2.4 Rational Functions Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Find the equation of any asymptotes for the following functions. Then, find the one-sided limits approaching the vertical asymptotes.

a) $f(x) = \dfrac{x+3}{x^2-4} = \dfrac{x+3}{(x-2)(x+2)}$

HA: $y = 0$

VA: $x = 2,\ x = -2$

Tests: $f(1.99) \cong -125$ $f(2.01) \cong 125$ $f(-1.99) \cong -25$ $f(-2.01) \cong 25$

Limits:

$$\lim_{x\to 2^-} f(x) = -\infty \qquad \lim_{x\to 2^+} f(x) = \infty$$
$$\lim_{x\to -2^-} f(x) = \infty \qquad \lim_{x\to -2^+} f(x) = -\infty$$

b) $y = \dfrac{x^2}{x^2-3x+2} = \dfrac{x^2}{(x-2)(x-1)}$

HA: $y = 1$

VA: $x = 2,\ x = 1$

Tests: $y(1.99) \cong -400$ $y(2.01) \cong 400$ $y(0.99) \cong 97$ $y(1.01) \cong -103$

Limits:

$$\lim_{x\to 2^+} y = \infty \qquad \lim_{x\to 2^-} y = -\infty$$
$$\lim_{x\to 1^+} y = -\infty \qquad \lim_{x\to 1^-} y = \infty$$

c) $y = 2x + \dfrac{1}{x} = \dfrac{2x^2+1}{x}$

SA: $y = 2x$

VA: $x = 0$

Tests: $y(-0.01) \cong -100$ $y(0.01) \cong 100$

Limits:

$$\lim_{x\to 0^+} y = \infty \qquad \lim_{x\to 0^-} y = -\infty$$

d) $g(x) = \dfrac{2x-3}{x^2-6x+9} = \dfrac{2x-3}{(x-3)^2}$

HA: $y = 0$

VA: $x = 3$

Tests: $g(2.99) = 29\,800$ $g(3.01) = 30\,200$

Limits:

$$\lim_{x\to 3^+} g(x) = \infty \qquad \lim_{x\to 3^-} g(x) = \infty$$

2) Find the derivative of each function. Then, determine whether the function has any local extrema.

a) $f(x) = \dfrac{2}{x+3}$

$$f'(x) = \frac{0(x+3) - 1(2)}{(x+3)^2}$$
$$f'(x) = \frac{-2}{(x+3)^2}$$

No critical points since $f'(x) \neq 0$ and $x = -3$ is not in the domain of $f(x)$. So no local extrema.

b) $h(x) = \dfrac{-3}{(x-2)^2}$

$$h'(x) = \frac{0(x-2)^2 - 2(x-2)(1)(-3)}{\left[(x-2)^2\right]^2}$$

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$$h'(x) = \frac{6(x-2)}{(x-2)^4}$$
$$h'(x) = \frac{6}{(x-2)^3}$$

$h'(x) \neq 0$ and $x = 2$ is not in the domain of $h(x)$. So no critical points and no local extrema.

3) Consider the function $f(x) = \frac{-2}{(x+1)^2}$ VA: $x = -1$

a) Find the intervals of increase and decrease for $f(x)$.

$$f'(x) = \frac{0(x+1)^2 - 2(x+1)(1)(-2)}{(x+1)^4}$$
$$f'(x) = \frac{4(x+1)}{(x+1)^4}$$
$$f'(x) = \frac{4}{(x+1)^3}$$

No critical points. Only use VA as a dividing point when testing.

Test	-2		0
$f'(x)$	$-$		$+$
$f(x)$	dec.		inc.

Increasing: $x > -1$

Decreasing: $x < -1$

b) Find the intervals of concavity for $f(x)$.

$$f''(x) = \frac{0(x+1)^3 - 3(x+1)^2(1)(4)}{(x+1)^6}$$
$$f''(x) = \frac{-12(x+1)^2}{(x+1)^6}$$
$$f''(x) = \frac{-12}{(x+1)^4}$$

$f''(x) \neq 0$. Only possible change in concavity is at the VA.

