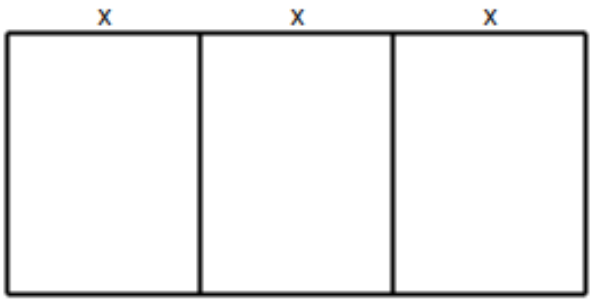
2.6 Optimization Problems Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) A rectangular pen is to be built with 1200 m of fencing. The pen is to be divided into three parts using two parallel partitions. Find the max possible area of the pen.



$$\text{width} = x$$
$$1200 = 4x + 2l$$
$$l = \frac{1200 - 4x}{2}$$
$$\text{length} = 600 - 2x$$
$$A(x) = x(600 - 2x)$$
$$A(x) = 600x - 2x^2$$

Critical Points:

$$A'(x) = 600 - 4x$$
$$0 = 600 - 4x$$
$$x = 150$$
$$A(150) = 45\,000 \text{ m}^2$$

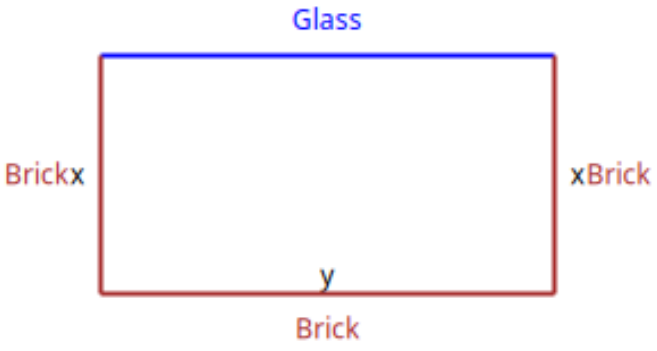
2nd Derivative Test:

$$A''(x) = -4$$
$$A''(150) = -4; \text{ concave down}$$

So $(150,\ 45\,000)$ is a max point.

The max area is 45 000 m^2

2) A showroom for a car dealership is to be built in the shape of a rectangle with brick on the back and sides, and glass on the front. The floor of the showroom is to have an area of 500 m^2. If a brick wall costs \$1200/m while a glass wall costs \$600/m, what dimensions would minimize the cost of the showroom? What is the min cost?



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$$xy = 500$$
$$y = \frac{500}{x}$$
$$C(x) = 600\left(\frac{500}{x}\right) + 1200\left(\frac{500}{x}\right) + 1200(2x)$$
$$C(x) = \frac{300\,000}{x} + \frac{600\,000}{x} + 2400x$$
$$C(x) = \frac{900\,000}{x} + 2400x$$

Critical Points:

$$C'(x) = -900\,000x^{-2} + 2400$$
$$0 = -\frac{900\,000}{x^2} + 2400$$
$$-2400x^2 = -900\,000$$
$$x^2 = 375$$
$$x \cong 19.36 \text{ m}$$
$$C(19.36) \cong 92\,951.6 \text{ dollars}$$

2nd Derivative Test:

$$C''(x) = \frac{1\,800\,000}{x^3}$$
$$C''(19.36) \cong 248$$

So concave up and $(19.36,\ 92\,951.6)$ is a MIN point. The dimensions that obtain a min cost of \$92 951.6 are 19.36 m by 25.83 m.

3) A soup can is to have a capacity of 250 cm^3 and the diameter of the can must be no less than 4 cm and no greater than 8 cm. What are the dimensions of the can that can be constructed using the LEAST amount of material?

