4.2 Vector Addition Lesson with solutions

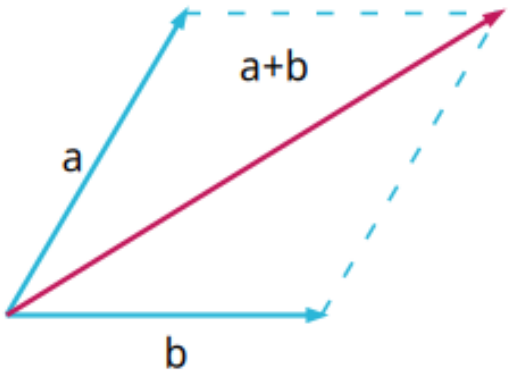
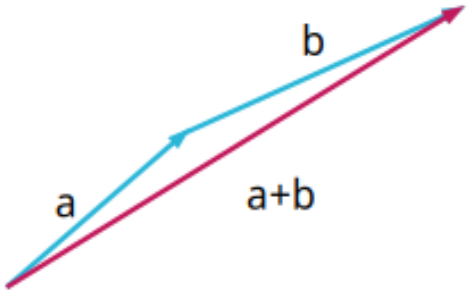
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The course material originated from www.jensenmath.ca

Part 1: Adding Vectors

When you add two or more vectors, you are finding a single vector, called the RESULTANT, that has the same effect as the original vectors applied one after the other.

Two methods:

Parallelogram	Tip to Tail (triangle)
To determine the sum of any two vectors $\vec{a}$ and $\vec{b}$, arranged tail-to-tail, complete the parallelogram formed by the two vectors. Their sum is the vector that is the diagonal of the constructed parallelogram.	The sum of vectors $\vec{a}$ and $\vec{b}$ can also be found by translating the tail of vector $\vec{b}$ to the head of vector $\vec{a}$. The resultant is the vector from the tail of $\vec{a}$ to the head of $\vec{b}$.
	

What if we add opposite vectors?

When two opposite vectors are added, the resultant is the zero vector. This means that the combined effect of a vector and its opposite is the zero vector.

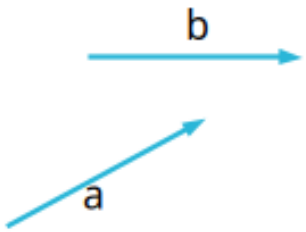


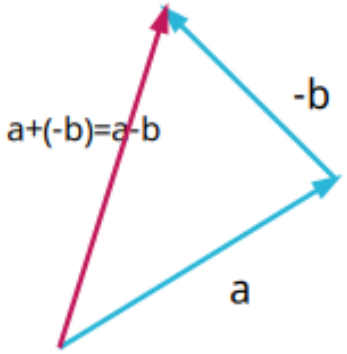
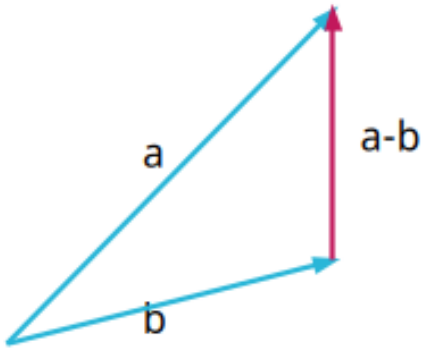
$\overrightarrow{AB} + \overrightarrow{BA} = \vec{0}$

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Part 2: Difference of 2 Vectors

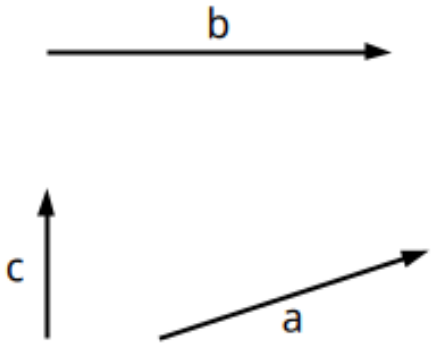
If you want to determine the difference between two vectors, $\vec{a} - \vec{b}$, there are two options:

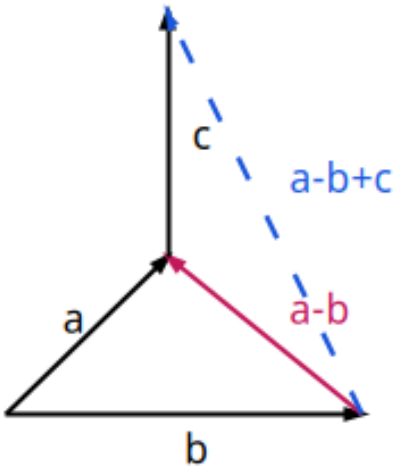


Adding the Opposite	Tail to Tail
The difference between $\vec{a}$ and $\vec{b}$ is found by adding the opposite of vector $\vec{b}$ to $\vec{a}$ using the triangle law of addition.	Another way to think about $\vec{a} - \vec{b}$ is to arrange the vectors tail to tail. In this case, $\vec{a} - \vec{b}$ is the vector that must be added to $\vec{b}$ to get $\vec{a}$
	

