2.5 Curve Sketching Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) For each function, determine the coordinates of the local extrema. Classify each point as a local max or min.

a) $y = 2x - 3x^2$

$$y' = 2 - 6x$$
$$0 = 2 - 6x$$
$$x = \frac{1}{3}$$
$$y\left(\frac{1}{3}\right) = \frac{1}{3}$$

Critical point: $\left(\frac{1}{3}, \frac{1}{3}\right)$

2nd derivative test:

$$y'' = -6$$

$y''\left(\frac{1}{3}\right) = -6$; concave down

$\left(\frac{1}{3}, \frac{1}{3}\right)$ is a local MAX

b) $y = 2t^3 + 6t^2 + 6t + 4$

$$y' = 6t^2 + 12t + 6$$
$$0 = 6(t^2 + 2t + 1)$$
$$0 = 6(t + 1)^2$$

CN: $t = -1$

CP: $(-1, 2)$

2nd DT:

$$y'' = 12t + 12$$
$$y''(-1) = 0$$

So 2nd DT fails; check 1st DT.

Test	-2	0
y'	+	+
y	incr.	incr.

$(-1, 2)$ is NOT a local max or min

2) For each function, determine the coordinates of any points of inflection.

a) $f(x) = 2x^3 - 4x^2$

$$f'(x) = 6x^2 - 8x$$
$$f''(x) = 12x - 8$$
$$0 = 12x - 8$$
$$x = \frac{2}{3}$$
$$f\left(\frac{2}{3}\right) = -\frac{32}{27}$$

Possible POI is $\left(\frac{2}{3}, -\frac{32}{27}\right)$

Test	0	1
$f''(x)$	−	+
$F(x)$	con. down	con. up

POI: $\left(\frac{2}{3}, -\frac{32}{27}\right)$

b) $f(x) = 3x^5 - 5x^4 - 40x^3 + 120x^2$

$$f'(x) = 15x^4 - 20x^3 - 120x^2 + 240x$$
$$f''(x) = 60x^3 - 60x^2 - 240x + 240$$

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$$0 = 60(x^3 - x^2 - 4x + 4)$$
$$0 = x^2(x-1) - 4(x-1)$$
$$0 = (x-1)(x^2-4)$$
$$0 = (x-1)(x-2)(x+2)$$

$x_1 = 1$ $x_2 = 2$ $x_3 = -2$

$f(1) = 78$ $f(2) = 176$ $f(-2) = 624$

Test	-3	0	1.5	3
$f''(x)$	$-$	$+$	$-$	$+$
$F(x)$	down	up	down	up

POI's: $(-2, 624)$, $(1, 78)$, and $(2, 176)$

3) Use the algorithm for curve sketching to analyze the key features of each of the following functions and to sketch a graph of them.

a) $f(x) = x^4 - 8x^3$

1. No domain restrictions; no asymptotes

2.

$$0 = x^3(x-8)$$

$x_1 = 0$ $x_2 = 8$

x-int: $(0, 0)$, $(8, 0)$

$f(0) = 0$

y-int: $(0, 0)$

3.

$$f'(x) = 4x^3 - 24x^2$$
$$0 = 4x^2(x-6)$$

$x_1 = 0$ $x_2 = 6$

$f(0) = 0$ $f(6) = -432$

Critical points: $(0, 0)$, $(6, -432)$

4.

$$f''(x) = 12x^2 - 48x$$
$$0 = 12x(x-4)$$

$x_1 = 0$ $x_2 = 4$

$f(0) = 0$ $f(4) = -256$

Possible points of inflection: $(0, 0)$, $(4, -256)$

5/6.

Test	-1	1	5	7
$f'(x)$	$-$	$-$	$-$	$+$
$f''(x)$	$+$	$-$	$+$	$+$
$F(x)$	Con. UP Decreasing	Con. Down Decreasing	Con. UP Decreasing	Con. UP Increasing

POI, POI, local min

Increasing: $x > 6$

Decreasing: $x < 6$

7.