Domain: $2 \le r \le 4$

$$SA = 2\pi r^2 + 2\pi rh$$

From Volume Equation:

$$V = \pi r^2 h$$
$$250 = \pi r^2 h$$
$$h = \frac{250}{\pi r^2}$$
$$SA(r) = 2\pi r^2 + 2\pi r\left(\frac{250}{\pi r^2}\right)$$
$$SA(r) = 2\pi r^2 + \frac{500}{r}$$
$$SA'(r) = 4\pi r - \frac{500}{r^2}$$
$$0 = 4\pi r - \frac{500}{r^2}$$
$$r^3 = \frac{500}{4\pi}$$
$$r = 3.41$$

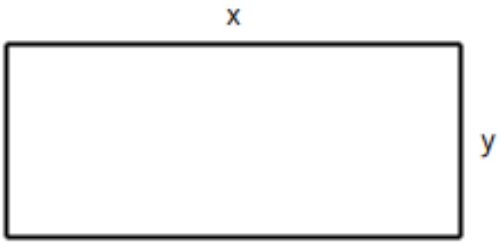
Test endpoints of domain and critical number:

$$SA(2) = 275.1 \text{ cm}^2$$
$$SA(3.41) = 219.7 \text{ cm}^2$$
$$SA(4) = 225.5 \text{ cm}^2$$

Therefore a radius = 3.41 cm and a height $= \frac{250}{\pi(3.41)^2} = 6.84$ cm will minimize the amount of material needed to make the can.

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4) A rectangular piece of paper with perimeter 100 cm is to be rolled to form a cylindrical tube. Find the dimensions of the paper that will produce a tube with maximum volume. What is the max volume?



$$x = 2\pi r \qquad r = \frac{x}{2\pi}$$
$$100 = 2x + 2y$$
$$y = \frac{100 - 2x}{2} = 50 - x$$
$$V(x) = \pi\left(\frac{x}{2\pi}\right)^2 (50 - x)$$
$$V(x) = \pi\left(\frac{x^2}{4\pi^2}\right)(50 - x)$$
$$V(x) = \frac{x^2}{4\pi}(50 - x)$$
$$V(x) = \frac{25}{\pi}x^2 - \frac{1}{4\pi}x^3$$

Critical Points:

$$V'(x) = \frac{25}{\pi}x - \frac{3}{4\pi}x^2$$
$$0 = x\left(\frac{25}{\pi} - \frac{3}{4\pi}x\right)$$
$$x_1 = 0$$
$$0 = \frac{25}{\pi} - \frac{3}{4\pi}x$$
$$\frac{3x}{4\pi} = \frac{25}{\pi}$$
$$3x = 100$$
$$x = \frac{100}{3}$$
$$V''(x) = \frac{25}{\pi} - \frac{3}{2\pi}x$$

$V''(0) = \frac{25}{\pi}$; concave up (min)

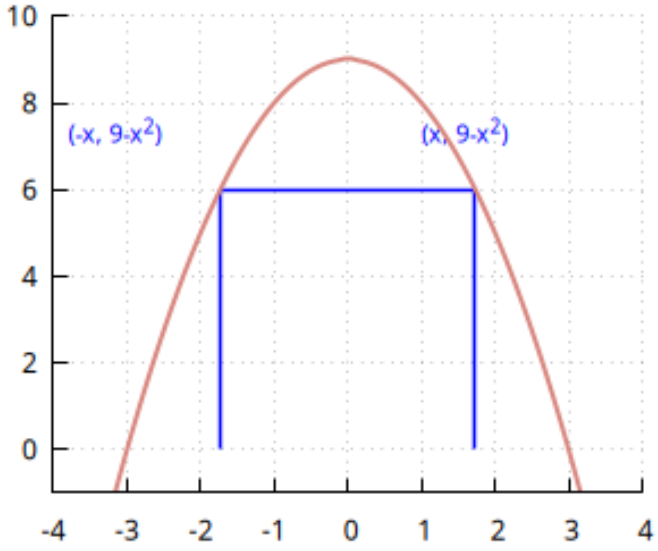
$V''\left(\frac{100}{3}\right) \cong -8$; concave down (max)

$$V\left(\frac{100}{3}\right) \cong 1473.66 \text{ cm}^3$$

A max volume of 1473.66 cm^3 can be obtained with a length of $\frac{100}{3}$ cm and width of $\frac{50}{3}$ cm.

