3.3 Derivatives of Exponential Functions Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine the derivative with respect to x for each function.

a) $g(x) = 4^x$

$$g'(x) = 4^x(\ln 4)$$

b) $f(x) = 11^x$

$$f'(x) = 11^x(\ln 11)$$

c) $y = \left(\frac{1}{2}\right)^x$

$$y' = \left(\frac{1}{2}\right)^x\left[\ln\left(\frac{1}{2}\right)\right]$$

d) $N(x) = -3e^x$

$$N'(x) = -3e^x$$

e) $h(x) = e^x$

$$h'(x) = e^x$$

f) $y = \pi^x$

$$y' = \pi^x[\ln(\pi)]$$

2) Find the first, second, and third derivatives of the function $f(x) = e^x$

$$f'(x) = f''(x) = f'''(x) = e^x$$

3) Calculate the instantaneous rate of change of the function $y = 5^x$ when $x = 2$.

$$y' = 5^x[\ln(5)]$$
$$y'(2) = 5^2[\ln(5)]$$
$$y'(2) \cong 40.2$$

4) Determine the slope of the graph of $y = \frac{1}{2}e^x$ at $x = 4$.

$$y' = \frac{1}{2}e^x$$
$$y'(4) = \frac{1}{2}e^4$$
$$y'(4) \cong 27.3$$

5) Determine the equation of the line that is tangent to $y = 8^x$ at the point on the curve where $x = \frac{1}{2}$.

Point:

$$y(0.5) = 8^{\frac{1}{2}}$$
$$y(0.5) = 2\sqrt{2}$$
$$(0.5,\ 2\sqrt{2})$$

Slope:

$$y' = 8^x[\ln(8)]$$
$$y'(0.5) = 8^{\frac{1}{2}}[\ln(8)]$$
$$y'(0.5) = 2\sqrt{2}[\ln(8)]$$
$$m = 2\sqrt{2}[\ln(8)]$$

Equation:

$$y = mx + b$$
$$2\sqrt{2} = 2\sqrt{2}\ln(8)\left(\frac{1}{2}\right) + b$$
$$2\sqrt{2} = \sqrt{2}\ln(8) + b$$
$$b = 2\sqrt{2} - \sqrt{2}\ln(8)$$
$$b = \sqrt{2}[2 - \ln(8)]$$
$$y = 2\sqrt{2}\ln(8)x + \sqrt{2}[2 - \ln(8)]$$

6) A fruit fly infestation is doubling every day. There are 10 flies when the infestation is first discovered.

a) Write an equation that relates the number of flies to time.

$$A(t) = 10(2)^t$$

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b) Determine the number of flies present after 1 week.

$$A(7) = 10(2)^7$$
$$A(7) = 1280 \text{ flies}$$

c) How fast is the fly population increasing after 1 week.

$$A'(t) = 10(2)^t \ln(2)$$
$$A'(7) = 10(2)^7 \ln(2)$$
$$A'(7) = 887.2 \text{ flies/day}$$

d) How long will it take for the fly population to reach 500?

$$500 = 10(2)^t$$
$$50 = 2^t$$
$$t = \log_2(50)$$
$$t \cong 5.64 \text{ days}$$

e) How fast is the fly population increasing at this point?

$$A'(5.64) = 10(2)^{5.64} \ln(2)$$
$$A'(5.64) \cong 346 \text{ flies/day}$$

7) Refer to question 6. At which point is the fly population increasing at a rate of

i) 20 flies per day?

$$20 = 10(2)^t \ln(2)$$
$$\frac{2}{\ln(2)} = 2^t$$
$$\log_2\left(\frac{2}{\ln(2)}\right) = t$$
$$t \cong 1.53 \text{ days}$$

ii) 2000 flies per day?

$$2000 = 10(2)^t \ln(2)$$
$$\frac{200}{\ln(2)} = 2^t$$
$$t = \log_2\left(\frac{200}{\ln(2)}\right)$$
$$t \cong 8.17 \text{ days}$$

8) Determine the equation of the line perpendicular to the tangent line to the function $f(x) = \frac{1}{2}e^x$ at the point on the curve where $x = \ln 3$

Point:

$$f(\ln 3) = \frac{1}{2}e^{\ln 3}$$
$$f(\ln 3) = \frac{1}{2}(3)$$
$$f(\ln 3) = \frac{3}{2}$$
$$(\ln 3,\ 1.5)$$

Slope:

$$f'(x) = \frac{1}{2}e^x$$
$$f'(\ln 3) = \frac{3}{2}$$
$$m = \frac{3}{2}$$
$$\perp m = -\frac{2}{3}$$

Equation:

$$y = mx + b$$
$$\frac{3}{2} = -\frac{2}{3}\ln 3 + b$$
$$\frac{3}{2} + \frac{2}{3}\ln 3 = b$$
$$y = -\frac{2}{3}x + \frac{3}{2} + \frac{2}{3}\ln 3$$

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