3.1 Derivatives of Sine and Cosine Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Find the derivative with respect to x for each function.

a) $y = 4\sin x$

$$\frac{dy}{dx} = 4\cos x$$

b) $f(x) = -3\cos x$

$$f'(x) = -3(-\sin x)$$
$$f'(x) = 3\sin x$$

c) $y = \cos x - \sin x$

$$\frac{dy}{dx} = -\sin x - \cos x$$

d) $y = x^2 - 3\sin x$

$$\frac{dy}{dx} = 2x - 3\cos x$$

e) $y = \cos x + 7\pi\sin x - 3x$

$$\frac{dy}{dx} = -\sin x + 7\pi\cos x - 3$$

f) $f(x) = \frac{\pi}{4}\cos x - \frac{\pi}{3}\sin x$

$$f'(x) = \frac{\pi}{4}(-\sin x) - \frac{\pi}{3}\cos x$$
$$f'(x) = -\frac{\pi}{4}\sin x - \frac{\pi}{3}\cos x$$

2) Find the equation of the line that is tangent to the function $y = \cos x$ and passes through the point $\left(\frac{\pi}{3}, \frac{1}{2}\right)$.

Slope:

$$y' = -\sin x$$
$$y'\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right)$$
$$y'\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$
$$m = -\frac{\sqrt{3}}{2}$$

Equation:

$$y = mx + b$$
$$\frac{1}{2} = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\pi}{3}\right) + b$$
$$\frac{1}{2} = -\frac{\sqrt{3}\pi}{6} + b$$
$$b = \frac{3}{6} + \frac{\sqrt{3}\pi}{6} = \frac{3+\sqrt{3}\pi}{6}$$
$$y = -\frac{\sqrt{3}}{2}x + \frac{3+\sqrt{3}\pi}{6}$$

3) Find the equation of the line that is tangent to the function $y = -4\sin x$ at $x = \frac{\pi}{4}$.

Point:

$$y\left(\frac{\pi}{4}\right) = -4\sin\left(\frac{\pi}{4}\right)$$
$$y\left(\frac{\pi}{4}\right) = -4\left(\frac{\sqrt{2}}{2}\right)$$
$$y\left(\frac{\pi}{4}\right) = -2\sqrt{2}$$
$$\left(\frac{\pi}{4}, -2\sqrt{2}\right)$$

Slope:

$$y' = -4\cos x$$
$$y'\left(\frac{\pi}{4}\right) = -4\cos\left(\frac{\pi}{4}\right)$$
$$y'\left(\frac{\pi}{4}\right) = -4\left(\frac{\sqrt{2}}{2}\right)$$
$$y'\left(\frac{\pi}{4}\right) = -2\sqrt{2}$$
$$m = -2\sqrt{2}$$

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Equation:

$$y = mx + b$$
$$-2\sqrt{2} = -2\sqrt{2}\left(\frac{\pi}{4}\right) + b$$
$$-2\sqrt{2} = -\frac{\pi\sqrt{2}}{2} + b$$
$$b = -2\sqrt{2} + \frac{\pi\sqrt{2}}{2}$$
$$y = -2\sqrt{2}x + \frac{\pi\sqrt{2}}{2} - 2\sqrt{2}$$

4) Determine an equation for the tangent to the function $f(x) = \tan x$ at $x = \frac{\pi}{4}$.

Point:

$$f\left(\frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right)$$
$$f\left(\frac{\pi}{4}\right) = 1$$
$$\left(\frac{\pi}{4}, 1\right)$$

Slope:

$$f'(x) = \sec^2 x$$
$$f'\left(\frac{\pi}{4}\right) = \sec^2\left(\frac{\pi}{4}\right)$$
$$f'\left(\frac{\pi}{4}\right) = \frac{1}{\cos^2\left(\frac{\pi}{4}\right)}$$
$$f'\left(\frac{\pi}{4}\right) = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2}$$
$$m = 2$$

Equation:

$$y = mx + b$$
$$1 = 2\left(\frac{\pi}{4}\right) + b$$
$$b = 1 - \frac{\pi}{2}$$
$$y = 2x + 1 - \frac{\pi}{2}$$

5) Find an equation of a line that is tangent to $y = 2\sin x$ and whose slope is a max value.

$y = 2\sin x$ has a max slope when $y' = 2\cos x$ is a max.

$$2 = 2\cos x$$
$$1 = \cos x$$
$$x = 0,\ 2\pi,\ 4\pi,\ 6\pi,\ \ldots$$

Max when $x = 2k\pi,\ k \in \mathbb{Z}$ (use any answer).

Point (using $x = 0$):

$$y(0) = 2\sin(0)$$
$$y(0) = 0$$
$$(0,\ 0)$$

Slope:

$$y'(0) = 2\cos(0)$$
$$y'(0) = 2(1)$$
$$y'(0) = 2$$

Equation:

$$y = mx + b$$
$$0 = 2(0) + b$$
$$b = 0$$
$$y = 2x$$

Note: there are an infinite number of solutions. The slope is at a max value at any $x = 2k\pi$ where $k \in \mathbb{Z}$. Depending on which x value you choose, you will get a different y-intercept.

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