2.1 Increasing / Decreasing Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Use critical numbers and the first derivative test to determine when the function is increasing or decreasing.

a) $f(x) = x^3 + 3x^2 + 1$

$$f'(x) = 3x^2 + 6x$$
$$0 = 3x(x+2)$$
$$x_1 = 0 \qquad x_2 = -2$$

Test value	-3	-1	1
$f'(x)$	+	−	+
$f(x)$	Increasing	Decreasing	Increasing

Increasing: $x < -2,\ x > 0$

Decreasing: $-2 < x < 0$

b) $f(x) = x^5 - 5x^4 + 100$

$$f'(x) = 5x^4 - 20x^3$$
$$0 = 5x^3(x-4)$$
$$x_1 = 0 \qquad x_2 = 4$$

Test value	-1	1	5
$f'(x)$	+	−	+
$f(x)$	inc.	dec.	inc.

Increasing: $x < 0,\ x > 4$

Decreasing: $0 < x < 4$

c) $f(x) = 3x^4 + 4x^3 - 12x^2$

$$f'(x) = 12x^3 + 12x^2 - 24x$$
$$0 = 12x(x^2 + x - 2)$$
$$0 = 12x(x+2)(x-1)$$
$$x_1 = 0 \qquad x_2 = -2 \qquad x_3 = 1$$

Test	-3	-1	0.5	2
$f'(x)$	−	+	−	+
$f(x)$	dec.	inc.	dec.	inc.

Increasing: $-2 < x < 0,\ x > 1$

Decreasing: $x < -2,\ 0 < x < 1$

d) $f(x) = (2x-1)^2(x^2-9)$

$$f'(x) = 2(2x-1)(2)(x^2-9) + 2x(2x-1)^2$$
$$0 = 2(2x-1)\left[2(x^2-9) + x(2x-1)\right]$$
$$0 = 2(2x-1)(4x^2 - x - 18)$$
$$0 = 2(2x-1)(4x-9)(x+2)$$
$$x_1 = \frac{1}{2} \qquad x_2 = \frac{9}{4} = 2.25 \qquad x_3 = -2$$

Test	-3	0	1	3
$f'(x)$	−	+	−	+
$f(x)$	dec.	inc.	dec.	inc.

Increasing: $-2 < x < 0.5,\ x > 2.25$

Decreasing: $x < -2,\ 0.5 < x < 2.25$

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2) Suppose that $f(x)$ is a differentiable function with the given derivative. Determine the values of x for which $f(x)$ is increasing and decreasing.

a) $f'(x) = (x-1)(x+2)(x+3)$

$$0 = (x-1)(x+2)(x+3)$$

$$x_1 = 1 \qquad x_2 = -2 \qquad x_3 = -3$$

Test value	-4	-2.5	0	2
$f'(x)$	$-$	$+$	$-$	$+$
$f(x)$	dec.	inc.	dec.	inc.

Increasing: $-3 < x < -2,\ x > 1$

Decreasing: $x < -3,\ -2 < x < 1$

b) $f'(x) = x^2 + 2x - 4$

$$0 = x^2 + 2x - 4$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{20}}{2} = -1 \pm \sqrt{5}$$

$x_1 \approx 1.24 \qquad x_2 \approx -3.24$

Test value	-4	0	2
$f'(x)$	$+$	$-$	$+$
$f(x)$	inc.	dec.	inc.

Increasing: $x < -1 - \sqrt{5},\ x > -1 + \sqrt{5}$

Decreasing: $-1 - \sqrt{5} < x < -1 + \sqrt{5}$

c) $f'(x) = x^3 + 3x^2 - 4x - 12$

$$0 = x^2(x+3) - 4(x+3)$$

$$0 = (x+3)(x^2 - 4)$$

$$0 = (x+3)(x-2)(x+2)$$

$$x_1 = -3 \qquad x_2 = 2 \qquad x_3 = -2$$

Test value	-4	-2.5	0	3
$f'(x)$	$-$	$+$	$-$	$+$
$f(x)$	dec.	inc.	dec.	inc.

