3.7 Unit 3 Pretest – Derivatives of Trig and Exponential Functions

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This review covers all of Unit 3: Derivatives of Trig and Exponential Functions, and is meant to be used to prepare for the unit test.

Section 1: 3.1 – Derivatives of Sine and Cosine

1) Find the derivatives of each of the following functions

a) $y = \cos x$

b) $f(x) = -2\sin x$

c) $y = \cos x - \sin x$

Section 2: 3.1 – Tangent Lines to Trig Functions

2) Determine the slope of the function $y = 4\sin x$ at $x = \frac{\pi}{3}$.

3) Find the equation of the line that is tangent to the curve $y = 2\sin\theta + 4\cos\theta$ at $\theta = \frac{\pi}{4}$.

Section 3: 3.2 – More Derivatives of Sine and Cosine

4) Differentiate each of the following:

a) $y = -\cos^2 x$

b) $y = \sin(2\theta) - 2\cos(2\theta)$

c) $f(\theta) = -\frac{\pi}{2}\sin(2\theta - \pi)$

d) $f(x) = \sin(\sin x)$

e) $f(x) = \cos(\cos x)$

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f) $f(\theta) = \cos(7\theta) - \cos(5\theta)$

g) $y = 3x\sin x$

h) $f(t) = 2t^2\cos(2t)$

i) $y = \pi t\sin(\pi t - 6)$

j) $y = \cos(\sin\theta) + \sin(\cos\theta)$

k) $f(x) = \cos^2(\sin x)$

l) $f(\theta) = \cos(7\theta) - \cos^2(5\theta)$

m) $y = 3\sin(2x) - 4\cos(2x)$

n) $y = \tan(3x)$

o) $y = \dfrac{1}{2 - \cos x}$

p) $y = x\tan(2x)$

q) $y = [\sin(2x)]e^{3x}$

r) $y = \cos^2(2x)$

Section 4: 3.2 – Tangent Lines (Maximum Slope)

5) Find an equation of a line that is tangent to the curve $f(x) = 2\cos(3x)$ and whose slope is a maximum.

Section 5: 3.6 – Applications: AC Voltage Signal

6) A voltage of the power supply in an AC-DC coupled circuit is given by the function $V(t) = 130\sin(5t) + 18$, where t is time, in seconds, and V is the voltage, in volts, at time t.

a) Find the maximum and minimum voltages and the times at which they occur.

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b) Determine the period, T, in seconds, the frequency, f, in hertz, and the amplitude, A, in volts, for this signal.

Section 6: 3.6 – Applications: Piston Motion

7) The movement of an engine piston can be modelled by $h = 4\sin t$ where h is the height of the piston, in cm, above the neutral position and t is time, in seconds.

a) Determine the velocity of the piston when $t = 5$.

b) Determine the acceleration of the piston when $t = 5$

Section 7: 3.6 – Applications: Pendulum

8) A simple pendulum has a length of 20 cm and a max horizontal displacement of 8 cm.

a) Determine a function that gives the horizontal position of the bob as a function of time.

b) Determine a function that gives the velocity of the bob as a function of time.

c) Find the max velocity of the bob. When does it occur?

Section 8: 3.3 – Derivatives of Exponential Functions

9) Differentiate each function with respect to x.

a) $f(x) = \left(\frac{1}{2}\right)^x$

b) $g(x) = -2e^x$

c) $y = 5^x$

d) $y = 5(2)^x$

e) $y = (52)^{2x}$

f) $y = -2(10)^{3x}$

g) $y = e^{3x^2-2x+1}$

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h) $y = (x-1)e^{2x}$

i) $y = 3x + e^{-x}$

j) $y = e^{x}\cos(2x)$

k) $g(x) = \left(\frac{1}{3}\right)^{4x} - 2e^{\sin x}$

Section 9: 3.3 – Tangent Lines to Exponential Functions

10) Find the equation of the line that is tangent to the curve $y = 2(3)^{x}$ at $x = 1$.

11) Find the equation of the line that is tangent to the curve $y = -3e^{x}$ at $x = \ln 2$.

Section 10: 3.5 – Tangent Lines to Logarithmic Functions

12) Find the equation of the tangent to $y = x\ln x$ that is perpendicular to $x + 3y - 9 = 0$.

Section 11: 3.2 – Tangent Lines to Trig Functions

13) Find the equation of the tangent to $y = 2\sin(\pi x)$ when $x = \frac{1}{2}$.