Local min: $(6, -432)$

Local max: NONE

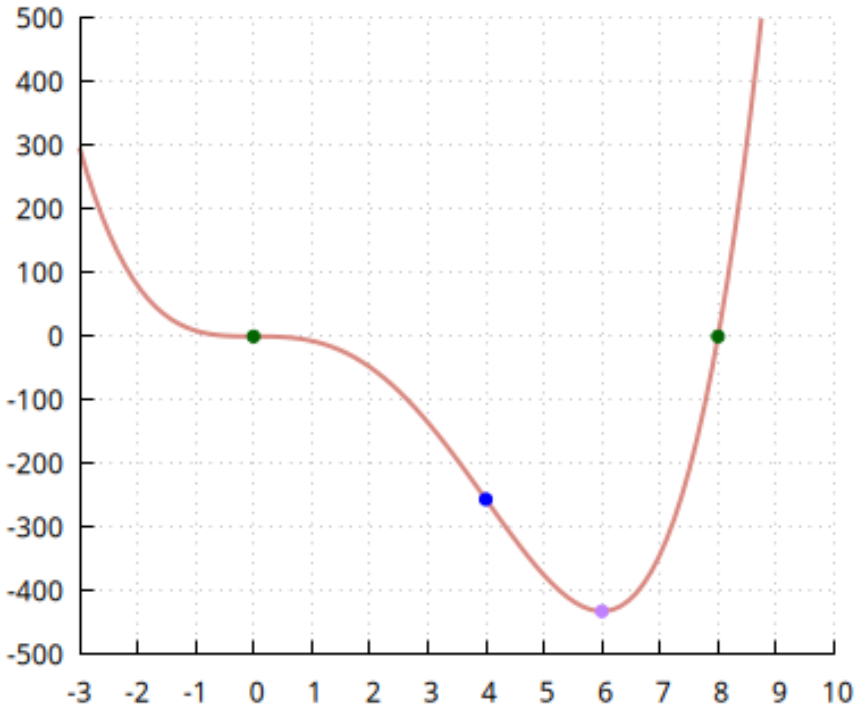
Points of inflection: $(0, 0)$ and $(4, -256)$

C.U.: $x < 0$, $x > 4$

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C.D.: $0 < x < 4$

8.



b) $g(x) = 3x^3 + 7x^2 + 3x - 1$

1. No restrictions on the domain; no asymptotes

2.

Test: $g(-1) = 0$, so $x + 1$ is a factor

$$\begin{array}{rrrrr} -1 & 3 & 7 & 3 & -1 \\ & & -3 & -4 & 1 \\ & 3 & 4 & -1 & 0 \end{array}$$

$$0 = (x+1)(3x^2 + 4x - 1)$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-4 \pm \sqrt{28}}{6}$$

$x_1 = -1 \qquad x_2 \cong 0.215 \qquad x_3 \cong -1.549$

x-int: $(-1, 0)$, $(0.215, 0)$, and $(-1.549, 0)$

$g(0) = -1$

y-int: $(0, -1)$

3.

$$g'(x) = 9x^2 + 14x + 3$$

$$0 = 9x^2 + 14x + 3$$

$$x = \frac{-14 \pm \sqrt{(14)^2 - 4(9)(3)}}{2(9)}$$

$$x = \frac{-14 \pm \sqrt{88}}{18}$$

$x_1 \cong -0.26 \qquad x_2 \cong -1.30$

$g(-0.26) = -1.36 \qquad g(-1.3) = 0.34$

Critical points: $(-0.26, -1.36)$, $(-1.3, 0.34)$

4.

$$g''(x) = 18x + 14$$

$$0 = 18x + 14$$

$$x = -\frac{7}{9} \cong -0.78$$

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$$g\left(-\frac{7}{9}\right) = -\frac{124}{243} \cong -0.51$$

Possible POI: $(-0.78, -0.51)$

5/6.

Test	-2	-1	-0.5	0
$g'(x)$	$+$	$-$	$-$	$+$
$g''(x)$	$-$	$-$	$+$	$+$
$g(x)$	Con. Down Increasing	Con. Down Decreasing	Con. UP Decreasing	Con. UP Increasing

local max, POI, local min

Increasing: $x < -1.3,\ x > -0.26$

Decreasing: $-1.3 < x < -0.26$

7.

