3.4 Differentiation Rules for Exponential Functions Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1)

a) Rewrite the function $y = b^x$ with base e.

$$y = e^{\ln(b^x)}$$
$$y = e^{x \ln b}$$

b) Find the derivative of your function in part a) and simplify.

$$y' = e^{x \ln b}[\ln b]$$
$$y' = e^{\ln(b^x)}[\ln b]$$
$$y' = b^x \ln b$$

2) Differentiate with respect to x.

a) $y = e^{-3x}$

$$y' = e^{-3x}(-3)$$
$$y' = -3e^{-3x}$$

b) $f(x) = e^{4x-5}$

$$f'(x) = e^{4x-5}(4)$$
$$f'(x) = 4e^{4x-5}$$

c) $y = e^{2x} - e^{-2x}$

$$y' = e^{2x}(2) - e^{-2x}(-2)$$
$$y' = 2e^{2x} + 2e^{-2x}$$

d) $y = 2^x + 3^x$

$$y' = 2^x \ln(2) + 3^x \ln(3)$$

e) $f(x) = 3e^{2x} - 2^{3x}$

$$f'(x) = 3e^{2x}(2) - 2^{3x} \ln(2)(3)$$
$$f'(x) = 6e^{2x} - 3\ln(2)(2^{3x})$$

f) $y = 4xe^x$

$$y' = 4(e^x) + e^x(4x)$$
$$y' = 4e^x + 4xe^x$$

g) $y = 5^x e^{-x}$

$$y' = 5^x(\ln 5)(e^{-x}) + (e^{-x})(-1)(5^x)$$
$$y' = 5^x(\ln 5)(e^{-x}) - e^{-x}(5^x)$$
$$y' = 5^x(e^{-x})(\ln 5 - 1)$$

h) $f(x) = xe^{2x} + 2e^{-3x}$

$$f'(x) = 1(e^{2x}) + (e^{2x})(2)x - 6e^{-3x}$$
$$f'(x) = e^{2x} + 2xe^{2x} - 6e^{-3x}$$
$$f'(x) = e^{2x}(1 + 2x - 6e^{-5x})$$

3) Determine the derivative with respect to x for each function.

a) $y = e^{-x} \sin x$

$$y' = -e^{-x} \sin x + \cos x(e^{-x})$$
$$y' = e^{-x}(-\sin x + \cos x)$$
$$y' = e^{-x}(\cos x - \sin x)$$

b) $y = e^{\cos x}$

$$y' = e^{\cos x}(-\sin x)$$
$$y' = -\sin x(e^{\cos x})$$

c) $f(x) = e^{2x}(x^2 - 3x + 2)$

$$f'(x) = 2e^{2x}(x^2 - 3x + 2) + (2x - 3)(e^{2x})$$
$$f'(x) = e^{2x}[2x^2 - 6x + 4 + 2x - 3]$$
$$f'(x) = e^{2x}(2x^2 - 4x + 1)$$

Copyright © 2026 Annie Bernatchez—All rights reserved.

d) $g(x) = 2x^2 e^{\cos(2x)}$

$$g'(x) = 4x[e^{\cos(2x)}] + e^{\cos(2x)}[-\sin(2x)](2)(2x^2)$$
$$g'(x) = 4xe^{\cos(2x)}[1 - x\sin(2x)]$$

4) Identify the coordinates of any local extrema of the function $y = e^x - e^{2x}$

$$y' = e^x - 2e^{2x}$$
$$0 = e^x(1 - 2e^x)$$

$e^x \neq 0$ so $1 - 2e^x = 0$

$$1 = 2e^x$$
$$\frac{1}{2} = e^x$$
$$x = \ln\left(\frac{1}{2}\right)$$

First Derivative Test:

		$\ln\left(\frac{1}{2}\right)$
Test Value	-10	10
$y' = e^x - 2e^{2x}$	+	−
$y = e^x - e^{2x}$	Increasing	decreasing

Local max at $\left(\ln\left(\frac{1}{2}\right), \frac{1}{4}\right)$

5) Find an equation for the tangent to the curve $y = 2e^{2x} + 2x + 1$ when $x = 0$.

Point:

$$y(0) = 2e^{2(0)} + 2(0) + 1$$
$$y(0) = 2 + 0 + 1$$
$$y(0) = 3$$
$$(0,\ 3)$$

Slope:

$$y' = 4e^{2x} + 2$$
$$y'(0) = 4e^{2(0)} + 2$$
$$y'(0) = 4 + 2$$
$$y'(0) = 6$$
$$m = 6$$

Equation:

$$y = mx + b$$
$$3 = 6(0) + b$$
$$b = 3$$

y = 6x + 3

6) Find the equation of the tangent to $y = x\ln x$ that is parallel to $y = 3x + 7$.

$m = 3$

$$y' = (1)\ln(x) + \frac{1}{x}\ln e$$
$$y' = \ln(x) + 1$$
$$3 = \ln(x) + 1$$
$$2 = \ln(x)$$
$$x = e^2$$

Point:

$$y(e^2) = e^2\ln(e^2)$$
$$y(e^2) = 2e^2$$
$$(e^2,\ 2e^2)$$

Equation:

$$y = mx + b$$
$$2e^2 = 3(e^2) + b$$
$$2e^2 - 3e^2 = b$$
$$b = -e^2$$

y = 3x - e^2

Copyright © 2026 Annie Bernatchez—All rights reserved.

7) Find all local extrema for $y = \frac{1}{2}x(2)^{3x+1}$.

$$y' = \frac{1}{2}(2)^{3x+1} + \ln(2)(2)^{3x+1}(3)\left(\frac{1}{2}x\right)$$
$$y' = \frac{1}{2}(2)^{3x+1}[1 + 3x\ln(2)]$$
$$0 = \frac{1}{2}(2)^{3x+1}[1 + 3x\ln(2)]$$

$0 = 2^{3x+1}$ has no solution

$$0 = 1 + 3x\ln(2)$$
$$x = \frac{-1}{3\ln(2)}$$
$$x \cong -0.48$$
$$y(-0.48) \cong -0.18$$

1st derivative test:

	-0.48	
$-$		$+$
decreasing		increasing

Min at $(-0.48, -0.18)$

8) Continuous growth or decay follows the formula $A = ce^{kt}$, where c is the initial amount, and k is a rate factor. The mass of a radioactive substance is 1000 g on day 1, and only 100 g after 100 days. Find:

a) k, then write the equation with c and k

$$100 = 1000e^{k(100)}$$
$$\frac{1}{10} = e^{100k}$$
$$\ln\left(\frac{1}{10}\right) = 100k$$
$$k = \frac{\ln\left(\frac{1}{10}\right)}{100}$$
$$k \cong -0.023$$
$$A(t) = 1000e^{-0.023t}$$

b) the half-life,

$$500 = 1000e^{-0.023t}$$
$$\frac{1}{2} = e^{-0.023t}$$
$$\ln\left(\frac{1}{2}\right) = -0.023t$$
$$t \cong 30.14 \text{ days}$$

c) the amount that remains after 300 days, and

$$A(300) = 1000e^{-0.023(300)}$$
$$A(300) \cong 1 \text{ gram}$$

d) the rate of decay after 50 days.

$$A'(t) = 1000e^{-0.023t}(-0.023)$$
$$A'(t) = -23e^{-0.023t}$$
$$A'(50) = -23e^{-0.023(50)}$$
$$A'(50) \cong -7.28 \text{ g/day}$$

Copyright © 2026 Annie Bernatchez—All rights reserved.