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5) Find the area of the largest rectangle that can be inscribed between the x-axis and the graph defined by $y = 9 - x^2$.



$$A(x) = 2x(9 - x^2)$$
$$A(x) = 18x - 2x^3$$
$$A'(x) = 18 - 6x^2$$
$$0 = 18 - 6x^2$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

Can't have $x < 0$

$$A(\sqrt{3}) \cong 20.78$$

2nd derivative test

$$A''(x) = -12x$$
$$A''(\sqrt{3}) \cong -20.8; \text{ concave down (max)}$$

So $(\sqrt{3},\ 20.78)$ is a max point. The max area is about 20.78 units^2.

6) For an outdoor concert, a ticket price of \$30 typically attracts 5000 people. For each \$1 increase in the ticket price, 100 fewer people will attend. The revenue, R, is the product of the number of people attending and the price per ticket. Let x equal the number of \$1 increases in price. Find the ticket price that maximizes the revenue. What is the max revenue?

$$R(x) = (30 + x)(5000 - 100x)$$
$$R'(x) = 1(5000 - 100x) + (-100)(30 + x)$$
$$R'(x) = 2000 - 200x$$
$$0 = 2000 - 200x$$
$$x = 10$$

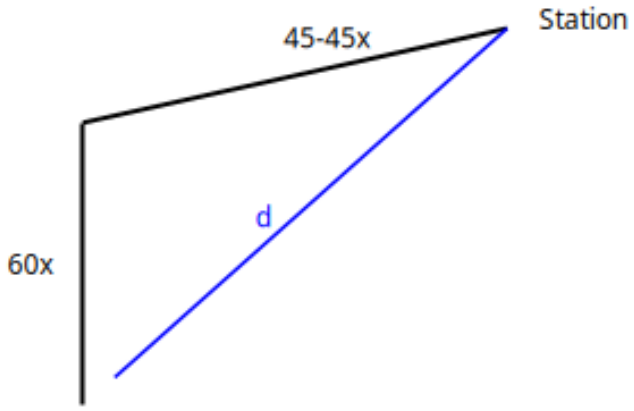
2nd derivative test:

$$R''(x) = -200$$
$$R''(10) = -200; \text{ concave down}$$
$$R(10) = (30 + 10)(5000 - 100(10))$$
$$R(10) = (40)(4000)$$
$$R(10) = 160\,000$$

A \$40 ticket will maximize the revenue to \$160 000.

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7) A train leaves the station at 10:00 a.m. and travels due south at a speed of 60 km/h. Another train has been heading due west at 45 km/h and reaches the same station at 11:00 a.m. At what time were the two trains closest together?



$$d(x) = \sqrt{(45 - 45x)^2 + (60x)^2}$$
$$d(x) = (2025 - 4050x + 2025x^2 + 3600x^2)^{\frac{1}{2}}$$
$$d(x) = (5625x^2 - 4050x + 2025)^{\frac{1}{2}}$$
$$d'(x) = \frac{1}{2}(5625x^2 - 4050x + 2025)^{-\frac{1}{2}}(11\,250x - 4050)$$
$$d'(x) = \frac{5625x - 2025}{\sqrt{5625x^2 - 4050x + 2025}}$$
$$0 = 5625x - 2025$$
$$x = 0.36$$

1st derivative test:

	0.36	
$d'(x)$	−	+
$d(x)$	decreasing	increasing

So they are closest together 0.36 hours after 10:00 a.m. (10:22 a.m.)

Answer Key

1) 45 000 m^2

2) 19.4 m by 25.8 m; min cost is \$92 952

3) $r = 3.41$ cm and $h = 6.83$ cm

4) $\frac{50}{3}$ cm by $\frac{100}{3}$ cm; volume is 1473.7 cm^3

5) $12\sqrt{3}$ units^2

6) \$40; max revenue is \$160 000

7) 0.36 hours after the first train left the station (10:22 am)

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