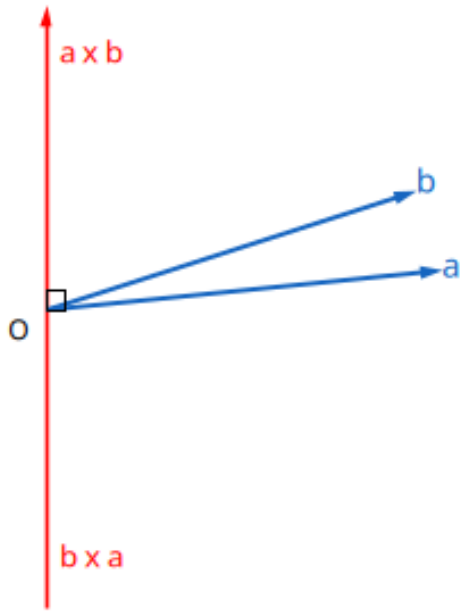
5.5 Cross Product of Vectors Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

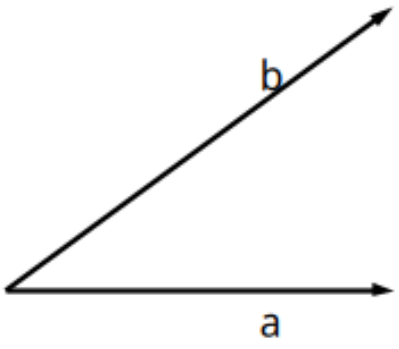
The course material originated from www.jensenmath.ca

The cross product of two vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^3$ is the vector that is **perpendicular** to these vectors such that the vectors $\vec{a}$, $\vec{b}$, and $\vec{a}\times\vec{b}$ form a right-handed system.

Right handed system tells you to point your hand along vector $\vec{a}$ and curl your fingers towards vector $\vec{b}$. Your thumb will be pointing in the direction of $\vec{a}\times\vec{b}$. Notice that $\vec{a}\times\vec{b}$ and $\vec{b}\times\vec{a}$ point in opposite directions.



Sometimes the direction of the cross product is defined by either 'in to the page' or 'out of the page':

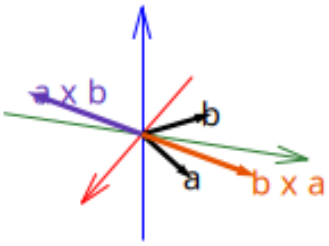


$\vec{a}\times\vec{b}$	$\vec{b}\times\vec{a}$
"Out of the page"	"In to the page"

3D visualization:

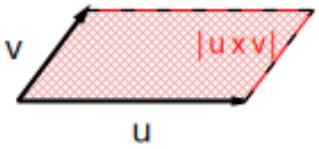
www.geogebra.org/3d/jyqcr3bf

Copyright © 2026 Annie Bernatchez—All rights reserved.



Properties of Cross Product:

- $\vec{u}\times\vec{v} = -(\vec{v}\times\vec{u})$
- $\vec{u}\times(\vec{v}+\vec{w}) = \vec{u}\times\vec{v} + \vec{u}\times\vec{w}$
- $(\vec{u}+\vec{v})\times\vec{w} = \vec{u}\times\vec{w} + \vec{v}\times\vec{w}$
- If $\vec{u}\times\vec{v} = 0$, and $\vec{u}$ and $\vec{v}$ are non-zero, then $\vec{u}$ and $\vec{v}$ are collinear.
- $k(\vec{u}\times\vec{v}) = (k\vec{u})\times\vec{v} = \vec{u}\times(k\vec{v})$
- $|\vec{u}\times\vec{v}| =$ the area of the parallelogram defined by $\vec{u}$ and $\vec{v}$



Part 2: Cross Product of Geometric Vectors

Formula: $\vec{a}\times\vec{b} = (|\vec{a}||\vec{b}|\sin\theta)\hat{n}$

θ is the angle between the vectors

$\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Example 1: If $|\vec{u}| = 30$, $|\vec{v}| = 20$, the angle between $\vec{u}$ and $\vec{v}$ is 40°, and $\vec{u}$ and $\vec{v}$ are in the plane of the page, find…

a) $\vec{u} \times \vec{v}$

$\vec{u} \times \vec{v} = (|\vec{u}||\vec{v}| \sin\theta)\hat{n}$

$\vec{u} \times \vec{v} = [(30)(20)\sin 40]\hat{n}$

$\vec{u} \times \vec{v} \cong 385.7\hat{n}$

$\vec{u} \times \vec{v} \cong 385.7$ out of the page

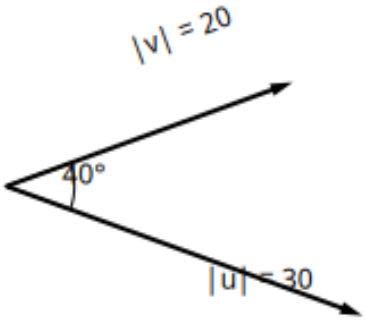
b) $\vec{v} \times \vec{u}$

$\vec{v} \times \vec{u} = (|\vec{v}||\vec{u}| \sin\theta)\hat{n}$

$\vec{v} \times \vec{u} = [(20)(30)\sin 40](-\hat{n})$

$\vec{v} \times \vec{u} \cong -385.7\hat{n}$

$\vec{v} \times \vec{u} \cong 385.7$ in to the page



Part 3: Cross Product of Algebraic Vectors

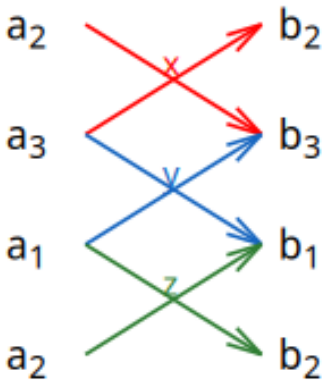
Formula:

