3.7 Unit 3 Pretest – Derivatives of Trig and Exponential Functions with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

This review covers all of Unit 3: Derivatives of Trig and Exponential Functions, and is meant to be used to prepare for the unit test.

Section 1: 3.1 – Derivatives of Sine and Cosine

1) Find the derivatives of each of the following functions

a) $y = \cos x$

$$\frac{dy}{dx} = -\sin x$$

b) $f(x) = -2\sin x$

$$f'(x) = -2\cos x$$

c) $y = \cos x - \sin x$

$$y' = -\sin x - \cos x$$

Section 2: 3.1 – Tangent Lines to Trig Functions

2) Determine the slope of the function $y = 4\sin x$ at $x = \frac{\pi}{3}$.

$$y' = 4\cos x$$
$$y'\left(\frac{\pi}{3}\right) = 4\cos\left(\frac{\pi}{3}\right)$$
$$y'\left(\frac{\pi}{3}\right) = 4\left(\frac{1}{2}\right)$$
$$y'\left(\frac{\pi}{3}\right) = 2$$

3) Find the equation of the line that is tangent to the curve $y = 2\sin\theta + 4\cos\theta$ at $\theta = \frac{\pi}{4}$.

Point:

$$y\left(\frac{\pi}{4}\right) = 2\sin\left(\frac{\pi}{4}\right) + 4\cos\left(\frac{\pi}{4}\right)$$
$$y\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2}\right) + 4\left(\frac{\sqrt{2}}{2}\right)$$
$$y\left(\frac{\pi}{4}\right) = \sqrt{2} + 2\sqrt{2}$$
$$y\left(\frac{\pi}{4}\right) = 3\sqrt{2}$$
$$\left(\frac{\pi}{4}, 3\sqrt{2}\right)$$

Slope:

$$y' = 2\cos\theta - 4\sin\theta$$
$$y'\left(\frac{\pi}{4}\right) = 2\cos\left(\frac{\pi}{4}\right) - 4\sin\left(\frac{\pi}{4}\right)$$
$$y'\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2}\right) - 4\left(\frac{\sqrt{2}}{2}\right)$$
$$y'\left(\frac{\pi}{4}\right) = \sqrt{2} - 2\sqrt{2}$$
$$y'\left(\frac{\pi}{4}\right) = -\sqrt{2}$$
$$m = -\sqrt{2}$$

Equation:

$$y = m\theta + b$$
$$3\sqrt{2} = -\sqrt{2}\left(\frac{\pi}{4}\right) + b$$
$$b = 3\sqrt{2} + \frac{\pi}{4}\sqrt{2}$$
$$y = -\sqrt{2}\theta + 3\sqrt{2} + \frac{\pi}{4}\sqrt{2}$$

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Section 3: 3.2 – More Derivatives of Sine and Cosine

4) Differentiate each of the following:

a) $y = -\cos^2 x$

$$y = -(\cos x)^2$$
$$y' = -2\cos x(-\sin x)$$
$$y' = 2\sin x\cos x$$
$$y' = \sin(2x)$$

b) $y = \sin(2\theta) - 2\cos(2\theta)$

$$y' = [\cos(2\theta)](2) - 2[-\sin(2\theta)](2)$$
$$y' = 2\cos(2\theta) + 4\sin(2\theta)$$

c) $f(\theta) = -\frac{\pi}{2}\sin(2\theta - \pi)$

$$f'(\theta) = -\frac{\pi}{2}[\cos(2\theta - \pi)](2)$$
$$f'(\theta) = -\pi\cos(2\theta - \pi)$$

d) $f(x) = \sin(\sin x)$

$$f'(x) = [\cos(\sin x)](\cos x)$$

e) $f(x) = \cos(\cos x)$

$$f'(x) = [-\sin(\cos x)](-\sin x)$$
$$f'(x) = [\sin(\cos x)](\sin x)$$

f) $f(\theta) = \cos(7\theta) - \cos(5\theta)$

$$f'(\theta) = [-\sin(7\theta)](7) - [-\sin(5\theta)](5)$$
$$f'(\theta) = -7\sin(7\theta) + 5\sin(5\theta)$$

g) $y = 3x\sin x$

$$y' = 3\sin x + \cos x(3x)$$
$$y' = 3\sin x + 3x\cos x$$

h) $f(t) = 2t^2\cos(2t)$

$$f'(t) = 4t\cos(2t) + [-\sin(2t)](2)(2t^2)$$
$$f'(t) = 4t\cos(2t) - 4t^2\sin(2t)$$

