3.2 More Derivatives of Sine and Cosine Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine the derivative of each function.

a) $y = \sin(4x)$

$$y' = \cos(4x)(4)$$
$$y' = 4\cos(4x)$$

b) $f(x) = \sin(2x + \pi)$

$$f'(x) = \cos(2x + \pi)(2)$$
$$f'(x) = 2\cos(2x + \pi)$$

c) $y = -2\sin(3\theta)$

$$y' = -2\cos(3\theta)(3)$$
$$y' = -6\cos(3\theta)$$

d) $y = \sin^2 x$

$$y = (\sin x)^2$$
$$y' = 2(\sin x)(\cos x)$$
$$y' = 2\sin x\cos x$$

e) $f(x) = \cos^2 x - \sin^2 x$

$$f(x) = (\cos x)^2 - (\sin x)^2$$
$$f'(x) = 2\cos x(-\sin x) - 2\sin x(\cos x)$$
$$f'(x) = -2\cos x\sin x - 2\sin x\cos x$$
$$f'(x) = -2(\cos x\sin x + \sin x\cos x)$$
$$f'(x) = -2(2\sin x\cos x)$$
$$f'(x) = -2\sin(2x)$$

f) $y = 3\sin^2(2t - 4) - 2\cos^2(3t + 1)$

$$y = 3[\sin(2t - 4)]^2 - 2[\cos(3t + 1)]^2$$
$$y' = 3(2)[\sin(2t - 4)]\cos(2t - 4)(2) - 2(2)[\cos(3t + 1)][-\sin(3t + 1)](3)$$
$$y' = 12\sin(2t - 4)\cos(2t - 4) + 12\cos(3t + 1)\sin(3t + 1)$$
$$y' = 6\sin[2(2t - 4)] + 6\sin[2(3t + 1)]$$
$$y' = 6\sin(4t - 8) + 6\sin(6t + 2)$$

g) $f(t) = \sin^2(\cos t)$

$$f(t) = [\sin(\cos t)]^2$$
$$f'(t) = 2[\sin(\cos t)][\cos(\cos t)](-\sin t)$$
$$f'(t) = -2\sin(t)\sin(\cos t)\cos(\cos t)$$

h) $f(x) = -x^2\sin(3x - \pi)$

$$f'(x) = -2x\sin(3x - \pi) + [\cos(3x - \pi)](3)(-x^2)$$
$$f'(x) = -2x\sin(3x - \pi) - 3x^2\cos(3x - \pi)$$

i) $f(\theta) = \sin^2\theta\cos^2\theta$

$$f(\theta) = (\sin\theta)^2(\cos\theta)^2$$
$$f'(\theta) = 2\sin\theta\cos\theta\cos^2\theta + 2\cos\theta(-\sin\theta)\sin^2\theta$$
$$f'(\theta) = 2\sin\theta\cos^3\theta - 2\cos\theta\sin^3\theta$$

j) $y = x^{-1}\cos^2 x$

$$y = x^{-1}(\cos x)^2$$
$$y' = -1x^{-2}(\cos^2 x) + 2\cos x(-\sin x)x^{-1}$$
$$y' = -\frac{\cos^2 x}{x^2} - \frac{2\cos x\sin x}{x}$$

k) $y = 2\tan x - \tan(2x)$

$$y' = 2\sec^2 x - \sec^2(2x)(2)$$
$$y' = 2\sec^2 x - 2\sec^2(2x)$$

l) $y = (\tan x + \cos x)^2$

$$y' = 2(\tan x + \cos x)(\sec^2 x - \sin x)$$

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2) Find the slope of the function $y = 2\cos x \sin(2x)$ at $x = \frac{\pi}{2}$.

$$y' = -2\sin x[\sin(2x)] + \cos(2x)(2)(2\cos x)$$
$$y' = -2\sin x[\sin(2x)] + 4\cos(2x)\cos x$$
$$y'\left(\frac{\pi}{2}\right) = -2\sin\left(\frac{\pi}{2}\right)[\sin(\pi)] + 4\cos(\pi)\cos\left(\frac{\pi}{2}\right)$$
$$y'\left(\frac{\pi}{2}\right) = -2(1)(0) + 4(-1)(0)$$
$$y'\left(\frac{\pi}{2}\right) = 0$$

$m = 0$

3) Find the equation of the line that is tangent to $y = x^2\sin(2x)$ at $x = -\pi$.

Slope:

$$y' = 2x\sin(2x) + \cos(2x)(2)x^2$$
$$y' = 2x\sin(2x) + 2x^2\cos(2x)$$
$$y'(-\pi) = 2(-\pi)\sin(-2\pi) + 2(-\pi)^2\cos(-2\pi)$$
$$y'(-\pi) = -2\pi(0) + 2\pi^2(1)$$
$$y'(-\pi) = 2\pi^2$$

Point:

$$y(-\pi) = (-\pi)^2\sin(-2\pi)$$
$$y(-\pi) = 0$$
$$(-\pi, 0)$$

Equation:

$$y = mx + b$$
$$0 = 2\pi^2(-\pi) + b$$
$$b = 2\pi^3$$
$$y = 2\pi^2x + 2\pi^3$$

4) Determine $\frac{d^2y}{dx^2}$ for $y = x^2\cos x$.

$$\frac{dy}{dx} = 2x\cos x + (-\sin x)(x^2)$$
$$\frac{dy}{dx} = 2x\cos x - x^2\sin x$$
$$\frac{d^2y}{dx^2} = 2\cos x + (-\sin x)(2x) - [2x\sin x + \cos x(x^2)]$$
$$\frac{d^2y}{dx^2} = 2\cos x - 2x\sin x - 2x\sin x - x^2\cos x$$
$$\frac{d^2y}{dx^2} = 2\cos x - 4x\sin x - x^2\cos x$$

5)

a) Write $y = \csc x$ in terms of $\sin x$ as a reciprocal function.

$$y = \frac{1}{\sin x}$$

b) Write the function in terms of a negative power of $\sin x$

$$y = (\sin x)^{-1}$$

c) Use the power rule and chain rule to find the derivative of $y = \csc x$

$$y' = -1(\sin x)^{-2}(\cos x)$$
$$y' = -\frac{\cos x}{\sin^2 x}$$
$$y' = -\frac{\cos x}{(\sin x)(\sin x)}$$
$$y' = -\cot x\left(\frac{1}{\sin x}\right)$$
$$\frac{dy}{dx} = -\csc x\cot x$$

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