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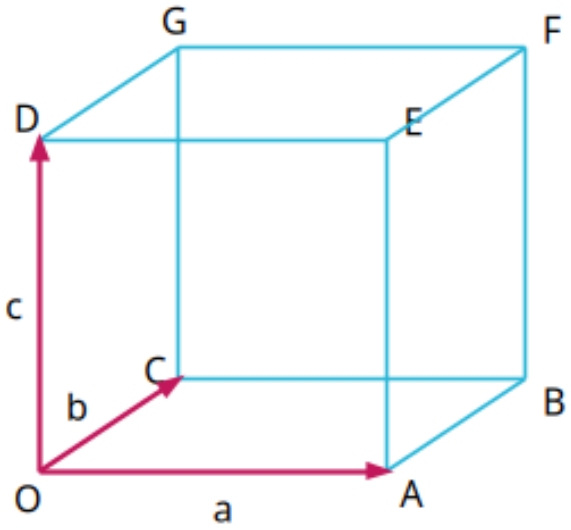
Example 1: Suppose you are given the vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ as shown below. Using these three vectors, sketch $\vec{a} - \vec{b} + \vec{c}$





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Example 2: In the rectangular box shown below, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{b}$, and $\overrightarrow{OD} = \vec{c}$. Express each of the following vectors in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.



a) $\overrightarrow{BC} = -\vec{a}$

b) $\overrightarrow{GF} = \vec{a}$

c) $\overrightarrow{OB} = \vec{a} + \vec{b}$

d) $\overrightarrow{AC} = \vec{b} - \vec{a}$

e) $\overrightarrow{BG} = \vec{c} + (-\vec{a}) = \vec{c} - \vec{a}$

f) $\overrightarrow{OF} = \vec{a} + \vec{b} + \vec{c}$

Part 3: Properties of Vector Addition

Commutative Property	$\vec{u} + \vec{v} = \vec{v} + \vec{u}$
Associative Property	$(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
Identity Property	$\vec{v} + \vec{0} = \vec{v} = \vec{0} + \vec{v}$

Example 3: Simplify each of the following

a) $(\vec{u} + \vec{v}) - \vec{u}$

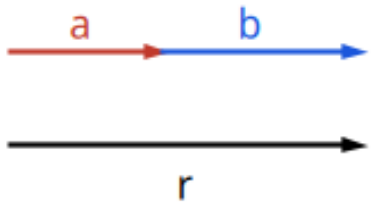
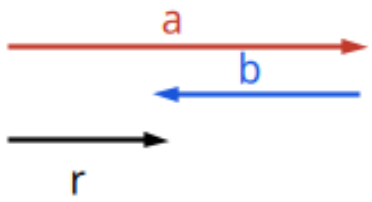
$$= (\vec{v} + \vec{u}) + (-\vec{u})$$
$$= \vec{v} + [\vec{u} + (-\vec{u})]$$
$$= \vec{v} + \vec{0}$$
$$= \vec{v}$$

b) $[(\vec{p} + \vec{q}) - \vec{p}] - \vec{q}$

$$= [\vec{q} + (\vec{p} - \vec{p})] - \vec{q}$$
$$= (\vec{q} + \vec{0}) - \vec{q}$$
$$= \vec{q} - \vec{q}$$
$$= \vec{0}$$

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Part 4: Solving Problems involving Vector Addition and Subtraction

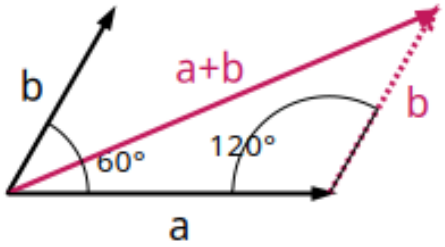
Diagram	Rule
	If you have two vectors acting in the same direction, the overall magnitude is equal to the sum of the two individual magnitudes. $\lvert\vec{r}\rvert = \lvert\vec{a}\rvert + \lvert\vec{b}\rvert$
	If you have two vectors acting in opposite directions, the overall magnitude is equal to the difference of the two individual magnitudes. $\lvert\vec{r}\rvert = \lvert\vec{a}\rvert - \lvert\vec{b}\rvert$

However, not all forces act in the same or opposite direction. Therefore, we will need some trigonometry to determine the magnitude of resultant vectors.

