# 3.5 Implicit Differentiation and Derivatives of Log Functions Lesson

This document was composed and formatted by Annie Bernatchez.

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## Part 1: Chain Rule Using Leibniz Notation

If $y$ is a function of $u$ and $u$ is a function of $x$, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$.

**Example 1:** Suppose we wish to differentiate $y = (5 + 2x)^{10}$ in order to calculate $\frac{dy}{dx}$. We make a substitution and let $u = 5 + 2x$ so that $y = u^{10}$

## Part 2: Implicit Differentiation

Up until now, most functions were written in the form in which $y$ was defined explicitly as a function of $x$, such as $y = x^3 - 4x$. In that equation, $y$ is isolated and is expressed EXPLICITLY as a function of $x$.

Functions can also be defined implicitly by relations, such as a circle $x^2 + y^2 = 16$. In this case, $y$ is not isolated or explicitly defined in terms of $x$. You could rearrange to isolate for $y$ but often this is very difficult or impossible. It will be necessary to use the chain rule to implicitly differentiate when it is in implicit form.

$\frac{d}{dx}f(y) = \frac{d}{dy}f(y) \times \frac{dy}{dx}$

**Example 2:** Differentiate each of the following using implicit differentiation

a) $x^2 + y^2 = 16$

b) $y^2 + x^3 - y^3 + 6 = 3y$

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## Part 3: Derivative of Logarithms

Proof of the derivative of $y = \log_a x$

Start by writing in inverse form:

Now use implicit differentiation to differentiate with respect to $x$

**Rule:**

$\frac{d}{dx}\log_a x = \frac{1}{x\ln a}$

**If you need chain rule:**

$\frac{d}{dx}\log_a[f(x)] = \frac{1}{f(x)\ln a}f'(x) = \frac{f'(x)}{f(x)\ln a}$

**Example 3:** Differentiate each of the following with respect to $x$.

a) $y = 2\ln(1 + x^2)$

b) $f(x) = 1 - \log_4(2x - 1)$

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