4.3 Multiplication of a Vector by a Scalar Lesson

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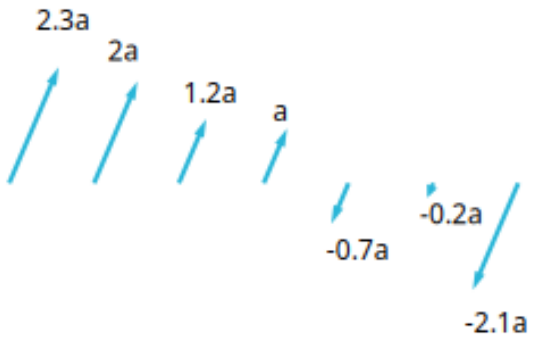
The course material originated from www.jensenmath.ca

Part 1: Scalar Multiplication

In this section we will multiply a vector, $\vec{a}$, by a number k to produce a new vector $k\vec{a}$. The number k used for multiplication is called a _____ and can be any real number.

Multiplying a vector by different values of a scalar k can affect the **magnitude** AND **direction** of the vector.

When two vectors are parallel, these vectors are described as being _____. They are described this way because they can be translated to be on the same straight line. Scalar multiples of vectors are colinear.



Rules:

1) If $k > 0$, then $k\vec{a}$ is in the same direction as $\vec{a}$ with magnitude $k|\vec{a}|$

a) If $0 < k < 1$, $\vec{a}$ is shortened $\rightarrow \left|\frac{1}{2}\vec{a}\right| = \frac{1}{2}|\vec{a}|$

b) If $k > 1$, $\vec{a}$ is lengthened $\rightarrow |2\vec{a}| = 2|\vec{a}|$

2) If $k < 0$, then $k\vec{a}$ is in the opposite direction as $\vec{a}$ with magnitude $|k|\vec{a}|$

a) If $-1 < k < 0$, $\vec{a}$ is shortened and changes to the opposite direction $\rightarrow \left|-\frac{1}{2}\vec{a}\right| = \frac{1}{2}|\vec{a}|$

b) If $k < -1$, $\vec{a}$ is lengthened and changes to the opposite direction $\rightarrow |-2\vec{a}| = 2|\vec{a}|$

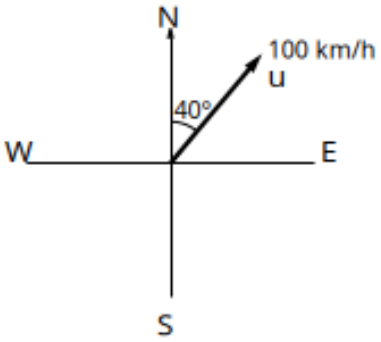
3) If $k = 0$, the result is the zero vector $\vec{0}$

4) The distributive property for vector multiplication is $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$

Example 1: An airplane is heading due north at 1000 km/h. The airplane's velocity is represented by $\vec{v}$. Draw the vectors $-\vec{v}$, $\frac{1}{2}\vec{v}$, and $-\frac{1}{2}\vec{v}$ and give an interpretation for each.

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Example 2: Consider vector $\vec{u}$ with magnitude $|\vec{u}| = 100$ km/h, at a quadrant bearing of $N40°E$. Draw a vector with each scalar multiplication. Describe the resulting vector.



a) $3\vec{u}$

b) $0.5\vec{u}$

c) $-2\vec{u}$

Vector Properties for Scalar Multiplication

Distributive Property: For any scalar $k \in \mathbb{R}$ and any vectors $\vec{u}$ and $\vec{v}$, $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$

Associative Property: For any scalars $a, b \in \mathbb{R}$ and any vector $\vec{v}$, $(ab)\vec{v} = a(b\vec{v})$

Identity Property: For any vector $\vec{v}$, $1\vec{v} = \vec{v}$

Example 3: If $\vec{x} = 3\vec{i} - 4\vec{j} + \vec{k}$, $\vec{y} = \vec{j} - 5\vec{k}$, and $\vec{z} = -\vec{i} - \vec{j} + 4\vec{k}$, determine each of the following in terms of $\vec{i}$, $\vec{j}$, and $\vec{k}$.

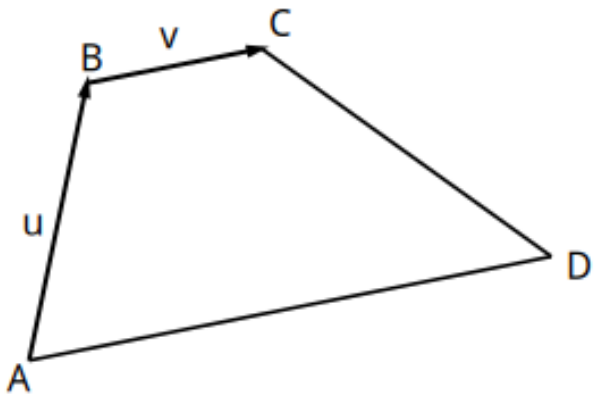
a) $\vec{x} + \vec{y}$

b) $\vec{x} - \vec{y}$

c) $\vec{x} - 2\vec{y} + 3\vec{z}$

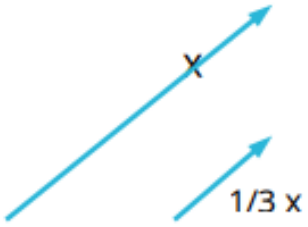
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Example 4: In trapezoid ABCD, $BC \parallel AD$ and $AD = 3BC$. Let $\overrightarrow{AB} = \vec{u}$ and $\overrightarrow{BC} = \vec{v}$. Express $\overrightarrow{AD}$, $\overrightarrow{BD}$, and $\overrightarrow{CD}$ as linear combinations of $\vec{u}$ and $\vec{v}$.



Part 2: Creating Unit Vectors

It is sometimes useful to multiply the nonzero vector $\vec{x}$ by the scalar $\frac{1}{|\vec{x}|}$. When we multiply $\vec{x}$ by $\frac{1}{|\vec{x}|}$, we get the vector $\frac{1}{|\vec{x}|}\vec{x}$. This is a vector of length one and is called a _____, which points in the same direction as $\vec{x}$. The concept of unit vectors will prove to be very useful when we discuss applications of vectors.



Example 5: Given that $|\vec{u}| = 4$ and $|\vec{v}| = 5$ and the angle between $\vec{u}$ and $\vec{v}$ is 120°, determine the unit vector in the same direction as $\vec{u} + \vec{v}$.

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