3.6 Applications of Exponential and Trigonometric Derivatives Lesson

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Part 1: Review of Differentiating Exponential Functions

Rule	Derivative
Exponential Functions If $h(x) = b^{g(x)}$	$h'(x) = b^{g(x)} \times \ln b \times g'(x)$
Trig Functions If $f(x) = \sin x$ $g(x) = \cos x$ $h(x) = \tan x$	$f'(x) = \cos x$ $g'(x) = -\sin x$ $h'(x) = \sec^2 x$
Log Functions If $g(x) = \log_a[f(x)]$	$g'(x) = \frac{f'(x)}{f(x)\ln a}$

The Greek letter λ, lambda, indicates the disintegration constant, which is related to how fast a radioactive substance decays.

Example 1: Differentiate $y = 3^x e^{\sin x}$ with respect to x.

Example 2: A radioactive isotope of gold, Au-198, is used in the diagnosis and treatment of liver disease. Suppose that a 6.0 mg sample of Au-198 is injected into a liver, and that this sample decays to 4.6 mg after 1 day. Assume the amount of Au-198 remaining after t days is given by $N(t) = N_0 e^{-\lambda t}$.

a) Determine the disintegration constant for Au-198

b) Determine the half-life of Au-198

c) Write the equation that gives the amount of Au-198 remaining as a function of time in terms of its half-life.

d) How fast is the sample decaying after 3 days?

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Example 3: A power supply delivers a voltage signal that consists of an alternating current (AC) component and a direct current (DC) component. Where t is the time, in seconds, the voltage, in volts, at time t is given by the function $V(t) = 5\sin t + 12$.

a) What are the max and min voltages? At which times do these values occur?

b) Determine the period, T, in seconds, frequency, f, in hertz, and amplitude, A, in volts, for this signal.

Example 4: For small amplitudes, and ignoring the effects of friction, a pendulum is an example of simple harmonic motion. Simple harmonic motion is motion that can be modelled by a sinusoidal function, and the graph of a function modelling simple harmonic motion has a constant amplitude.

The period of a simple pendulum depends only on its length and can be found using the relation $T = 2\pi\sqrt{\frac{l}{g}}$, where T is the period, in seconds, l is the length of the pendulum, in meters, and g is the acceleration due to gravity. On or near the surface of Earth, g has a constant value of 9.8 m/s^2.

Under these conditions, the horizontal position of the bob as a function of time can be described by the function $h(t) = A\cos\left(\frac{2\pi t}{T}\right)$, where A is the amplitude of the pendulum, t is time, in seconds, and T is the period of the pendulum, in seconds.

Find the max speed of the bob and the time at which that speed first occurs for a pendulum having a length of 1.0 meters and an amplitude of 5 cm.

Example 5: The vertical displacement of a SUV's body after passing over a bump is modelled by the function $h(t) = e^{-0.5t}\sin t$, where h is the vertical displacement, in meters, at time t, in seconds.

a) Use technology to generate a rough sketch of the graph of the function.

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b) Determine when the max displacement of the sport utility vehicle's body occurs.

c) Determine the maximum displacement.

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