Test	-2		0
$f''(x)$	$-$		$-$
$f(x)$	con. down		con. down

Concave down: $x < -1, x > -1$

Concave up: never

4) Consider the function $h(x) = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)}$

a) Write the equations of the asymptotes

HA: $y = 0$

VA: $x = 2, x = -2$

b) Make a table showing the increasing and decreasing intervals for the function

$$h'(x) = \frac{0(x^2-4) - 2x(1)}{(x^2-4)^2}$$
$$h'(x) = \frac{-2x}{(x^2-4)^2}$$
$$0 = -2x$$
$$x = 0$$
$$h(0) = -0.25$$

Critical point $(0, -0.25)$

Test	-3	-1	1	3
$f'(x)$	$+$	$+$	$-$	$-$
$F(x)$	inc.	inc.	dec.	dec.

Increasing: $x < -2, -2 < x < 0$

Decreasing: $0 < x < 2, x > 2$

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c) How can you use the table from part b) to determine the behavior of $f(x)$ near the vertical asymptotes?

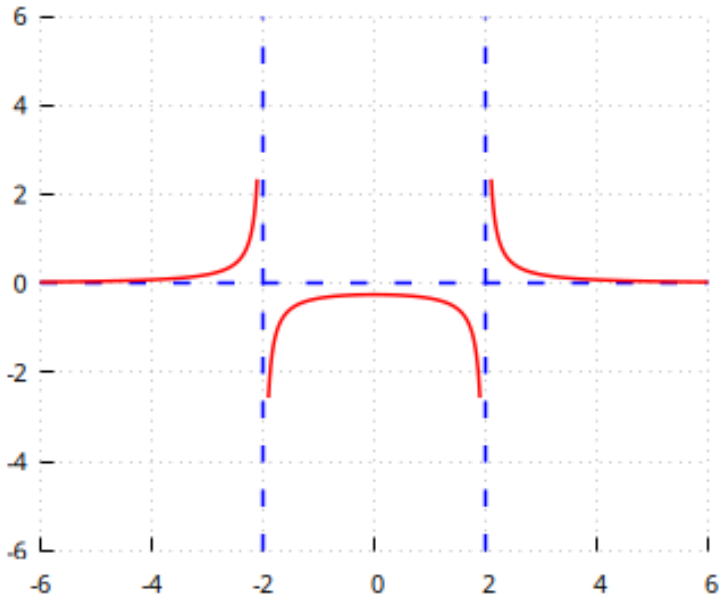
Since the curve is increasing to the left of $x = -2$, $\lim_{x\to -2^-} h(x) = \infty$

Since the curve is increasing to the right of $x = -2$, $\lim_{x\to -2^+} h(x) = -\infty$

Since the curve is decreasing to the left of $x = 2$, $\lim_{x\to 2^-} h(x) = -\infty$

Since the curve is decreasing to the right of $x = 2$, $\lim_{x\to 2^+} h(x) = \infty$

d) Sketch a graph of the function.



Answer Key

1) a) VA: $x = 2$ and $x = -2$; HA: $y = 0$; $\lim_{x\to 2^+} = \infty$, $\lim_{x\to 2^-} = -\infty$, $\lim_{x\to -2^+} = -\infty$, $\lim_{x\to -2^-} = \infty$

b) VA: $x = 1$ and $x = 2$; HA: $y = 1$; $\lim_{x\to 2^+} = \infty$, $\lim_{x\to 2^-} = -\infty$, $\lim_{x\to 1^+} = -\infty$, $\lim_{x\to 1^-} = \infty$

c) VA: $x = 0$; SA: $y = 2x$; $\lim_{x\to 0^+} = \infty$, $\lim_{x\to 0^-} = -\infty$

d) VA: $x = 3$; HA: $y = 0$; $\lim_{x\to 3^+} = \infty$, $\lim_{x\to 3^-} = \infty$

2) a) $f'(x) = \frac{-2}{(x+3)^2}$; no local extrema b) $h'(x) = \frac{6}{(x-2)^3}$; no local extrema

3) a) decreasing when $x < -1$, increasing when $x > -1$ b) concave down when $x < -1$ or $x > -1$

4) a) VA: $x = 2$ and $x = -2$; HA: $y = 0$ b) increasing when $x < -2$ or $-2 < x < 0$; decreasing when $0 < x < 2$ or $x > 2$ c) Since the curve is increasing to the left of $x = -2$, $\lim_{x\to -2^-} = \infty$ Since the curve is increasing to the right of $x = -2$, $\lim_{x\to -2^+} = -\infty$ Since the curve is decreasing to the left of $x = 2$, $\lim_{x\to 2^-} = -\infty$ Since the curve is decreasing to the right of $x = 2$, $\lim_{x\to 2^+} = \infty$

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