3.5 Implicit Differentiation and Derivatives of Log Functions Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y.

a) $2x^3 = 2y^2 + 5$

$$6x^2 = 4y\frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

b) $5y^2 = 2x^3 - 5y$

$$10y\frac{dy}{dx} = 6x^2 - 5\frac{dy}{dx}$$
$$10y\frac{dy}{dx} + 5\frac{dy}{dx} = 6x^2$$
$$\frac{dy}{dx}(10y + 5) = 6x^2$$
$$\frac{dy}{dx} = \frac{6x^2}{10y + 5}$$

c) $5x^3 = -3xy + 2$

$$15x^2 = -3y + \frac{dy}{dx}(-3x)$$
$$\frac{15x^2 + 3y}{-3x} = \frac{dy}{dx}$$
$$\frac{dy}{dx} = \frac{-5x^2 - y}{x}$$

d) $2x^3 = (3xy + 1)^2$

$$6x^2 = 2(3xy + 1)\left[3y + \frac{dy}{dx}(3x)\right]$$
$$6x^2 = (6xy + 2)\left[3y + \frac{dy}{dx}(3x)\right]$$
$$6x^2 = 18xy^2 + 18x^2y\frac{dy}{dx} + 6y + 6x\frac{dy}{dx}$$
$$6x^2 - 18xy^2 - 6y = \frac{dy}{dx}(18x^2y + 6x)$$
$$\frac{dy}{dx} = \frac{x^2 - 3xy^2 - y}{3x^2y + x}$$

e) $x^3 - 3x^2y + 4xy^2 = 12$

$$3x^2 - 6xy + \frac{dy}{dx}(-3x^2) + 4y^2 + 2y\frac{dy}{dx}(4x) = 0$$
$$\frac{dy}{dx}(8xy - 3x^2) = -3x^2 + 6xy - 4y^2$$
$$\frac{dy}{dx} = \frac{-3x^2 + 6xy - 4y^2}{8xy - 3x^2}$$

f) $4\sin(2y)\cos x = 2$

$$4\cos(2y)\left(2\frac{dy}{dx}\right)\cos x + (-\sin x)[4\sin(2y)] = 0$$
$$8\cos(2y)\cos x\frac{dy}{dx} = 4\sin x\sin(2y)$$
$$\frac{dy}{dx} = \frac{4\sin x\sin(2y)}{8\cos(2y)\cos x}$$
$$\frac{dy}{dx} = \frac{\sin x\sin(2y)}{2\cos x\cos(2y)}$$

g) $y^2 = \frac{x^2 - 4}{x^2 + 4}$

$$2y\frac{dy}{dx} = \frac{2x(x^2 + 4) - 2x(x^2 - 4)}{(x^2 + 4)^2}$$
$$2y\frac{dy}{dx} = \frac{2x(x^2 + 4 - x^2 + 4)}{(x^2 + 4)^2}$$
$$2y\frac{dy}{dx} = \frac{16x}{(x^2 + 4)^2}$$
$$\frac{dy}{dx} = \frac{8x}{y(x^2 + 4)^2}$$

2) Find the equation of the tangent line to $(x + y)^3 = x^3 + y^3$ at the point $(-1, 1)$.

Slope:

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$$3(x+y)^2\left(1+\frac{dy}{dx}\right)=3x^2+3y^2\frac{dy}{dx}$$
$$3(x^2+2xy+y^2)\left(1+\frac{dy}{dx}\right)=3x^2+3y^2\frac{dy}{dx}$$
$$3x^2+6xy+3y^2+(3x^2+6xy+3y^2)\frac{dy}{dx}=3x^2+3y^2\frac{dy}{dx}$$
$$3x^2\frac{dy}{dx}+6xy\frac{dy}{dx}=-6xy-3y^2$$
$$\frac{dy}{dx}(3x^2+6xy)=-3y(2x+y)$$
$$\frac{dy}{dx}=\frac{-3y(2x+y)}{3x(x+2y)}$$
$$\frac{dy}{dx}=\frac{-y(2x+y)}{x(x+2y)}$$
$$\left.\frac{dy}{dx}\right|_{\substack{x=-1\\y=1}}=\frac{-1[2(-1)+1]}{-1[-1+2(1)]}$$
$$\left.\frac{dy}{dx}\right|_{\substack{x=-1\\y=1}}=\frac{-1(-1)}{-1(1)}=\frac{1}{-1}=-1$$

Equation:

$$y=mx+b$$
$$1=-1(-1)+b$$
$$b=0$$

y = -x

3) Differentiate each of the following with respect to x.

a) $y=\dfrac{\ln x}{2x+3}$

$$y'=\frac{\frac{1}{x}(2x+3)-2\ln x}{(2x+3)^2}$$
$$y'=\frac{2x+3-2x\ln x}{x(2x+3)^2}$$

b) $f(x)=e^{x^7}$

$$f'(x)=e^{x^7}(7x^6)$$
$$f'(x)=7x^6e^{x^7}$$

c) $f(x)=\ln\left(\dfrac{x^2+1}{x^3-x}\right)$

Use the log rule $\ln\left(\frac{a}{b}\right)=\ln a-\ln b$:

$$f(x)=\ln(x^2+1)-\ln(x^3-x)$$
$$f'(x)=\frac{2x}{x^2+1}-\frac{3x^2-1}{x^3-x}$$

d) $y=\log_2(4x^2)$

$$y'=\frac{1}{4x^2\ln(2)}(8x)$$
$$y'=\frac{2}{x\ln(2)}$$

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