i) $y = \pi t\sin(\pi t - 6)$

$$y' = \pi\sin(\pi t - 6) + [\cos(\pi t - 6)](\pi)(\pi t)$$
$$y' = \pi\sin(\pi t - 6) + \pi^2 t\cos(\pi t - 6)$$

j) $y = \cos(\sin\theta) + \sin(\cos\theta)$

$$y' = [-\sin(\sin\theta)](\cos\theta) + [\cos(\cos\theta)](-\sin\theta)$$
$$y' = -\sin(\sin\theta)\cos\theta - \cos(\cos\theta)\sin\theta$$

k) $f(x) = \cos^2(\sin x)$

$$f(x) = [\cos(\sin x)]^2$$
$$f'(x) = 2[\cos(\sin x)][-\sin(\sin x)](\cos x)$$
$$f'(x) = -2\cos x[\cos(\sin x)][\sin(\sin x)]$$

l) $f(\theta) = \cos(7\theta) - \cos^2(5\theta)$

$$f'(\theta) = [-\sin(7\theta)](7) - 2\cos(5\theta)[-\sin(5\theta)](5)$$
$$f'(\theta) = -7\sin(7\theta) + 10\cos(5\theta)\sin(5\theta)$$

m) $y = 3\sin(2x) - 4\cos(2x)$

$$y' = 3[\cos(2x)](2) - 4[-\sin(2x)](2)$$
$$y' = 6\cos(2x) + 8\sin(2x)$$

n) $y = \tan(3x)$

$$y' = [\sec^2(3x)](3)$$
$$y' = 3\sec^2(3x)$$

o) $y = \frac{1}{2 - \cos x}$

$$y = (2 - \cos x)^{-1}$$
$$y' = -1(2 - \cos x)^{-2}(\sin x)$$

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$$y' = \frac{-\sin x}{(2-\cos x)^2}$$

p) $y = x\tan(2x)$

$$y' = 1\tan(2x) + [\sec^2(2x)](2)(x)$$
$$y' = \tan(2x) + 2x\sec^2(2x)$$

q) $y = [\sin(2x)]e^{3x}$

$$y' = 2\cos(2x)e^{3x} + 3e^{3x}[\sin(2x)]$$
$$y' = e^{3x}[2\cos(2x) + 3\sin(2x)]$$

r) $y = \cos^2(2x)$

$$y = [\cos(2x)]^2$$
$$y' = 2[\cos(2x)][-\sin(2x)](2)$$
$$y' = -4\cos(2x)\sin(2x)$$

Section 4: 3.2 – Tangent Lines (Maximum Slope)

5) Find an equation of a line that is tangent to the curve $f(x) = 2\cos(3x)$ and whose slope is a maximum.

$$f'(x) = 2[-\sin(3x)](3)$$
$$f'(x) = -6\sin(3x)$$

Max slope is 6. When?

$$6 = -6\sin(3x)$$
$$-1 = \sin(3x)$$
$$3x = \frac{3\pi}{2}$$
$$x = \frac{\pi}{2}$$

Point:

$$f\left(\frac{\pi}{2}\right) = 2\cos\left(\frac{3\pi}{2}\right)$$
$$f\left(\frac{\pi}{2}\right) = 0$$
$$\left(\frac{\pi}{2}, 0\right)$$

Equation:

$$y = mx + b$$
$$0 = (6)\left(\frac{\pi}{2}\right) + b$$
$$b = -3\pi$$

y = 6x - 3π (answers may vary)

Section 5: 3.6 – Applications: AC Voltage Signal

6) A voltage of the power supply in an AC-DC coupled circuit is given by the function $V(t) = 130\sin(5t) + 18$, where t is time, in seconds, and V is the voltage, in volts, at time t.

a) Find the maximum and minimum voltages and the times at which they occur.

$$\text{Max} = 18 + 130 = 148$$
$$148 = 130\sin(5t) + 18$$
$$1 = \sin(5t)$$
$$5t = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$$
$$t = \frac{\pi}{10}, \frac{5\pi}{10}, \frac{9\pi}{10}, \ldots$$

Max of 148 V when $t = \frac{\pi + 4\pi k}{10}$, $k \in \mathbb{Z}$, $k \geq 0$

$$\text{Min} = 18 - 130 = -112\ \text{V}$$
$$-112 = 130\sin(5t) + 18$$
$$-1 = \sin(5t)$$
$$5t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$$
$$t = \frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}, \ldots$$

