) + 2(5) = 0$

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are perpendicular, so $\triangle ABC$ is a right-angled triangle. $\angle A$ is the right angle.

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 14: 5.2 – Perpendicular Vectors

14) Find a vector that is perpendicular to $\vec{u} = [9, 2]$. Verify that the vectors are perpendicular.

Let $\vec{v} = [-2, 9]$.

$\vec{u} \cdot \vec{v} = 9(-2) + 2(9)$

$\vec{u} \cdot \vec{v} = 0$, so $\vec{u}$ and $\vec{v}$ are perpendicular.

Section 15: 5.2 – Solving for k (Perpendicular Vectors)

15) Determine the value of k so that $\vec{u} = [2, 5]$ and $\vec{v} = [k, 4]$ are perpendicular.

$[2, 5] \cdot [k, 4] = 0$

$2k + 20 = 0$

$k = -10$

Section 16: 5.2 – Solving for k (Perpendicular Vectors)

16) Determine the value of k so that $\vec{u} = [k, 3]$ and $\vec{v} = [k, 2k]$ are perpendicular.

$[k, 3] \cdot [k, 2k] = 0$

$k^2 + 6k = 0$

$k(k + 6) = 0$

$k = 0$ or $k = -6$

Section 17: 5.3 – Work

17) Determine the work done by force $\vec{F}$, in Newtons, for an object moving along the vector $\vec{s}$, in meters.

a) $\vec{F} = [5, 2]$, $\vec{s} = [7, 4]$

$W = 5(7) + 2(4)$

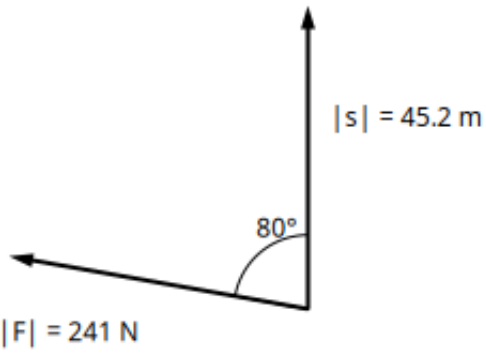
$W = 43\ J$

b) $\vec{F} = [100, 400]$, $\vec{s} = [12, 27]$

$W = 100(12) + 400(27)$

$W = 12\,000\ J$

c)



$W = 241(45.2)\cos(80°)$

$W \cong 1891.6\ J$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 18: 5.3 – Angle Between Two Vectors

18) Calculate the angle between the vectors in each pair.

a) $\vec{p} = [7, 8]$, $\vec{q} = [4, 3]$ $\cos\theta = \frac{7(4)+8(3)}{\sqrt{7^2+8^2}\sqrt{4^2+3^2}} = \frac{52}{(\sqrt{113})(5)}$ $\theta \cong 11.9°$	b) $\vec{t} = [-7, 2]$, $\vec{u} = [6, 11]$ $\cos\theta = \frac{-7(6)+2(11)}{\sqrt{(-7)^2+2^2}\sqrt{6^2+11^2}} = \frac{-20}{(\sqrt{53})(\sqrt{157})}$ $\theta \cong 102.7°$

Section 19: 5.3 – Projection of a Vector

19) Determine the projection of $\vec{u}$ on $\vec{v}$

a) $\lvert\vec{u}\rvert = 56$, $\lvert\vec{v}\rvert = 100$, angle θ between $\vec{u}$ and $\vec{v}$ is 125°. $proj_{\vec{v}}\vec{u} = \lvert\vec{u}\rvert\cos\theta(\hat{v}) = [56\cos(125°)](\hat{v})$ $proj_{\vec{v}}\vec{u} \cong -32.1\hat{v}$	b) $\vec{u} = [7, 1]$, $\vec{v} = [9, -3]$ $proj_{\vec{v}}\vec{u} = \frac{7(9)+1(-3)}{9^2+(-3)^2}[9, -3] = \frac{60}{90}[9, -3]$ $proj_{\vec{v}}\vec{u} = [6, -2]$

