3.6 Applications of Exponential and Trigonometric Derivatives Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

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1) A 100-mg sample of thorium-233 (Th-233) is placed into a nuclear reactor. After 10 min, the sample has decayed to 73 mg. Use the equation $N(t) = N_0 e^{-\lambda t}$ to answer the following questions:

a) Determine the disintegration constant λ for Th-233.

$$73 = 100e^{-10\lambda}$$
$$0.73 = e^{-10\lambda}$$
$$\ln(0.73) = -10\lambda$$
$$\lambda \cong 0.031$$

b) Determine the half-life of Th-233

$$50 = 100e^{-0.031t}$$
$$0.5 = e^{-0.031t}$$
$$\ln(0.5) = -0.031t$$
$$t \cong 22 \text{ minutes}$$

c) Write the equation that gives the amount of Th-233 remaining as a function of time, in terms of half-life.

$$N(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{22}}$$

d) How fast is the sample decaying after 5 min?

$$N'(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{22}} \ln\left(\frac{1}{2}\right)\left(\frac{1}{22}\right)$$
$$N'(5) \cong -2.7 \text{ mg/min}$$

2) Radon-222 (Rn-222) is a radioactive element that spontaneously decays into polonium-218 (Po-218) with a half-life of 3.8 days. The atoms of these two substances have approximately the same mass. Suppose that the initial sample of radon has a mass of 100 mg.

The mass of radon, in milligrams, as a function of time is given by the function $M_{Rn}(t) = M_0\left(\frac{1}{2}\right)^{\frac{t}{3.8}}$, where M_0 is the initial mass of radon and M_{Rn} is the mass of radon after time t, in days.

a) How much radon will remain after

i) 1 day?

$$M_{Rn}(1) = 100\left(\frac{1}{2}\right)^{\frac{1}{3.8}}$$
$$M_{Rn}(1) \cong 83.3 \text{ mg}$$

ii) 1 week?

$$M_{Rn}(7) = 100\left(\frac{1}{2}\right)^{\frac{7}{3.8}}$$
$$M_{Rn}(7) \cong 27.9 \text{ mg}$$

b) At what rate is the radon decaying at each of these times?

$$M(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{3.8}}$$
$$M'(t) = 100\left(\frac{1}{2}\right)^{\frac{t}{3.8}} \ln\left(\frac{1}{2}\right)\left(\frac{1}{3.8}\right)$$
$$M'(1) \cong -15.2 \text{ mg/day}$$
$$M'(7) \cong -5.1 \text{ mg/day}$$

c) How long will it take for a sample of radon to decay to 25% of its initial mass?

$$25 = 100\left(\frac{1}{2}\right)^{\frac{t}{3.8}}$$
$$0.25 = \left(\frac{1}{2}\right)^{\frac{t}{3.8}}$$
$$\log_{0.5}(0.25) = \frac{t}{3.8}$$
$$t \cong 7.6 \text{ days}$$

3) Consider a car shock absorber modelled by the equation $h(t) = e^{-0.5t}\sin t$, where $h(t)$ represents the vertical displace-

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ment, in meters, as a function of time, t, in seconds. Determine when the maximum vertical velocity, in m/s, occurs and its value, given that $v(t) = h'(t)$.

$$v(t) = h'(t) = -0.5e^{-0.5t}\sin t + \cos t(e^{-0.5t})$$
$$v(t) = -0.5e^{-0.5t}(\sin t - 2\cos t)$$
$$a(t) = v'(t) = 0.25e^{-0.5t}(\sin t - 2\cos t) + (\cos t + 2\sin t)(-0.5e^{-0.5t})$$
$$a(t) = 0.25e^{-0.5t}[\sin t - 2\cos t - 2(\cos t + 2\sin t)]$$
$$0 = 0.25e^{-0.5t}(-3\sin t - 4\cos t)$$
$$0 = -3\sin t - 4\cos t$$
$$4\cos t = -3\sin t$$
$$-\frac{4}{3} = \tan t$$
$$t \cong -0.927,\ 2.214,\ 5.356,\ \ldots$$

Test critical numbers (and $t = 0$, since $t \geq 0$):

$$v(0) = 1 \text{ m/s}$$
$$v(2.214) \cong 0.0687 \text{ m/s}$$
$$v(5.356) \cong -0.33 \text{ m/s}$$

Max velocity of 1 m/s at 0 seconds.

4) Rocco and Biff are two koala bears that are foraging for food together in a eucalyptus tree. Suddenly, a gust of wind causes Rocco to lose his grip and begin to fall. He quickly grabs a nearby vine and begins to swing away from the tree. Rocco's horizontal displacement as a function of time is given by the equation $x(t) = 5\cos\left(\frac{\pi t}{2}\right)e^{-0.1t}$ where x is Rocco's horizontal displacement from the bottom of his swing arc, in meters, at time t, in seconds.

a) Biff can grab Rocco if Rocco swings back to within 1 meter from where he started falling. Will Biff be able to rescue Rocco? Explain, using mathematical reasoning.