Min of −112 V when $t = \frac{3\pi + 4\pi k}{10}$, $k \in \mathbb{Z}$, $k \geq 0$

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b) Determine the period, T, in seconds, the frequency, f, in hertz, and the amplitude, A, in volts, for this signal.

$$T = \frac{2\pi}{5} \text{ seconds}$$
$$f = \frac{5}{2\pi} \text{ Hz}$$
$$A = 130 \text{ V}$$

Section 6: 3.6 – Applications: Piston Motion

7) The movement of an engine piston can be modelled by $h = 4\sin t$ where h is the height of the piston, in cm, above the neutral position and t is time, in seconds.

a) Determine the velocity of the piston when $t = 5$.

$$v(t) = h'(t) = 4\cos t$$
$$v(5) = 4\cos(5)$$
$$v(5) \cong 1.13 \text{ cm/s}$$

b) Determine the acceleration of the piston when $t = 5$

$$a(t) = v'(t) = -4\sin t$$
$$a(5) = -4\sin(5)$$
$$a(5) \cong 3.8 \text{ cm/s}^2$$

Section 7: 3.6 – Applications: Pendulum

8) A simple pendulum has a length of 20 cm and a max horizontal displacement of 8 cm.

a) Determine a function that gives the horizontal position of the bob as a function of time.

$$T = 2\pi\sqrt{\frac{0.2}{9.8}}$$
$$h(t) = A\cos\left(\frac{2\pi t}{T}\right)$$
$$h(t) = 8\cos\left[\frac{2\pi t}{2\pi\sqrt{\frac{0.2}{9.8}}}\right]$$
$$h(t) = 8\cos(7t)$$

b) Determine a function that gives the velocity of the bob as a function of time.

$$h'(t) = 8[-\sin(7t)](7)$$
$$h'(t) = -56\sin(7t)$$

c) Find the max velocity of the bob. When does it occur?

$$\text{Max} = 0 + 56 = 56 \text{ cm/s}$$
$$56 = -56\sin(7t)$$
$$-1 = \sin(7t)$$
$$7t = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$$
$$t = \frac{3\pi}{14}, \frac{7\pi}{14}, \frac{11\pi}{14}, \ldots$$

$t = \frac{3\pi + 4\pi k}{14}$ sec; $k \in \mathbb{Z}$, $k \geq 0$

Section 8: 3.3 – Derivatives of Exponential Functions

9) Differentiate each function with respect to x.

a) $f(x) = \left(\frac{1}{2}\right)^x$

$$f'(x) = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right)$$

b) $g(x) = -2e^x$

$$g'(x) = -2e^x$$

c) $y = 5^x$

$$y' = 5^x \ln(5)$$

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d) $y = 5(2)^x$

$$y' = 5(2)^x \ln(2)$$

e) $y = (52)^{2x}$

$$y' = (52)^{2x}[\ln(52)](2)$$
$$y' = 2(52)^{2x}\ln(52)$$

f) $y = -2(10)^{3x}$

$$y' = -2(10)^{3x}[\ln(10)](3)$$
$$y' = -6(10)^{3x}\ln(10)$$

g) $y = e^{3x^2-2x+1}$

$$y' = e^{3x^2-2x+1}(6x-2)$$

h) $y = (x-1)e^{2x}$

$$y' = 1e^{2x} + 2e^{2x}(x-1)$$
$$y' = e^{2x}[1+2x-2]$$
$$y' = e^{2x}(2x-1)$$

i) $y = 3x + e^{-x}$

$$y' = 3 + e^{-x}(-1)$$
$$y' = 3 - e^{-x}$$

j) $y = e^x \cos(2x)$

$$y' = e^x\cos(2x) + [-\sin(2x)](2)e^x$$
$$y' = e^x[\cos(2x) - 2\sin(2x)]$$

k) $g(x) = \left(\frac{1}{3}\right)^{4x} - 2e^{\sin x}$

$$g'(x) = \left(\frac{1}{3}\right)^{4x}\left[\ln\left(\frac{1}{3}\right)\right](4) - 2e^{\sin x}(\cos x)$$
$$g'(x) = 4\ln\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{4x} - 2e^{\sin x}(\cos x)$$

Section 9: 3.3 – Tangent Lines to Exponential Functions

10) Find the equation of the line that is tangent to the curve $y = 2(3)^x$ at $x = 1$.