Section 20: 5.3 – Projection of a Vector

20) Find the magnitude of the projection of $\vec{a}$ on $\vec{b}$ and also the vector projection of $\vec{a}$ on $\vec{b}$ if $\vec{a} = [6, -1]$ and $\vec{b} = [11, 5]$

$\lvert proj_{\vec{b}}\vec{a}\rvert = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert} = \frac{6(11)+(-1)(5)}{\sqrt{11^2+5^2}} = \frac{61}{\sqrt{146}}$

$proj_{\vec{b}}\vec{a} = \lvert proj_{\vec{b}}\vec{a}\rvert(\hat{b}) = \frac{61}{\sqrt{146}}\left(\frac{1}{\sqrt{146}}\right)[11, 5] = \frac{61}{146}[11, 5]$

$proj_{\vec{b}}\vec{a} = \left[\frac{671}{146}, \frac{305}{146}\right]$

Section 21: 5.3 – Work

21) A superhero pulls herself 15 m up the side of a wall with a force of 500 N, at an angle of 12° to the vertical. What is the work done?

$W = 15(500)\cos(12°)$

$W \cong 7336.1\ J$

Section 22: 5.3 – Work

22) A crate is dragged 3 meters along a smooth level floor by a 30 N force, applied at 25° to the floor. Then, it is pulled 4 meters up a ramp inclined at 20° to the horizontal, using the same force. Then, the crate is dragged a further 5 meters along a level platform using the same force again. Determine the total work done in moving the crate.

The angle between the 30 N force and the direction of travel is 25° on the level floor, 5° on the ramp (25°-20°), and 25° on the level platform again.

$W = 3(30)\cos(25°) + 4(30)\cos(5°) + 5(30)\cos(25°)$

$W \cong 337.1\ J$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 23: 5.3 – Work Against Gravity

23) How much work is done against gravity by a worker who carries a 25-kg carton up a 6 meter long set of stairs, inclined at 30°.

$|\vec{F}| = mg = (25)(9.8) = 245\ N$

$W = \vec{F} \cdot \vec{s} = 245(6)\cos(60°)$

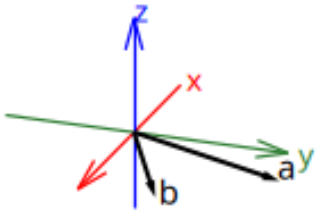
$W = 735\ J$

Section 24: 5.4 – Plotting Vectors in 3-Space

24) Draw each position vector. Then, determine its exact magnitude.

$\vec{a} = [-1,\ 5,\ -2]$

$\vec{b} = [-2,\ 0,\ -4]$



$|\vec{a}| = \sqrt{(-1)^2 + 5^2 + (-2)^2} = \sqrt{30}$

$|\vec{a}| = \sqrt{30}$

$|\vec{b}| = \sqrt{(-2)^2 + 0^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$

$|\vec{b}| = 2\sqrt{5}$

Section 25: 5.4 – Collinear Vectors in 3-Space

25) Are the vectors $\vec{u} = [6,\ -2,\ -5]$ and $\vec{v} = [-12,\ 4,\ 10]$ collinear? Explain.

$6 = k(-12) \Rightarrow k = -\frac{1}{2}$

$-2 = k(4) \Rightarrow k = -\frac{1}{2}$

$-5 = k(10) \Rightarrow k = -\frac{1}{2}$

Yes. $\vec{v} = -2\vec{u}$, so $\vec{u}$ and $\vec{v}$ are collinear.

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 26: 5.4 – Collinear Vectors in 3-Space

26) Find a and b such that $\vec{u} = [a, 3, 6]$ and $\vec{v} = [-8, 12, b]$ are collinear.

$3 = k(12) \Rightarrow k = \frac{1}{4}$

$a = \frac{1}{4}(-8) = -2$

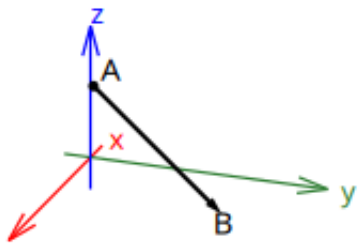
$6 = \frac{1}{4}(b) \Rightarrow b = 24$

$a = -2, \ b = 24$

Section 27: 5.4 – Vectors Between Points in 3-Space

27) Draw the vector $\overrightarrow{AB}$ joining each pair of points. Then, write the vector in the form $[x, y, z]$ and determine its magnitude.