Local min: $(-0.26, -1.36)$

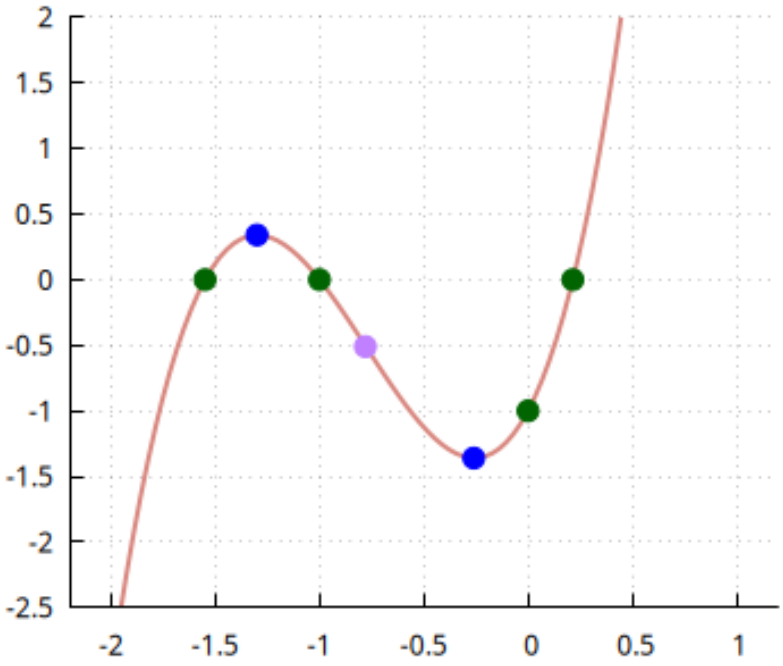
Local max: $(-1.3, 0.34)$

Point of inflection: $(-0.78, -0.51)$

C.U.: $x > -0.78$

C.D.: $x < -0.78$

8.



c) $h(x) = 2x^4 - 26x^2 + 72$

1. No restrictions; no asymptotes

2.

$$0 = 2x^4 - 26x^2 + 72$$
$$0 = x^4 - 13x^2 + 36$$
$$0 = (x^2 - 9)(x^2 - 4)$$
$$0 = (x - 3)(x + 3)(x - 2)(x + 2)$$

$x_1 = -3 \quad x_2 = -2 \quad x_3 = 2 \quad x_4 = 3$

x-int: $(-3, 0), (-2, 0), (2, 0), (3, 0)$

$h(0) = 72$

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y-int: $(0,\ 72)$

3.

$$h'(x) = 8x^3 - 52x$$
$$0 = 4x(2x^2 - 13)$$

$x_1 = 0 \qquad 2x^2 - 13 = 0 \Rightarrow x = \pm\sqrt{6.5}$

$h(0) = 72$

$x_2 \cong 2.55 \qquad x_3 \cong -2.55$

$h(2.55) \cong -12.5 \qquad h(-2.55) \cong -12.5$

Critical points: $(0,\ 72),\ (2.55,\ -12.5),\ (-2.55,\ -12.5)$

4.

$$h''(x) = 24x^2 - 52$$
$$0 = 24x^2 - 52$$
$$x = \pm\sqrt{\frac{13}{6}}$$

$x_1 \cong 1.47 \qquad x_2 \cong -1.47$

$h(1.47) \cong 25.16 \qquad h(-1.47) \cong 25.16$

Possible POI's: $(1.47,\ 25.16),\ (-1.47,\ 25.16)$

5/6.

Test	-3	-2	-1	1	2	3
$h'(x)$	$-$	$+$	$+$	$-$	$-$	$+$
$h''(x)$	$+$	$+$	$-$	$-$	$+$	$+$
$h(x)$	CU dec.	CU inc.	CD inc.	CD dec.	CU dec.	CU inc.

min, POI, max, POI, min

Increasing: $-2.55 < x < 0,\ x > 2.55$

Decreasing: $x < -2.55,\ 0 < x < 2.55$

7.