Point:

$$y(1) = 2(3)^1$$
$$y(1) = 6$$

Slope:

$$y' = 2(3)^x\ln(3)$$
$$y'(1) = 2(3)^1\ln(3)$$
$$y'(1) = 6\ln(3)$$
$$m = 6\ln(3)$$

Equation:

$$y = mx + b$$
$$6 = 6\ln(3)(1) + b$$
$$b = 6 - 6\ln(3)$$
$$y = 6\ln(3)x + 6 - 6\ln(3)$$

11) Find the equation of the line that is tangent to the curve $y = -3e^x$ at $x = \ln 2$.

Point:

$$y(\ln 2) = -3e^{\ln 2}$$
$$y(\ln 2) = -3(2)$$
$$y(\ln 2) = -6$$

Slope:

$$y' = -3e^x$$
$$y'(\ln 2) = -3e^{\ln 2}$$
$$y'(\ln 2) = -6$$
$$m = -6$$

Equation:

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$$y = mx + b$$
$$-6 = -6[\ln(2)] + b$$
$$b = -6 + 6\ln(2)$$
$$y = -6x - 6 + 6\ln(2)$$

Section 10: 3.5 – Tangent Lines to Logarithmic Functions

12) Find the equation of the tangent to $y = x\ln x$ that is perpendicular to $x + 3y - 9 = 0$.

Slope:

$$x + 3y - 9 = 0$$
$$3y = -x + 9$$
$$y = -\frac{1}{3}x + 3$$

$\perp m = 3$

When is tangent slope 3?

$$y = x\ln(x)$$
$$y' = 1\ln(x) + \frac{1}{x}(x)$$
$$y' = \ln(x) + 1$$
$$3 = \ln(x) + 1$$
$$2 = \ln(x)$$
$$x = e^2$$

Point:

$$y(e^2) = e^2\ln(e^2)$$
$$y(e^2) = e^2(2)$$
$$y(e^2) = 2e^2$$
$$(e^2,\ 2e^2)$$

Equation:

$$y = mx + b$$
$$2e^2 = 3e^2 + b$$
$$b = -e^2$$
$$y = 3x - e^2$$

Section 11: 3.2 – Tangent Lines to Trig Functions

13) Find the equation of the tangent to $y = 2\sin(\pi x)$ when $x = \frac{1}{2}$.

Slope:

$$y' = 2[\cos(\pi x)](\pi)$$
$$y' = 2\pi\cos(\pi x)$$
$$y'\left(\frac{1}{2}\right) = 2\pi\cos\left(\frac{\pi}{2}\right)$$
$$y'\left(\frac{1}{2}\right) = 0$$
$$m = 0$$

Point:

$$y\left(\frac{1}{2}\right) = 2\sin\left(\frac{\pi}{2}\right)$$
$$y\left(\frac{1}{2}\right) = 2$$

Equation:

$$y = mx + b$$
$$2 = 0\left(\frac{1}{2}\right) + b$$
$$b = 2$$
$$y = 2$$

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Section 12: 3.4 – Local Extrema of Exponential Functions

14) Find all local extrema for $f(x) = x^2e^{2x}$

$$f'(x) = 2x(e^{2x}) + 2e^{2x}(x^2)$$
$$f'(x) = xe^{2x}(2 + 2x)$$

Critical numbers:

$$x_1 = 0 \qquad x_2 = -1$$
$$f(0) = 0 \qquad f(-1) = e^{-2}$$

Test value	-2	-0.5	1
$f'(x)$	+	−	+
$f(x)$	Increasing	Decreasing	Increasing

Max at $(-1, e^{-2})$

Min at $(0, 0)$

15) Find all local extrema for $y = \frac{1}{2}x(2)^{3x+1}$

$$y' = \frac{1}{2}(2)^{3x+1} + (2)^{3x+1}[\ln(2)](3)\left(\frac{1}{2}x\right)$$
$$0 = \frac{1}{2}(2)^{3x+1}[1 + 3x\ln(2)]$$
$$0 = 1 + 3x\ln(2)$$
$$x = \frac{-1}{3\ln(2)}$$
$$x \cong -0.48$$
$$y(-0.48) \cong -0.18$$