If $\vec{a} = [a_1, a_2, a_3]$ and $\vec{b} = [b_1, b_2, b_3]$

$\vec{a} \times \vec{b} = [a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1]$

$= (a_2b_3 - a_3b_2)\vec{i} + (a_3b_1 - a_1b_3)\vec{j} + (a_1b_2 - a_2b_1)\vec{k}$

How to set it up:

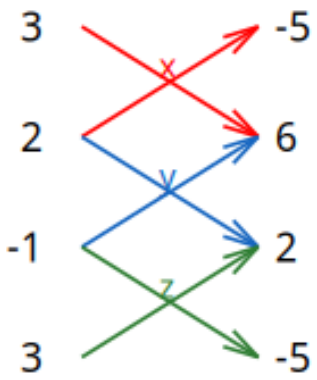


Example 2: If $\vec{p} = [-1, 3, 2]$ and $\vec{q} = [2, -5, 6]$, calculate each of the following:

Copyright © 2026 Annie Bernatchez—All rights reserved.

Example 2: If $\vec{p} = [-1, 3, 2]$ and $\vec{q} = [2, -5, 6]$, calculate each of the following:

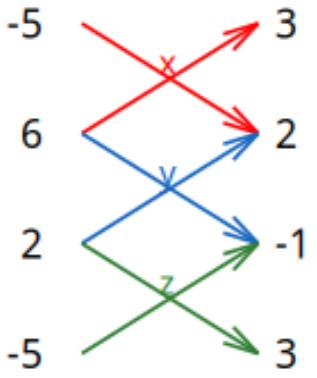
a) $\vec{p} \times \vec{q}$



$\vec{p} \times \vec{q} = [3(6) - 2(-5), 2(2) - (-1)(6), (-1)(-5) - (3)(2)]$

$\vec{p} \times \vec{q} = [28, 10, -1]$

b) $\vec{q} \times \vec{p}$



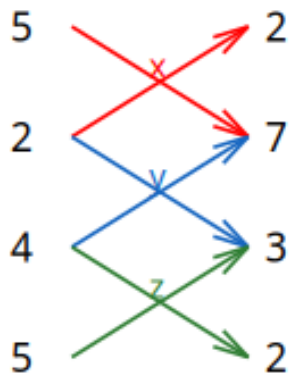
$\vec{q} \times \vec{p} = [(-5)(2) - 6(3), 6(-1) - 2(2), 2(3) - (-5)(-1)]$

$\vec{q} \times \vec{p} = [-28, -10, 1]$

Notice $\vec{q} \times \vec{p} = -1(\vec{p} \times \vec{q})$

Copyright © 2026 Annie Bernatchez—All rights reserved.

Example 3a: Determine the area of the parallelogram defined by the vectors $\vec{u} = [4, 5, 2]$ and $\vec{v} = [3, 2, 7]$.



$\vec{u} \times \vec{v} = [5(7) - 2(2),\ 2(3) - 4(7),\ 4(2) - 5(3)]$

$\vec{u} \times \vec{v} = [31,\ -22,\ -7]$

$|\vec{u} \times \vec{v}| = \sqrt{(31)^2 + (-22)^2 + (-7)^2}$

$|\vec{u} \times \vec{v}| = \sqrt{1494}$

$|\vec{u} \times \vec{v}| \cong 38.7$ units^2

Example 3b: Determine the angle between the vectors $\vec{u}$ and $\vec{v}$.

$|\vec{u} \times \vec{v}| = |\vec{u}||\vec{v}| \sin\theta$

$$\sin\theta = \frac{|\vec{u} \times \vec{v}|}{|\vec{u}||\vec{v}|}$$

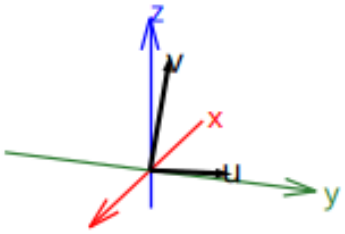
$$\sin\theta = \frac{\sqrt{1494}}{\left(\sqrt{4^2 + 5^2 + 2^2}\right)\left(\sqrt{3^2 + 2^2 + 7^2}\right)}$$

$$\sin\theta = \frac{\sqrt{1494}}{(\sqrt{45})(\sqrt{62})}$$

$\theta \cong 47°$ OR $\theta \cong 180 - 47 = 133°$

From an inspection of the vectors we can tell that $\theta \cong 47°$.

$\theta \cong 47°$



Copyright © 2026 Annie Bernatchez—All rights reserved.