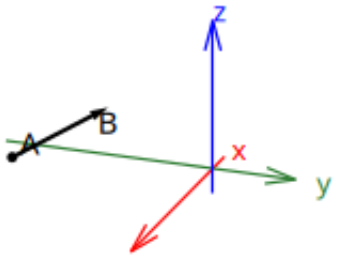
a) $A(2, 1, 3)$, $B(5, 7, 1)$



$\overrightarrow{AB} = [5-2, 7-1, 1-3] = [3, 6, -2]$

$|\overrightarrow{AB}| = \sqrt{3^2 + 6^2 + (-2)^2} = 7$

b) $A(3, -4, 1)$, $B(6, -1, 5)$



$\overrightarrow{AB} = [6-3, -1-(-4), 5-1] = [3, 3, 4]$

$|\overrightarrow{AB}| = \sqrt{3^2 + 3^2 + 4^2} = \sqrt{34}$

Section 28: 5.4 – Vector Operations in 3-Space

28) Given the vectors $\vec{a} = [-4, 1, 7]$, $\vec{b} = [2, 0, -3]$, and $\vec{c} = [1, -1, 5]$, simplify each expression.

a) $7\vec{a}$

$7\vec{a} = [-28, 7, 49]$

b) $3\vec{a} - 2\vec{b} + 4\vec{c}$

$= [-12, 3, 21] - [4, 0, -6] + [4, -4, 20]$

$3\vec{a} - 2\vec{b} + 4\vec{c} = [-12, -1, 47]$

c) $\vec{a} \cdot \vec{c}$

$= -4(1) + 1(-1) + 7(5)$

$\vec{a} \cdot \vec{c} = 30$

d) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$

$= [-2, 1, 4] \cdot [-6, 1, 10] = -2(-6) + 1(1) + 4(10)$

$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 53$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 29: 5.4 – Angle Between Vectors in 3-Space

29) Determine the angle between the vectors $\vec{g} = [6, 1, 2]$ and $\vec{h} = [-5, 3, 6]$

$\cos\theta = \dfrac{6(-5) + 1(3) + 2(6)}{\sqrt{6^2 + 1^2 + 2^2}\sqrt{(-5)^2 + 3^2 + 6^2}} = \dfrac{-15}{(\sqrt{41})(\sqrt{70})}$

$\theta \cong 106.3°$

Section 30: 5.4 – Orthogonal Vectors in 3-Space

30) Determine a vector that is orthogonal to $\vec{e} = [3, -1, 4]$

Let $\vec{f} = [0, 4, 1]$.

$\vec{e} \cdot \vec{f} = 3(0) + (-1)(4) + 4(1) = 0$

$\vec{f} = [0, 4, 1]$ (answers will vary) is orthogonal to $\vec{e}$.

Section 31: 5.4 – Classifying a Triangle in 3-Space

31) Identify the type of triangle with vertices $A(2, 3, -5)$, $B(-4, 8, 1)$, and $C(6, -4, 0)$.

$\overrightarrow{AB} = [-6, 5, 6]$, $|\overrightarrow{AB}| = \sqrt{97}$

$\overrightarrow{AC} = [4, -7, 5]$, $|\overrightarrow{AC}| = \sqrt{90}$

$\overrightarrow{BC} = [10, -12, -1]$, $|\overrightarrow{BC}| = \sqrt{245}$

$(\sqrt{97})^2 + (\sqrt{90})^2 = 187 \neq 245 = (\sqrt{245})^2$, so it is not a right triangle.

Since all three side lengths are different, the triangle is scalene.