Increasing: $-3 < x < -2,\ x > 2$

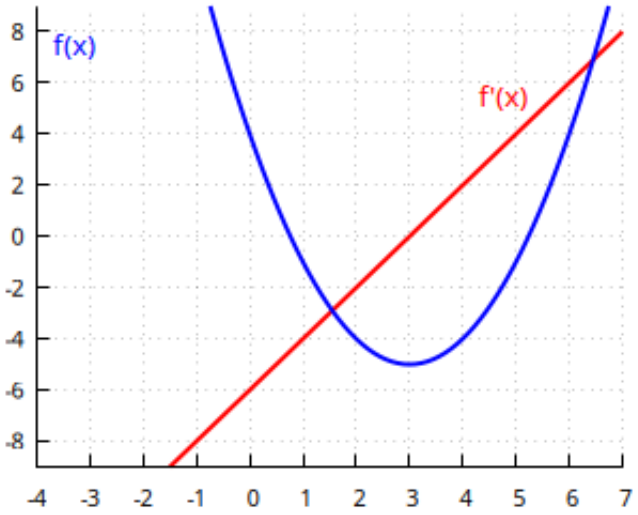
Decreasing: $x < -3,\ -2 < x < 2$

3) Given each graph of $f'(x)$, state the intervals of increase and decrease for the function $f(x)$. Then sketch a possible graph of $f(x)$.

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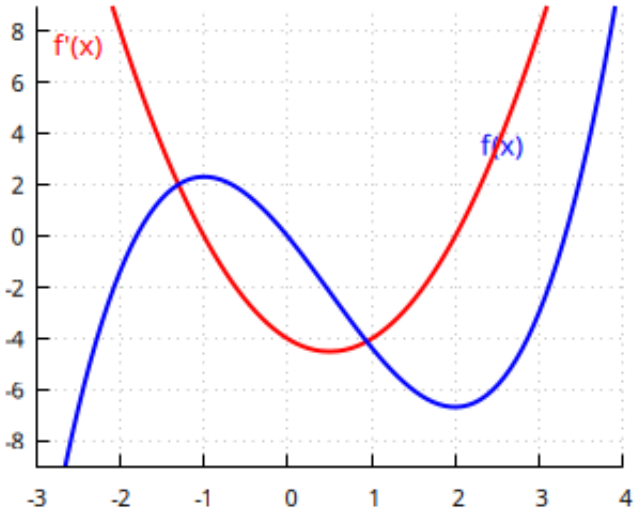
a)



Increasing: $x > 3$

Decreasing: $x < 3$

b)

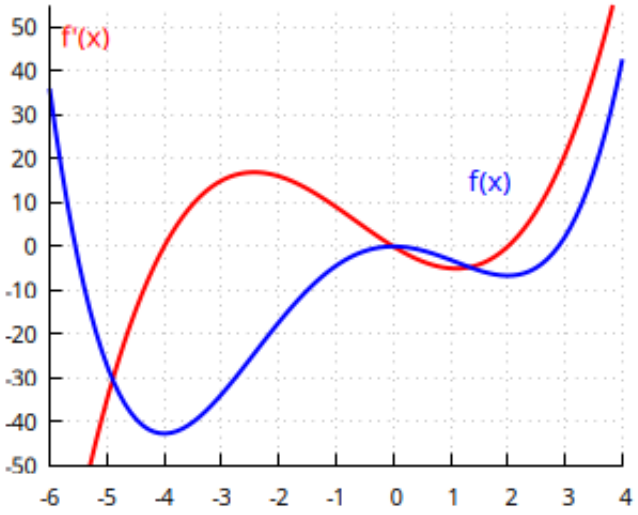


Increasing: $x < -1,\ x > 2$

Decreasing: $-1 < x < 2$

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c)