Rule	When to Use It	
Pythagorean Theorem $a^2 + b^2 = c^2$	Right Triangle Know: 2 sides Want: 3rd side	
SOH CAH TOA	Right Triangle Know: 2 sides Want: Angle (use inverse ratio)	Right Triangle Know: 1 side, 1 angle Want: Side
Sine Law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	Oblique Triangle (no right angle) Know: 2 sides and opposite angle Want: Angle	Oblique Triangle (no right angle) Know: 1 side and all angles Want: Side
Cosine Law $a^2 = b^2 + c^2 - 2bc(\cos A)$ $\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$	Oblique Triangle Know: 2 sides and contained angle Want: 3rd side (use top formula)	Oblique Triangle Know: All 3 sides Want: Angle (use bottom formula)

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Example 4: Given vectors $\vec{a}$ and $\vec{b}$ such that the angle between the two vectors is 60°, $|\vec{a}| = 3$, and $|\vec{b}| = 2$, determine $|\vec{a} + \vec{b}|$.



Note: "angle between vectors" means the angle between the vectors when placed tail to tail.
Translate them tip to tail to determine the resultant vector.

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(120)$$
$$|\vec{a} + \vec{b}|^2 = (3)^2 + (2)^2 - 2(3)(2)\cos(120)$$
$$|\vec{a} + \vec{b}|^2 = 13 - 12\left(-\frac{1}{2}\right)$$
$$|\vec{a} + \vec{b}|^2 = 19$$
$$|\vec{a} + \vec{b}| = \sqrt{19}$$

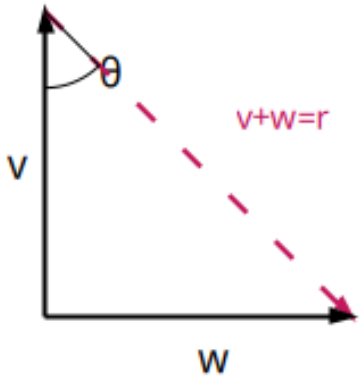
Example 5: An airplane heads due south at a speed of 300 km/h and meets a wind from the west at 100 km/h. What is the resultant velocity of the airplane (relative to the ground)?

Let $\vec{v}$ represent the velocity of the airplane without the wind.

Let $\vec{w}$ represent the velocity of the wind.

Let $\vec{r}$ represent the resultant velocity of airplane with wind taken into account relative to a fixed point on the ground.

Magnitude of resultant (speed):	Direction:
$\|\vec{r}\|^2 = \|\vec{v}\|^2 + \|\vec{w}\|^2$ $\|\vec{r}\|^2 = (300)^2 + (100)^2$ $\|\vec{r}\|^2 = 100\,000$ $\|\vec{r}\| \approx 316.23$ km/h	$\tan\theta = \frac{100}{300}$ $\theta = \tan^{-1}\left(\frac{1}{3}\right)$ $\theta \approx 18.4°$



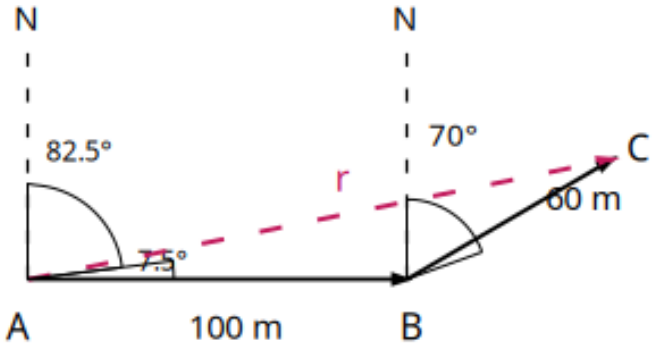
Therefore, the plane is heading $S18.4°E$ at a speed of 316.23 km/h.

Example 6: In an orienteering race, you walk 100 m due east and then walk $N70°E$ for 60 m. How far are you from

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your starting position, and at what bearing?

$$|\vec{r}|^2 = |\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 - 2|\overrightarrow{AB}||\overrightarrow{BC}|\cos(160)$$
$$|\vec{r}|^2 = (100)^2 + (60)^2 - 2(100)(60)\cos(160)$$
$$|\vec{r}| \approx 157.7 \text{ m}$$



$$\frac{157.7}{\sin 160} = \frac{60}{\sin A}$$
$$\sin A = \frac{60\sin 160}{157.7}$$
$$\angle A = \sin^{-1}\left(\frac{60\sin 160}{157.7}\right)$$
$$\angle A \approx 7.5°$$

You have travelled about 157.7 m at a quadrant bearing of $N82.5°E$.

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