Section 32: 5.5 – Magnitude of the Cross Product

32) Determine $\vec{u} \times \vec{v}$

$|\vec{v}| = 80$, $|\vec{u}| = 60$, 55° between

$\vec{u} \times \vec{v} = |\vec{u}||\vec{v}|\sin\theta\,(\hat{n}) = (60)(80)\sin(55°)\,(\hat{n})$

$\vec{u} \times \vec{v} \cong 3931.9\hat{n}$

Section 33: 5.5 – Cross Product of Algebraic Vectors

33) Determine $\vec{a} \times \vec{b}$ for each pair of vectors

a) $\vec{a} = [3, -2, 9]$, $\vec{b} = [1, 1, 6]$ $\vec{a} \times \vec{b} = [-2(6) - 9(1), 9(1) - 3(6), 3(1) - (-2)(1)]$ $\vec{a} \times \vec{b} = [-21, -9, 5]$	b) $\vec{a} = [-8, 10, 3]$, $\vec{b} = [2, 0, 5]$ $\vec{a} \times \vec{b} = [10(5) - 3(0), 3(2) - (-8)(5), -8(0) - 10(2)]$ $\vec{a} \times \vec{b} = [50, 46, -20]$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 34: 5.5 – Area of a Parallelogram

34) Determine the area of the parallelogram defined by each pair of vectors.

a) $\vec{p} = [6,\ 3,\ 8]$, $\vec{q} = [3,\ 3,\ 5]$ $\vec{p} \times \vec{q} = [3(5) - 8(3),\ 8(3) - 6(5),\ 6(3) - 3(3)]$ $\vec{p} \times \vec{q} = [-9,\ -6,\ 9]$ Area $= \lvert\vec{p} \times \vec{q}\rvert = \sqrt{198} = 3\sqrt{22}$ units^2	b) $\lvert\vec{v}\rvert = 27$, $\lvert\vec{u}\rvert = 43$, 32° between Area $= \lvert\vec{u}\rvert\lvert\vec{v}\rvert \sin\theta = 43(27)\sin(32°)$ Area $\cong 615.2$ units^2

Section 35: 5.5 – Cross Product Expressions

35) Given $\vec{a} = [2,\ -6,\ 3]$, $\vec{b} = [-1,\ 5,\ 8]$, and $\vec{c} = [-4,\ 5,\ 6]$, evaluate each of the following.

a) $\vec{a} \times (\vec{b} + \vec{c})$

$\vec{b} + \vec{c} = [-5,\ 10,\ 14]$

$\vec{a} \times (\vec{b} + \vec{c}) = [-6(14) - 3(10),\ 3(-5) - 2(14),\ 2(10) - (-6)(-5)]$

$\vec{a} \times (\vec{b} + \vec{c}) = [-114,\ -43,\ -10]$

b) $\vec{a} \times \vec{b} - \vec{a} \times \vec{c}$

$\vec{a} \times \vec{b} = [-6(8) - 3(5),\ 3(-1) - 2(8),\ 2(5) - (-6)(-1)] = [-63,\ -19,\ 4]$

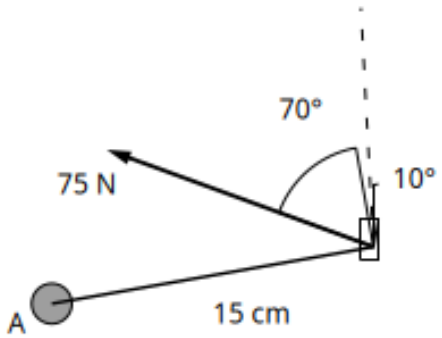
$\vec{a} \times \vec{c} = [-6(6) - 3(5),\ 3(-4) - 2(6),\ 2(5) - (-6)(-4)] = [-51,\ -24,\ -14]$

$\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = [-63,\ -19,\ 4] - [-51,\ -24,\ -14]$

$\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = [-12,\ 5,\ 18]$

Section 36: 5.6 – Torque

36) A bicycle pedal is pushed by a 75 N force, exerted as shown in the diagram. The shaft of the pedal is 15 cm long. Find the magnitude of the torque vector, in Newton-meters, about point A.



$\lvert\vec{\tau}\rvert = \lvert\vec{r}\rvert\lvert\vec{F}\rvert \sin\theta$

$\lvert\vec{\tau}\rvert = (0.15)(75)\sin(80°)$

$\lvert\vec{\tau}\rvert \cong 11.1\ N \cdot m$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Section 37: 5.6 – Work in 3-Space

37) A force of $\vec{F} = [3,\ 5,\ 12]$, in Newtons, is applied to lift a box, with displacement $\vec{s} = [2,\ 1,\ 6]$. Calculate the work against gravity and compare it to the work in the direction of travel.