Local min's: $(-2.55,\ -12.5)$ and $(2.55,\ -12.5)$

Local max: $(0,\ 72)$

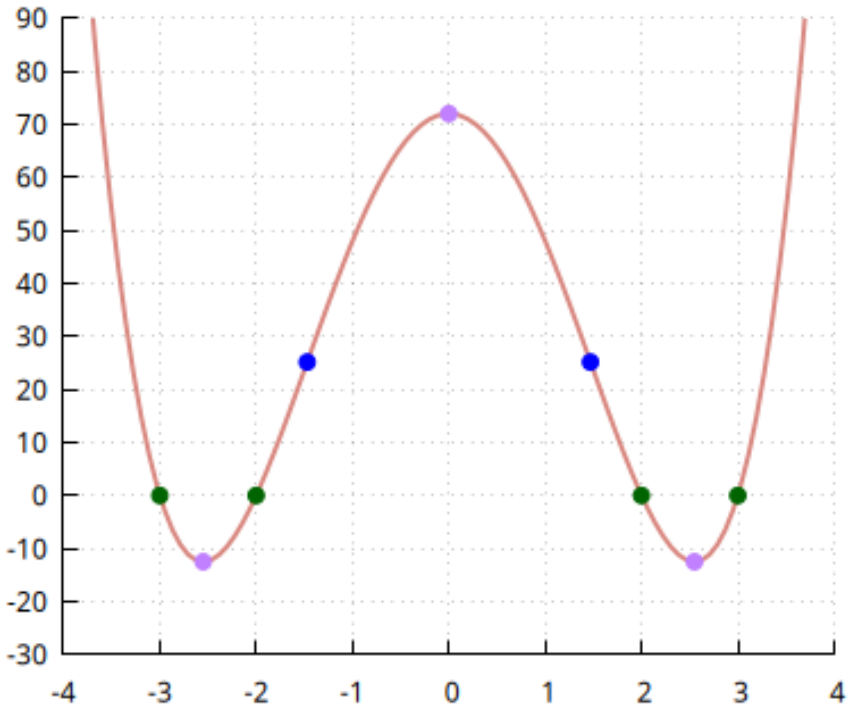
POI's: $(-1.47,\ 25.16)$ and $(1.47,\ 25.16)$

C.U.: $x < -1.47,\ x > 1.47$

C.D.: $-1.47 < x < 1.47$

8.

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d) $j(x) = \dfrac{x^2 + 2x - 4}{x^2}$

1. $x \neq 0$; VA at $x = 0$

HA at $y = 1$

2.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{20}}{2}$$
$$x = -1 \pm \sqrt{5}$$

$(1.24,\ 0)$ and $(-3.24,\ 0)$

y-int: $j(0) = -\dfrac{4}{0}$ is undefined, so no y-intercept.

3.

$$j'(x) = \frac{(2x + 2)(x^2) - 2x(x^2 + 2x - 4)}{x^4}$$
$$j'(x) = \frac{x\left[(2x + 2)(x) - 2(x^2 + 2x - 4)\right]}{x^4}$$
$$j'(x) = \frac{2x^2 + 2x - 2x^2 - 4x + 8}{x^3}$$
$$j'(x) = \frac{-2x + 8}{x^3}$$
$$0 = -2x + 8$$
$$x = 4$$

$x = 0$ is not a critical number because it is NOT in the domain of $j'(x)$

$j(4) = 1.25$

Critical #: $(4,\ 1.25)$

4.

$$j''(x) = \frac{-2(x^3) - 3x^2(-2x + 8)}{x^6}$$
$$j''(x) = \frac{-2x^3 + 6x^3 - 24x^2}{x^6}$$
$$j''(x) = \frac{4x^3 - 24x^2}{x^6}$$
$$j''(x) = \frac{4x^2(x - 6)}{x^6}$$
$$j''(x) = \frac{4(x - 6)}{x^4}$$

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$$0 = 4(x - 6)$$
$$x = 6$$

$j(6) = 1.22$

Possible POI is $(6,\ 1.22)$

5/6.

