3.3 Derivatives of Exponential Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Part 1: Review of e and $\ln x$

Properties of e:

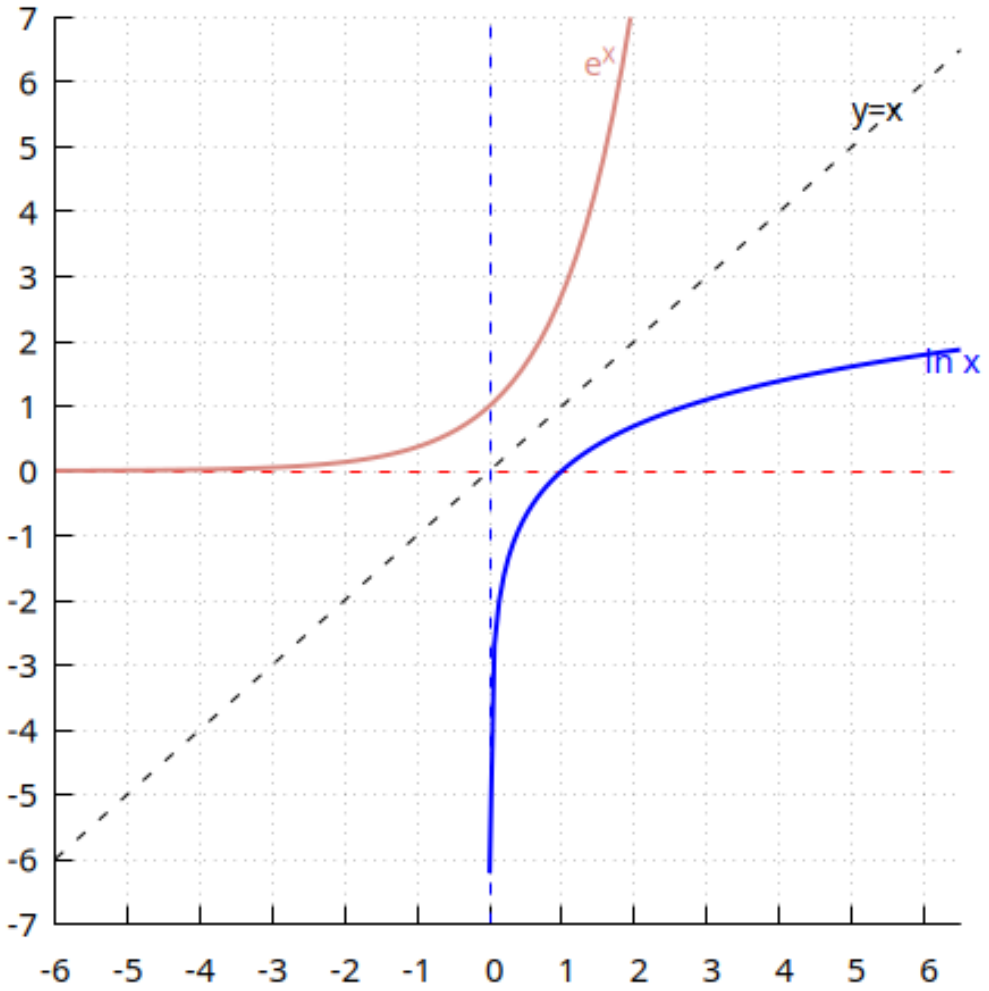
- $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.718\,281\,828\,459$
- e is an irrational number, similar to π. They are non-terminating and non-repeating.
- $\log_e x$ is known as the natural logarithm and can be written as ln x
- Many naturally occurring phenomena can be modelled using base-e exponential and logarithmic functions.
- $\log_e e = \ln e = 1$

Example 1: Graph the functions $y = e^x$ and $y = \ln x$

$x\,(y = e^x)$	y	$x\,(y = \ln x)$	y
-1	0.37	0.37	-1
0	1	1	0
1	2.72	2.72	1
HA	$y = 0$	VA	$x = 0$

Note: $y = \ln x$ is the inverse of $y = e^x$

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Example 2: The population of a bacterial culture as a function of time is given by the equation $P(t) = 200e^{0.094t}$, where P is the population after t days.

a) What is the initial population of the bacterial culture?

$$P(0) = 200e^{0.094(0)}$$
$$P(0) = 200$$

b) Estimate the population after 3 days.

$$P(3) = 200e^{0.094(3)}$$
$$P(3) \cong 265.2$$

c) How long will the bacterial culture take to double its population?

$$400 = 200e^{0.094t}$$
$$2 = e^{0.094t}$$
$$\ln 2 = \ln e^{0.094t}$$
$$\ln 2 = 0.094t \ln e$$
$$\ln 2 = 0.094t$$
$$t = \frac{\ln 2}{0.094}$$
$$t \cong 7.37 \text{ days}$$

d) Re-write this function as an exponential function having base 2.

$$P(t) = 200(2)^{\frac{t}{7.37}}$$

Use the exponential growth formula:

$$A(t) = A_0(2)^{\frac{t}{D}}$$

where D is the doubling period

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Part 2: Derivatives of Exponential Functions

Rule: If $f(x) = b^x$, $f'(x) = b^x \ln b$

Example 3: Determine the derivative of each function

a) $y = 2^x$

$$y' = 2^x \ln 2$$

b) $y = e^x$

$$y' = e^x \ln e$$
$$y' = e^x(1)$$
$$y' = e^x$$

c) $y = 3(2)^x$

$$y' = 3(2)^x \ln 2$$

d) $y = 3^x + 1$

$$y' = 3^x \ln 3$$

Notice that $\frac{d}{dx} e^x = e^x$

Example 4: Find the equation of the line that is tangent to the curve $y = 2e^x$ at $x = \ln 3$.

Slope of tangent line:

$$y' = 2e^x$$
$$y'(\ln 3) = 2e^{\ln 3}$$
$$y'(\ln 3) = 2(3)$$
$$y'(\ln 3) = 6$$
$$m = 6$$

Point on tangent line:

$$y = 2e^x$$

When $x = \ln 3$

$$y = 2e^{\ln 3}$$
$$y = 2(3)$$
$$y = 6$$

Equation of tangent line:

$$y = mx + b$$
$$6 = 6(\ln 3) + b$$
$$b = 6 - 6\ln 3$$

y = 6x + 6 - 6 ln 3

Example 5: A biologist is studying the increase in the population of a particular insect in a provincial park. The population triples every week. Assume the population continues to increase at this rate. Initially, there are 100 insects.

a) Determine the number of insects present after 4 weeks.

$$P(t) = 100(3)^t$$
$$P(4) = 100(3)^4$$
$$P(4) = 8100$$

b) How fast is the number of insects increasing

i) when they are initially discovered?

$$P'(t) = 100(3)^t \ln 3$$
$$P'(0) = 100(3)^0 \ln 3$$
$$P'(0) \cong 109.9$$

At the beginning, it is increasing by 109.9 insects per week

ii) at the end of 4 weeks?

$$P'(4) = 100(3)^4 \ln 3$$
$$P'(4) \cong 8898.8 \text{ insects per week}$$

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