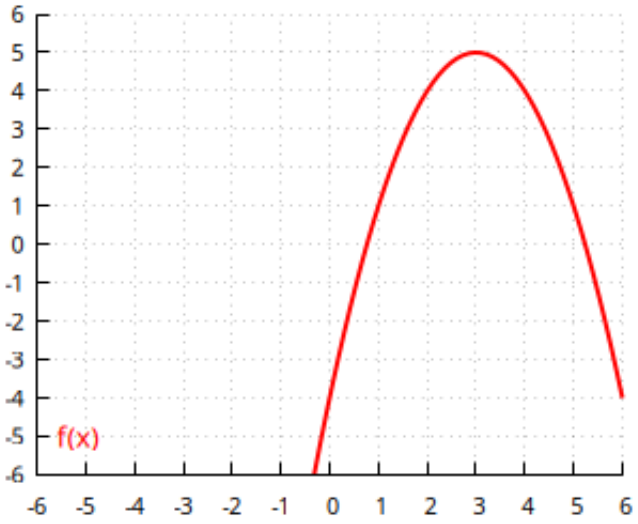


Increasing: $-4 < x < 0,\ x > 2$

Decreasing: $x < -4,\ 0 < x < 2$

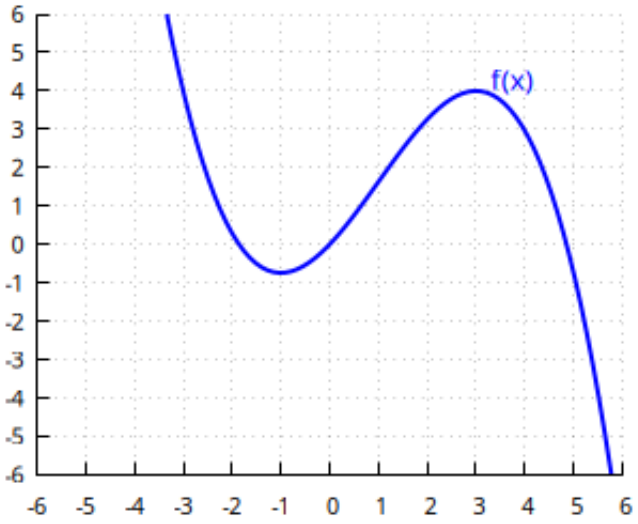
4) Sketch a continuous graph of $f(x)$ given each set of conditions.

a) $f'(x) > 0$ when $x < 3$, $f'(x) < 0$ when $x > 3$, $f(3) = 5$

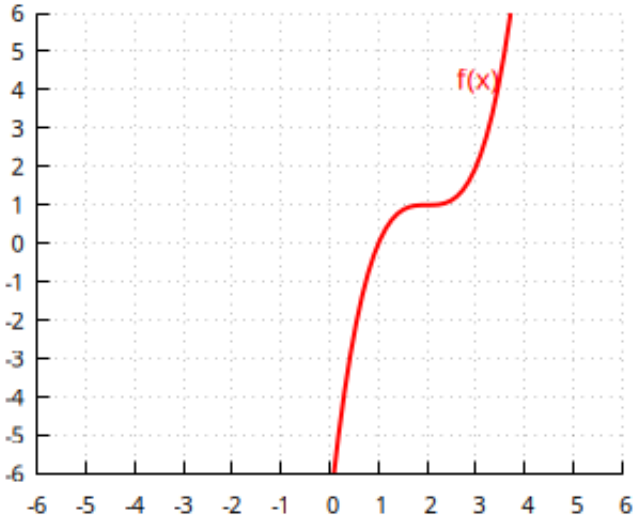


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b) $f'(x) > 0$ when $-1 < x < 3$, $f'(x) < 0$ when $x < -1$ and when $x > 3$, $f(-1) = -\frac{20}{27}$, $f(3) = 4$



c) $f'(x) > 0$ when $x \neq 2$, $f(2) = 1$



Answer Key

1) a) increasing: $x < -2,\ x > 0$; decreasing: $-2 < x < 0$ b) increasing: $x < 0,\ x > 4$; decreasing: $0 < x < 4$ c) increasing: $-2 < x < 0,\ x > 1$; decreasing: $x < -2,\ 0 < x < 1$ d) increasing: $-2 < x < 0.5,\ x > 2.25$; decreasing: $x < -2,\ 0.5 < x < 2.25$

2) a) increasing: $-3 < x < -2,\ x > 1$; decreasing: $x < -3,\ -2 < x < 1$ b) increasing: $x < -1 - \sqrt{5},\ x > -1 + \sqrt{5}$; decreasing: $-1 - \sqrt{5} < x < -1 + \sqrt{5}$ c) increasing: $-3 < x < -2,\ x > 2$; decreasing: $x < -3,\ -2 < x < 2$

3) a) increasing: $x > 3$; decreasing: $x < 3$ b) increasing: $x < -1,\ x > 2$; decreasing: $-1 < x < 2$ c) increasing: $-4 < x < 0,\ x > 2$; decreasing: $x < -4,\ 0 < x < 2$

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