Initial position:

$$x(0) = 5\cos(0) \cong 5 \text{ m}$$

Period:

$$\text{Period} = \frac{2\pi}{\frac{\pi}{2}} = 4 \text{ seconds}$$

Position after 4 seconds:

$$x(4) = 5\cos(2\pi)e^{-0.4} \cong 3.35 \text{ m}$$

He is 1.65 m away from where he started. Biff will NOT be able to rescue Rocco.

b) The other option Rocco has is to let go of the vine at the bottom of one of the swing arcs and drop to the ground. But Rocco will only feel safe doing this if his horizontal velocity at the bottom of the swing is less than 2 m/s. Assuming that Biff is unable to save his friend, how many times must Rocco swing back and forth on the vine before he can safely drop to the ground?

$$x(t) = 5e^{-0.1t}\cos\left(\frac{\pi t}{2}\right)$$
$$v(t) = x'(t) = -0.5e^{-0.1t}\cos\left(\frac{\pi t}{2}\right) - \frac{\pi}{2}\sin\left(\frac{\pi t}{2}\right)(5)e^{-0.1t}$$

When is he at the bottom?

$$0 = \cos\left(\frac{\pi t}{2}\right)$$
$$\frac{\pi t}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$
$$t = 1,\ 3,\ 5,\ 7,\ \ldots$$
$$t = 1 + 2k;\ \{k \in \mathbb{Z} | k \geq 0\}$$

Evaluating the speed at each bottom-crossing (the cosine term vanishes there, leaving only the sine term):

$|v(1)| \cong 7.11$ m/s $|v(3)| \cong 5.82$ m/s $|v(5)| \cong 4.76$ m/s $|v(7)| \cong 3.90$ m/s
$|v(9)| \cong 3.19$ m/s $|v(11)| \cong 2.61$ m/s $|v(13)| \cong 2.14$ m/s $|v(15)| \cong 1.75$ m/s

Number of swings $= \frac{15}{4} = 3.75$ times

5) The voltage signal from a standard North American wall socket can be described by the equation $V(t) = 170\sin(120\pi t)$, where t is time, in seconds, and V is the voltage, in volts, at time t.

a) Find the max and min voltage levels and the times at which they occur.

$$\text{Max} = 0 + 170 = 170 \text{ V}$$
$$170 = 170\sin(120\pi t)$$
$$1 = \sin(120\pi t)$$
$$120\pi t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$$

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$$t = \frac{1}{240}, \frac{5}{240}, \frac{9}{240}, \ldots$$

A max voltage of 170 V when $t = \frac{1+4k}{240}, \{k \in \mathbb{Z} | k \geq 0\}$

$$\text{Min} = 0 - 170 = -170 \text{ V}$$
$$-170 = 170\sin(120\pi t)$$
$$-1 = \sin(120\pi t)$$
$$120\pi t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$$
$$t = \frac{3}{240}, \frac{7}{240}, \frac{11}{240}, \ldots$$

A min voltage of -170 V when $t = \frac{3+4k}{240}, \{k \in \mathbb{Z} | k \geq 0\}$

b) For the given signal, determine

i) the period, T, in seconds

$$T = \frac{2\pi}{|k|} = \frac{2\pi}{120\pi} = \frac{1}{60} \text{ seconds}$$

ii) the frequency, f, in hertz

$$f = \frac{1}{\text{Period}} = 60 \text{ Hz}$$

iii) the amplitude, A, in volts

$$A = |a| = 170 \text{ V}$$

6) Consider a simple pendulum that has a length of 50 cm and a max horizontal displacement of 8 cm.

a) Find the period of the pendulum.

$$T = 2\pi\sqrt{\frac{0.5}{9.8}} \cong 1.42 \text{ seconds}$$

b) Determine a function that gives the horizontal position of the bob as a function of time.

$$h(t) = 8\cos\left(\frac{2\pi t}{2\pi\sqrt{\frac{0.5}{9.8}}}\right)$$
$$h(t) = 8\cos(4.43t)$$

c) Determine a function that gives the velocity of the bob as a function of time.

$$v(t) = h'(t) = 8[-\sin(4.43t)](4.43)$$
$$v(t) = -35.44\sin(4.43t)$$

d) Determine a function that gives the acceleration of the bob as a function of time.

$$a(t) = v'(t) = -35.44[\cos(4.43t)](4.43)$$
$$a(t) = -157\cos(4.43t)$$

e) Find the max velocity of the bob and the time at which it first occurs.

$$\text{Max} = 0 + 35.44 = 35.44 \text{ cm/s}$$
$$35.44 = -35.44\sin(4.43t)$$
$$-1 = \sin(4.43t)$$
$$4.43t = \frac{3\pi}{2}$$
$$t \cong 1.06 \text{ seconds}$$

Max velocity: 35.4 cm/s at time $t = 1.06$ s

7) A piston in an engine oscillates up and down from a rest position. The motion of this piston can be approximated by the function $h(t) = 0.05\cos(13t)$, where t is time, in seconds, and h is the displacement of the piston head from rest position, in meters, at time t.

a) Determine an equation for the velocity of the piston head as a function of time.

$$v(t) = h'(t) = 0.05[-\sin(13t)](13)$$
$$v(t) = -0.65\sin(13t)$$

b) Find the max and min velocities and the times at which they occur.

$$\text{Max} = 0 + 0.65 = 0.65 \text{ m/s}$$
$$0.65 = -0.65\sin(13t)$$
$$-1 = \sin(13t)$$
$$13t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$$
$$t = \frac{3\pi}{26}, \frac{7\pi}{26}, \frac{11\pi}{26}, \ldots$$

Max velocity of 0.65 m/s when $t = \frac{(4k+3)\pi}{26} \{k \in \mathbb{Z} | k \geq 0\}$

$$\text{Min} = 0 - 0.65 = -0.65 \text{ m/s}$$

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$$\begin{aligned} -0.65 &= -0.65\sin(13t) \\ 1 &= \sin(13t) \\ 13t &= \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots \\ t &= \frac{\pi}{26}, \frac{5\pi}{26}, \frac{9\pi}{26}, \ldots \end{aligned}$$

Min velocity of -0.65 m/s when $t = \frac{(4k+1)\pi}{26}\{k \in \mathbb{Z} | k \geq 0\}$

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