Work against gravity: $W = [3,\ 5,\ 12] \cdot [0,\ 0,\ 6] = 3(0) + 5(0) + 12(6)$

$W = 72\ J$

Work in direction of travel: $W = [3,\ 5,\ 12] \cdot [2,\ 1,\ 6] = 3(2) + 5(1) + 12(6)$

$W = 83\ J$

The work done in the direction of travel (83 J) is greater than the work done against gravity alone (72 J), since part of the force also acts in the horizontal direction of travel.

Section 38: 5.6 – Projection in 3-Space

38) Determine the projection, and its magnitude, of $\vec{u} = [3,\ 1,\ 4]$ on $\vec{v} = [6,\ 2,\ 7]$.

$|proj_{\vec{v}}\,\vec{u}| = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|} = \dfrac{3(6) + 1(2) + 4(7)}{\sqrt{6^2 + 2^2 + 7^2}} = \dfrac{48}{\sqrt{89}}$

$proj_{\vec{v}}\,\vec{u} = \dfrac{48}{\sqrt{89}}\left(\dfrac{[6,\ 2,\ 7]}{\sqrt{89}}\right) = \dfrac{48}{89}[6,\ 2,\ 7]$

$proj_{\vec{v}}\,\vec{u} = \left[\dfrac{288}{89},\ \dfrac{96}{89},\ \dfrac{336}{89}\right]$

Section 39: 5.6 – Triple Scalar Product

39) Given $\vec{a} = [-2,\ 3,\ 5]$, $\vec{b} = [4,\ 0,\ -1]$, and $\vec{c} = [2,\ -2,\ 3]$, evaluate $\vec{a} \cdot \vec{b} \times \vec{c}$

$\vec{b} \times \vec{c} = [0(3) - (-1)(-2),\ -1(2) - 4(3),\ 4(-2) - 0(2)]$

$\vec{b} \times \vec{c} = [-2,\ -14,\ -8]$

$\vec{a} \cdot (\vec{b} \times \vec{c}) = [-2,\ 3,\ 5] \cdot [-2,\ -14,\ -8]$

$= -2(-2) + 3(-14) + 5(-8)$

$\vec{a} \cdot \vec{b} \times \vec{c} = -78$

Section 40: 5.6 – Volume of a Parallelepiped

40) Find the volume of the parallelepiped, defined by the vectors $\vec{u} = [1,\ 4,\ 3]$, $\vec{v} = [2,\ 5,\ 6]$, and $\vec{w} = [1,\ 2,\ 7]$.

$\vec{u} \times \vec{v} = [4(6) - 3(5),\ 3(2) - 1(6),\ 1(5) - 4(2)]$

$\vec{u} \times \vec{v} = [9,\ 0,\ -3]$

$V = |(\vec{u} \times \vec{v}) \cdot \vec{w}| = |[9,\ 0,\ -3] \cdot [1,\ 2,\ 7]|$

$= |9(1) + 0(2) + (-3)(7)| = |-12|$

$V = 12\ \text{units}^3$

Section 41: 5.6 – Area of a Triangle Using the Cross Product

41) A triangle has vertices $A(-2,\ 1,\ 3)$, $B(7,\ 8,\ -4)$, and $C(5,\ 0,\ 2)$. Determine the area of $\triangle ABC$.

$\overrightarrow{AB} = [9,\ 7,\ -7]$, $\overrightarrow{AC} = [7,\ -1,\ -1]$

$\overrightarrow{AB} \times \overrightarrow{AC} = [7(-1) - (-7)(-1),\ -7(7) - 9(-1),\ 9(-1) - 7(7)]$

Copyright © 2026 Annie Bernatchez—All rights reserved.

$\overrightarrow{AB} \times \overrightarrow{AC} = [-14, \ -40, \ -58]$

$\text{Area} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{(-14)^2 + (-40)^2 + (-58)^2} = \frac{1}{2}\sqrt{5160}$

$\text{Area} = \sqrt{1290} \cong 35.9 \ \text{units}^2$

Copyright © 2026 Annie Bernatchez—All rights reserved.