Test value		-1	0
y'		−	+
y		decreasing	increasing

Min at $(-0.48, -0.18)$

Section 13: 3.6 – Applications: Radioactive Decay

16) Continuous growth or decay follows the formula $A = ce^{kt}$, where c is the initial amount, and k is a rate factor. The mass of a radioactive substance is 1000 g on day 1, and only 100 g after 100 days. Find…

a) k

$$100 = 1000e^{100k}$$
$$0.1 = e^{100k}$$
$$\ln(0.1) = 100k$$
$$k = \frac{\ln(0.1)}{100}$$
$$k \cong -0.023$$

b) the half-life

$$500 = 1000e^{-0.023t}$$
$$0.5 = e^{-0.023t}$$
$$\ln(0.5) = -0.023t$$
$$t \cong 30.1 \text{ days}$$

c) the amount that remains after 300 days

$$A(300) = 1000e^{-0.023(300)}$$
$$A(300) \cong 1.01 \text{ grams}$$

d) the rate of decay after 50 days

$$A(t) = 1000e^{-0.023t}$$
$$A'(t) = 1000e^{-0.023t}(-0.023)$$
$$A'(t) = -23e^{-0.023t}$$
$$A'(50) = -23e^{-0.023(50)}$$

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$$A'(50) \cong -7.3 \text{ g/day.}$$

Section 14: 3.5 – Implicit Differentiation

17) Find $\frac{dy}{dx}$ by implicit differentiation

a) $x^2 + y^2 = 36$

$$2x + 2y\left(\frac{dy}{dx}\right) = 0$$
$$\frac{dy}{dx} = \frac{-2x}{2y}$$
$$\frac{dy}{dx} = -\frac{x}{y}$$

b) $x^3 - xy + y^2 = 4$

$$3x^2 - \left[1y + \frac{dy}{dx}(x)\right] + 2y\left(\frac{dy}{dx}\right) = 0$$
$$3x^2 - y - x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$
$$-x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = y - 3x^2$$
$$\frac{dy}{dx}(-x + 2y) = y - 3x^2$$
$$\frac{dy}{dx} = \frac{y - 3x^2}{-x + 2y}$$

c) $2\sin x \cos y = 1$

$$2\cos x \cos y + (-\sin y)\left(\frac{dy}{dx}\right)(2)\sin x = 0$$
$$2\cos x \cos y = 2\sin x \sin y\left(\frac{dy}{dx}\right)$$
$$\frac{dy}{dx} = \frac{\cos x \cos y}{\sin x \sin y}$$

d) $(4 - x)y^2 = x^3$

$$-1y^2 + 2y\left(\frac{dy}{dx}\right)(4 - x) = 3x^2$$
$$\frac{dy}{dx}(2y)(4 - x) = 3x^2 + y^2$$
$$\frac{dy}{dx} = \frac{3x^2 + y^2}{2y(4 - x)}$$

Section 15: 3.5 – Implicit Differentiation: Tangent Lines

18) Use implicit differentiation to find an equation of the tangent line to the ellipse $\frac{x^2}{2} + \frac{y^2}{8} = 1$ at $(1, 2)$

Slope:

$$\frac{1}{2}x^2 + \frac{1}{8}y^2 = 1$$
$$x + \frac{1}{4}y\left(\frac{dy}{dx}\right) = 0$$
$$\frac{y}{4}\left(\frac{dy}{dx}\right) = -x$$
$$\frac{dy}{dx} = \frac{-4x}{y}$$
$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=2}} = \frac{-4(1)}{2}$$
$$\left.\frac{dy}{dx}\right|_{\substack{x=1\\y=2}} = -2$$
$$m = -2$$

Equation:

$$y = mx + b$$
$$2 = -2(1) + b$$
$$b = 4$$
$$y = -2x + 4$$

19) Find the slope of the tangent at $(2, 1)$ for the curve $y = 2^{x-2y}$

$$y' = 2^{x-2y}(\ln 2)(1 - 2y')$$

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$$y' = 2^{x-2y}(\ln 2) - 2^{x-2y}(\ln 2)(2y')$$
$$y' + 2y'(2)^{x-2y}(\ln 2) = 2^{x-2y}(\ln 2)$$
$$y'\left[1 + 2(2)^{x-2y}\ln 2\right] = 2^{x-2y}(\ln 2)$$
$$y' = \frac{2^{x-2y}(\ln 2)}{1 + 2(2)^{x-2y}\ln 2}$$
$$y'(2,\ 1) = \frac{2^0(\ln 2)}{1 + 2\ln 2}$$
$$m = \frac{\ln(2)}{1 + 2\ln(2)}$$

Section 16: 3.5 – Derivatives of Logarithmic Functions

20) Differentiate with respect to x

a) $y = \ln x$

$$y' = \frac{1}{x}$$

b) $y = \log_3 x$

$$y' = \frac{1}{x\ln 3}$$

c) $y = 4\log_5(2x - 1)$

$$y' = \frac{4}{(2x-1)\ln 5}(2)$$
$$y' = \frac{8}{(2x-1)\ln 5}$$

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