2.2 Maxima and Minima Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Find the critical numbers for each function

a) $f(x) = -x^2 + 6x + 2$

$$f'(x) = -2x + 6$$
$$0 = -2x + 6$$
$$x = 3$$

b) $f(x) = x^3 - 2x^2 + 3x$

$$f'(x) = 3x^2 - 4x + 3$$
$$0 = 3x^2 - 4x + 3$$
$$b^2 - 4ac = (-4)^2 - 4(3)(3) = -20$$

$b^2 - 4ac < 0$, so no critical numbers

c) $g(x) = 2x^3 - 3x^2 - 12x + 5$

$$g'(x) = 6x^2 - 6x - 12$$
$$0 = 6(x^2 - x - 2)$$
$$0 = 6(x - 2)(x + 1)$$
$$x_1 = -1 \qquad x_2 = 2$$

d) $y = x - \sqrt{x}$

$$\frac{dy}{dx} = 1 - \frac{1}{2}x^{-\frac{1}{2}}$$
$$\frac{dy}{dx} = 1 - \frac{1}{2\sqrt{x}}$$
$$0 = 1 - \frac{1}{2\sqrt{x}}$$
$$\frac{1}{2\sqrt{x}} = 1$$
$$1 = 2\sqrt{x}$$
$$\left(\frac{1}{2}\right)^2 = x$$
$$x = \frac{1}{4}$$

2) Determine the absolute extreme values of each function on the given interval.

a) $y = 3x^2 - 12x + 7,\ 0 \le x \le 4$

$$\frac{dy}{dx} = 6x - 12$$
$$0 = 6x - 12$$

$x = 2$ is a critical number

$$y(0) = 3(0)^2 - 12(0) + 7 = 7$$
$$y(2) = 3(2)^2 - 12(2) + 7 = -5$$
$$y(4) = 3(4)^2 - 12(4) + 7 = 7$$

Absolute min: $(2, -5)$

Absolute max: $(0, 7)$ and $(4, 7)$

b) $g(x) = 2x^3 - 3x^2 - 12x + 2,\ -3 \le x \le 3$

$$g'(x) = 6x^2 - 6x - 12$$
$$0 = 6(x^2 - x - 2)$$
$$0 = 6(x - 2)(x + 1)$$

Critical numbers: $x_1 = 2 \qquad x_2 = -1$

$$g(-3) = 2(-3)^3 - 3(-3)^2 - 12(-3) + 2 = -43$$
$$g(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 2 = 9$$
$$g(2) = 2(2)^3 - 3(2)^2 - 12(2) + 2 = -18$$
$$g(3) = 2(3)^3 - 3(3)^2 - 12(3) + 2 = -7$$

Absolute min: $(-3, -43)$

Absolute max: $(-1, 9)$

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c) $f(x) = x^3 + x,\ 0 \le x \le 10$

$$f'(x) = 3x^2 + 1$$
$$0 = 3x^2 + 1$$

$x = \pm\sqrt{-\frac{1}{3}}$, so no critical numbers

$$f(0) = 0$$
$$f(10) = 10^3 + 10 = 1010$$

Absolute min: $(0,\ 0)$

Absolute max: $(10,\ 1010)$

3) Find and classify the critical points of each function as a local max, local min, or neither.

a) $y = 4x - x^2$

$$y' = 4 - 2x$$
$$0 = 4 - 2x$$
$$x = 2$$
$$y(2) = 4(2) - (2)^2 = 4$$

$(2,\ 4)$ is a critical point

Test value		1	3
$f'(x)$		+	−
$f(x)$		inc.	dec.

$(2,\ 4)$ is a local MAX

b) $f(x) = (x-1)^4$

$$f'(x) = 4(x-1)^3(1)$$
$$0 = 4(x-1)^3$$
$$0 = (x-1)^3$$
$$x = 1$$
$$f(1) = (1-1)^4 = 0$$

$(1,\ 0)$ is a critical point

Test value		0	2
$f'(x)$		−	+
$f(x)$		dec.	inc.

$(1,\ 0)$ is a local MIN

c) $g(x) = 2x^3 - 24x + 5$

$$g'(x) = 6x^2 - 24$$
$$0 = 6(x^2 - 4)$$
$$0 = 6(x-2)(x+2)$$
$$x_1 = 2 \qquad x_2 = -2$$
$$g(2) = -27 \qquad g(-2) = 37$$

Critical points: $(2,\ -27)$ and $(-2,\ 37)$

Test	-3		0		3
$f'(x)$	+		−		+
$f(x)$	inc.		dec.		inc.