Test	-1	1	5	7
$j'(x)$	−	+	−	−
$j''(x)$	−	−	−	+
$j(x)$	CD decrease	CD increase	CD decrease	CU decrease

VA, Max, POI

Increasing: $0 < x < 4$

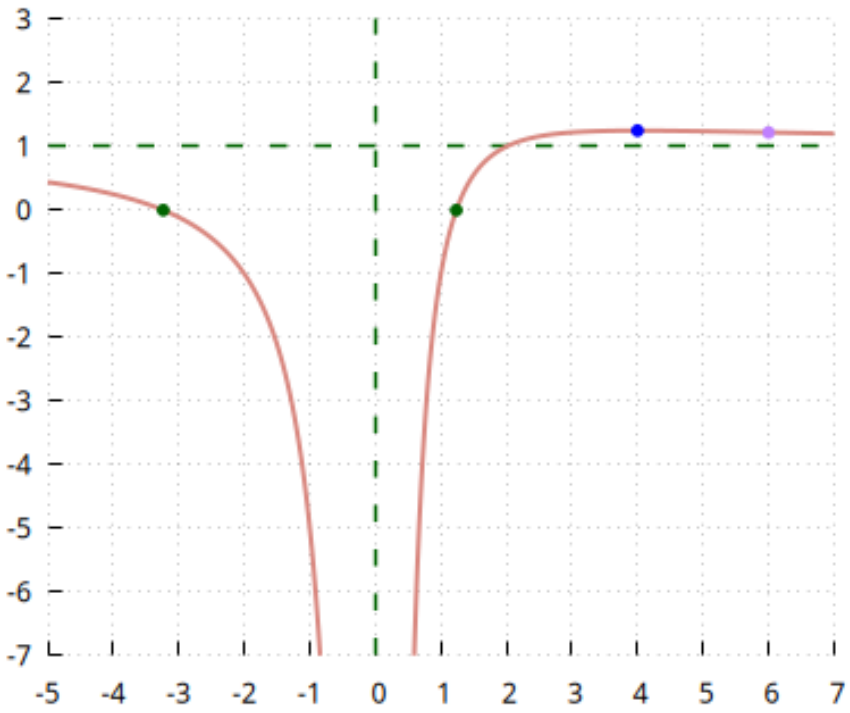
Decreasing: $x < 0,\ x > 4$

7.

CU: $x > 6$

CD: $x < 0,\ 0 < x < 6$

8.



Answer Key

1) a) max: $\left(\frac{1}{3},\ \frac{1}{3}\right)$ b) no local extrema; $(-1,\ 2)$ is an inflection point NOT a max or min

2) a) $\left(\frac{2}{3},\ -\frac{32}{27}\right)$ b) $(-2,\ 624)$, $(2,\ 176)$, and $(0,\ 0)$

3) a) x-int: $(0,\ 0)$ and $(8,\ 0)$; y-int: $(0,\ 0)$; local max: none; local min: $(6,\ -432)$; POI: $(0,\ 0)$ and $(4,\ -256)$; increasing: $x > 6$; decreasing: $x < 0$ and $0 < x < 6$; concave up: $x < 0$ and $x > 4$; concave down: $0 < x < 4$

b) x-int: $(-1,\ 0)$, $(0.215,\ 0)$, and $(-1.549,\ 0)$; y-int: $(0,\ -1)$; local max: $(-1.3,\ 0.34)$; local min: $(-0.26,\ -1.36)$; POI: $(-0.78,\ -0.51)$; increasing: $x < -1.3$ and $x > -0.26$; decreasing: $-1.3 < x < -0.26$; concave up: $x > -0.78$; concave down: $x < -0.78$

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c) x-int: $(-3, 0)$, $(-2, 0)$, $(2, 0)$ and $(3, 0)$; y-int: $(0, 72)$; local max: $(0, 72)$; local min: $(-2.55, -12.5)$ and $(2.55, -12.5)$; POI: $(-1.47, 25.16)$ and $(1.47, 25.16)$; increasing: $-2.55 < x < 0$ and $x > 2.55$; decreasing: $x < -2.55$ and $0 < x < 2.55$; concave up: $x < -1.47$ and $x > 1.47$; concave down: $-1.47 < x < 1.47$

d) VA: $x = 0$; HA: $y = 1$; x-int: $(-3.24, 0)$, and $(1.24, 0)$; y-int: none; local max: $(4, 1.25)$; local min: none; POI: $(6, 1.22)$; increasing: $0 < x < 4$; decreasing: $x < 0$, and $x > 4$; concave up: $x > 6$; concave down: $x < 0$ and $0 < x < 6$

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