5.6 Applications of Dot and Cross Product Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: Torque

Torque, τ, is a measure of the force acting on an object that causes it to rotate. Torque is the cross product of the force and the torque arm. Torque is measured in $N \cdot m$ (Newton meters).

$\vec{\tau} = \vec{r} \times \vec{F}$

$|\vec{\tau}| = |\vec{r}||\vec{F}| \sin\theta$

$\vec{F}$ is the force acting on the object

$\vec{r}$ represent the arm and points from the axis of rotation to the point where the force is applied

θ is the angle between $\vec{r}$ and $\vec{F}$

Use the right-hand rule to determine the direction of the torque vector.

Example 1: A wrench is used to tighten a bolt. A force of 60 N is applied in a clockwise direction at 80° to the handle, 20 cm from the centre of the bolt.

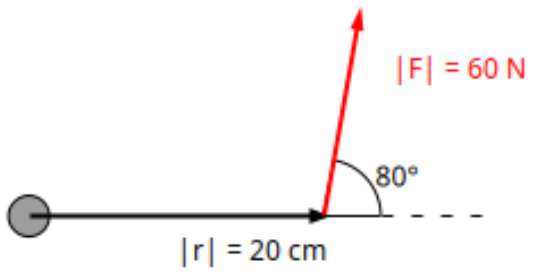
a) Calculate the magnitude of the torque
Note: make sure to convert displacement vector to meters.
$|\vec{\tau}| = |\vec{r}||\vec{F}| \sin\theta$
$|\vec{\tau}| = (0.2)(60)\sin 80$

$|\vec{\tau}| \cong 11.8\ N \cdot m$

b) In what direction does the torque vector point?

Downward into the material.

Therefore, the bolt is being tightened.



Part 2: Projection and Work

Projections in $\mathbb{R}^3$ are similar to projections in $\mathbb{R}^2$, so you can use the same formula: $proj_{\vec{b}}\,\vec{a} = \frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}(\vec{b})$

Example 2: For the vectors $\vec{v} = [4,\ 2,\ 7]$ and $\vec{u} = [6,\ 3,\ 8]$

a) Find $proj_{\vec{u}}\,\vec{v}$

$proj_{\vec{u}}\,\vec{v} = \frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}}(\vec{u})$

$proj_{\vec{u}}\,\vec{v} = \frac{4(6) + 2(3) + 7(8)}{6^2 + 3^2 + 8^2}[6,\ 3,\ 8]$

$proj_{\vec{u}}\,\vec{v} = \frac{86}{109}[6,\ 3,\ 8]$

Copyright © 2026 Annie Bernatchez—All rights reserved.

b) Find the magnitude of the projection

$|proj_{\vec{u}}\ \vec{v}| = \frac{|\vec{v}\cdot\vec{u}|}{|\vec{u}|}$

$|proj_{\vec{u}}\ \vec{v}| = \frac{86}{\sqrt{109}}$

Or just find the magnitude of the vector found in part a): $\left|\frac{86}{109}[6,\ 3,\ 8]\right| = \frac{86}{109}|[6,\ 3,\ 8]| = \frac{86}{109}\sqrt{109} = \frac{86}{\sqrt{109}}$

Reminder: Work performed is the product of the magnitude of the displacement travelled by an object and the magnitude of the force applied in the direction of the motion (use dot product to find this). The units are joules (J).

Example 3: A force with units in newtons and defined by $\vec{F} = [300,\ 700,\ 500]$ acts on an object with displacement, in meters, defined by $\vec{d} = [3,\ 1,\ 12]$.

a) Determine the work done in the direction of travel.

$W = \vec{F}\cdot\vec{d}$

$W = 300(3) + 700(1) + 500(12)$

$W = 7600\ J$

b) Determine the work done against gravity, which is a force in the direction of the negative z-axis.

Use only the vertical components to calculate the work against gravity.

$W = [0,\ 0,\ 500]\cdot[0,\ 0,\ 12]$

$W = 0(0) + 0(0) + 500(12)$

$W = 6000\ J$

Part 3: Triple Scalar Product and Volume of a Parallelepiped

The Triple Scalar Product

Certain situations require a combination of the dot and cross products. The triple scalar product, $\vec{a}\cdot\vec{b}\times\vec{c}$, is one such combination. Because the dot product is a scalar, this combination is only meaningful if the cross product is performed first.

Example 4: Consider the vectors $\vec{u} = [4,\ 3,\ 1]$, $\vec{v} = [2,\ 5,\ 6]$, and $\vec{w} = [10,\ -3,\ -14]$. Evaluate the expression $\vec{u}\times\vec{v}\cdot\vec{w}$

$\vec{u}\times\vec{v}\cdot\vec{w} = [4,\ 3,\ 1]\times[2,\ 5,\ 6]\cdot[10,\ -3,\ -14]$

$\vec{u}\times\vec{v}\cdot\vec{w} = [3(6) - 1(5),\ 1(2) - 4(6),\ 4(5) - 3(2)]\cdot[10,\ -3,\ -14]$

$\vec{u}\times\vec{v}\cdot\vec{w} = [13,\ -22,\ 14]\cdot[10,\ -3,\ -14]$

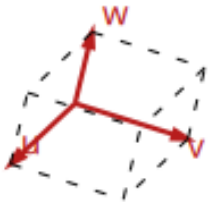
$\vec{u}\times\vec{v}\cdot\vec{w} = 13(10) + (-22)(-3) + 14(-14)$

$\vec{u}\times\vec{v}\cdot\vec{w} = 0$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Volume of a Parallelepiped

A parallelepiped is a three-dimensional figure formed by six parallelograms. In general, the volume of the parallelepiped defined by the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ is given by $V = |\vec{w} \cdot \vec{u} \times \vec{v}|$.



Example 5: Find the volume of the parallelepiped defined by $\vec{a} = [6,\ 3,\ -2]$, $\vec{b} = [-4,\ 6,\ 9]$ and $\vec{c} = [3,\ 3,\ -11]$

$V = |\vec{a} \cdot \vec{b} \times \vec{c}|$

$V = |[6,\ 3,\ -2] \cdot [-4,\ 6,\ 9] \times [3,\ 3,\ -11]|$

$V = |[6,\ 3,\ -2] \cdot [6(-11) - 9(3),\ 9(3) - (-4)(-11),\ (-4)(3) - 6(3)]|$

$V = |[6,\ 3,\ -2] \cdot [-93,\ -17,\ -30]|$

$V = |6(-93) + 3(-17) + (-2)(-30)|$

$V = |-549|$

$V = 549\ \text{units}^3$

Copyright © 2026 Annie Bernatchez—All rights reserved.