Section 12: 3.4 – Local Extrema of Exponential Functions

14) Find all local extrema for $f(x) = x^{2}e^{2x}$

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15) Find all local extrema for $y = \frac{1}{2}x(2)^{3x+1}$

Section 13: 3.6 – Applications: Radioactive Decay

16) Continuous growth or decay follows the formula $A = ce^{kt}$, where c is the initial amount, and k is a rate factor. The mass of a radioactive substance is 1000 g on day 1, and only 100 g after 100 days. Find…

a) k

b) the half-life

c) the amount that remains after 300 days

d) the rate of decay after 50 days

Section 14: 3.5 – Implicit Differentiation

17) Find $\frac{dy}{dx}$ by implicit differentiation

a) $x^2 + y^2 = 36$

b) $x^3 - xy + y^2 = 4$

c) $2\sin x \cos y = 1$

d) $(4 - x)y^2 = x^3$

Section 15: 3.5 – Implicit Differentiation: Tangent Lines

18) Use implicit differentiation to find an equation of the tangent line to the ellipse $\frac{x^2}{2} + \frac{y^2}{8} = 1$ at $(1, 2)$

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19) Find the slope of the tangent at $(2,\ 1)$ for the curve $y = 2^{x-2y}$

Section 16: 3.5 – Derivatives of Logarithmic Functions

20) Differentiate with respect to x

a) $y = \ln x$

b) $y = \log_3 x$

c) $y = 4\log_5(2x - 1)$

Answer Key

1) a) $\frac{dy}{dx} = -\sin x$ b) $f'(x) = -2\cos x$ c) $y' = -\sin x - \cos x$

2) 2

3) $y = -\sqrt{2}\theta + \frac{\sqrt{2}}{4}\pi + 3\sqrt{2}$

4) a) $y' = \sin(2x)$ b) $y' = 2\cos(2\theta) + 4\sin(2\theta)$ c) $f'(\theta) = -\pi\cos(2\theta - \pi)$ d) $f'(x) = [\cos(\sin x)](\cos x)$ e) $f'(x) = [\sin(\cos x)](\sin x)$

f) $f'(\theta) = -7\sin(7\theta) + 5\sin(5\theta)$ **g)** $y' = 3x\cos x + 3\sin x$ **h)** $f'(t) = -4t^2\sin(2t) + 4t\cos(2t)$ **i)** $y' = \pi^2 t[\cos(\pi t - 6)] + \pi\sin(\pi t - 6)$ **j)** $y' = -\sin(\sin\theta)(\cos\theta) - \cos(\cos\theta)(\sin\theta)$ **k)** $f'(x) = -2\cos x[\sin(\sin x)][\cos(\sin x)]$ **l)** $f'(\theta) = -7\sin(7\theta) + 10\cos(5\theta)[\sin(5\theta)]$ **m)** $y' = 6\cos(2x) + 8\sin(2x)$ **n)** $y' = 3\sec^2(3x)$ **o)** $y' = \frac{-\sin x}{(2 - \cos x)^2}$

p) $y' = 2x\sec^2(2x) + \tan(2x)$ **q)** $y' = e^{3x}[3\sin(2x) + 2\cos(2x)]$ **r)** $y' = -4\cos(2x)\sin(2x)$

5) $y = 6x - 3\pi$ (answers may vary)

6) a) max voltage: 148 V at time $t = \frac{(4k+1)\pi}{10},\ k \in \mathbb{Z},\ k \geq 0$

min voltage: -112 V at time $t = \frac{(4k+3)\pi}{10},\ k \in \mathbb{Z},\ k \geq 0$

b) period: $T = \frac{2\pi}{5}$ seconds; frequency: $f = \frac{5}{2\pi}$ Hz; amplitude: $A = 130$ V

7) a) 1.13 cm/s b) 3.8 cm/s^2

8) a) $h(t) = 8\cos(7t)$ b) $v(t) = -56\sin(7t)$ c) max velocity of 56 cm/s when $t = \frac{3+4k}{14}\pi,\ k \in \mathbb{Z},\ k \geq 0$

9) a) $f'(x) = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right)$ b) $g'(x) = -2e^x$ c) $y' = 5^x\ln(5)$ d) $y' = 5(2)^x\ln(2)$ e) $y' = 2(52)^{2x}\ln(52)$

f) $y = -6(10)^{3x}\ln(10)$ **g)** $y' = (e^{3x^2-2x+1})(6x - 2)$ **h)** $f'(x) = e^{2x}(2x - 1)$ **i)** $y' = 3 - e^{-x}$ **j)** $y' = e^x[-2\sin(2x) + \cos(2x)]$

k) $g'(x) = \left[4\ln\left(\frac{1}{3}\right)\right]\left(\frac{1}{3}\right)^{4x} - (2e^{\sin x})(\cos x)$

10) $y = 6x\ln(3) - 6\ln(3) + 6$

11) $y = -6x + 6\ln(2) - 6$

12) $y = 3x - e^2$

13) $y = 2$

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14) $y' = 2xe^{2x}(1+x)$; min at $(0,\ 0)$, max at $\left(-1,\ \frac{1}{e^2}\right)$

15) $(-0.48,\ -0.18)$ is a local minimum

16) a) $k \sim -0.023$ b) $t \sim 30$ days c) $A(300) \cong 1g$ d) $A'(50) \cong -7.3$ g/day

17) a) $y' = -\frac{x}{y}$ b) $y' = \frac{y-3x^2}{2y-x}$ c) $y' = \frac{\cos(x)\cos(y)}{\sin(x)\sin(y)}$ d) $y' = \frac{-y^2-3x^2}{(2x-8)y}$

18) $y = -2x+4$

19) $\frac{\ln(2)}{2\ln(2)+1}$

20) a) $y' = \frac{1}{x}$ b) $y' = \frac{1}{x\ln 3}$ c) $y' = \frac{8}{(2x-1)\ln 5}$

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