$(-2,\ 37)$ is a local MAX

$(2,\ -27)$ is a local MIN

d) $y = \frac{1}{4}x^4 - \frac{2}{3}x^3$

$$\frac{dy}{dx} = x^3 - 2x^2$$
$$0 = x^2(x-2)$$
$$x_1 = 0 \qquad x_2 = 2$$
$$y(0) = 0 \qquad y(2) = -\frac{4}{3}$$

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Critical points: $(0,\ 0)$ and $\left(2,\ -\frac{4}{3}\right)$

Test value	-1		1		3
$f'(x)$	$-$		$-$		$+$
$f(x)$	dec.		dec.		inc.

$(0,\ 0)$ is neither; $\left(2,\ -\frac{4}{3}\right)$ is a local MIN

4)a) Find the critical numbers of $f(x) = 2x^3 - 3x^2 - 12x + 5$

$$f'(x) = 6x^2 - 6x - 12$$
$$0 = 6(x^2 - x - 2)$$
$$0 = 6(x - 2)(x + 1)$$

Critical #'s: $x_1 = 2 \qquad x_2 = -1$

$$f(2) = -15 \qquad f(-1) = 12$$

Critical points: $(2,\ -15)$ and $(-1,\ 12)$

b) Find any local extrema of $f(x)$.

Test	-2		0		3
$f'(x)$	$+$		$-$		$+$
$f(x)$	inc.		dec.		inc.

$(-1,\ 12)$ is a local MAX

$(2,\ -15)$ is a local MIN

c) Find the absolute extrema of $f(x)$ in the interval $[-2,\ 4]$.

$$f(-2) = 1$$
$$f(-1) = 12$$
$$f(2) = -15$$
$$f(4) = 37$$

Absolute MIN: $(2,\ -15)$

Absolute MAX: $(4,\ 37)$

5) A section of rollercoaster is in the shape of $f(x) = -x^3 - 2x^2 + x + 15$, where x is between -2 and 2.

a) Find all local extrema and explain what portions of the rollercoaster they represent.

$$f'(x) = -3x^2 - 4x + 1$$
$$0 = -3x^2 - 4x + 1$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(-3)(1)}}{2(-3)}$$
$$x = \frac{4 \pm 2\sqrt{7}}{-6} = \frac{-2 \pm \sqrt{7}}{3}$$
$$x_1 \approx -1.55 \qquad x_2 \approx 0.22$$
$$f(-1.55) \approx 12.37 \qquad f(0.22) \approx 15.11$$

Critical points: $(-1.55,\ 12.37)$ and $(0.22,\ 15.11)$

Test	-2		0		1
$f'(x)$	$-$		$+$		$-$
$f(x)$	dec.		inc.		dec.

The coaster starts going down a hill at $x = -2$, reaches a min at $(-1.55,\ 12.37)$, goes up to a max at $(0.22,\ 15.11)$, then continues down until $x = 2$.

b) Is the highest point of this section of the ride at the beginning, the end, or neither?

$$f(-2) = 13$$
$$f(2) = 1$$

The absolute max is at $(0.22,\ 15.11)$; NOT at the beginning or end.

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Answer Key

1) a) $x = 3$ b) no critical numbers c) $x = -1, 2$ d) $x = \frac{1}{4}$

2) a) absolute max at $(0, 7)$ and $(4, 7)$; absolute min at $(2, -5)$ b) absolute max at $(-1, 9)$; absolute min at $(-3, -43)$ c) absolute max at $(10, 1010)$; absolute min at $(0, 0)$

3) a) $(2, 4)$ is a local max b) $(1, 0)$ is a local min c) $(-2, 37)$ is a local max; $(2, -27)$ is a local min d) $(0, 0)$ is neither; $\left(2, -\frac{4}{3}\right)$ is a local min

4) a) $x = -1, 2$ b) $(-1, 12)$ is a local max; $(2, -15)$ is a local min c) $(2, -15)$ is the absolute min, $(4, 37)$ is the absolute max

5) a) The coaster starts down a hill from $x = -2$, reaching a local min at the bottom of a hill at $(-1.55, 12.37)$. It then increases height until it reaches a local max at the top of a hill at $(0.22, 15.11)$. It then continues downward until $x = 2$. b) The highest point is at $(0.22, 15.11)$, not either of the endpoints.

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