2.7 Unit 2 Pretest – Curve Sketching with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

This review covers all of Unit 2: Curve Sketching, and is meant to be used to prepare for the unit test.

Section 1: 2.1 – Increasing / Decreasing

1) Find the increasing and decreasing intervals for each function.

a) $f(x) = 7 + 6x - x^2$

$$f'(x) = 6 - 2x$$
$$0 = 6 - 2x$$
$$x = 3$$

Test	4
$f'(x)$	$-$
$f(x)$	dec.

Increasing: $x < 3$

Decreasing: $x > 3$

b) $y = x^3 - 48x + 5$

$$y' = 3x^2 - 48$$
$$0 = 3x^2 - 48$$
$$x^2 = 16$$
$$x = \pm 4$$

Test	-5	0
y'	$+$	$-$

Increasing: $x < -4,\ x > 4$

Decreasing: $-4 < x < 4$

c) $g(x) = x^4 - 18x^2$

$$g'(x) = 4x^3 - 36x$$
$$0 = 4x(x^2 - 9)$$
$$0 = 4x(x - 3)(x + 3)$$
$$x_1 = 0 \qquad x_2 = 3 \qquad x_3 = -3$$

Test	-4	-1	1
$g'(x)$	$-$	$+$	$-$

Increasing: $-3 < x < 0,\ x > 3$

Decreasing: $x < -3,\ 0 < x < 3$

d) $f(x) = x^3 + 10x - 9$

$$f'(x) = 3x^2 + 10$$
$$0 = 3x^2 + 10$$

No critical #'s

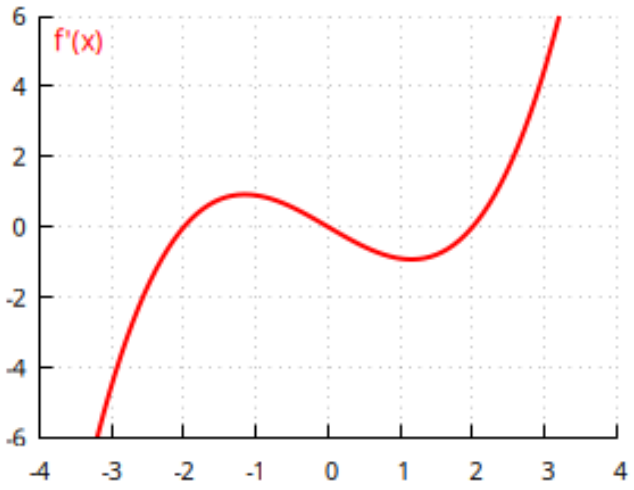
$f'(x) > 0$ for $x \in \mathbb{R}$, so $f(x)$ is always increasing.

Section 2: 2.1 – Reading a Derivative Graph

2) Given the graph of $f'(x)$, state the intervals of increase and decrease for the function $f(x)$. Then sketch a possible

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graph of $f(x)$.

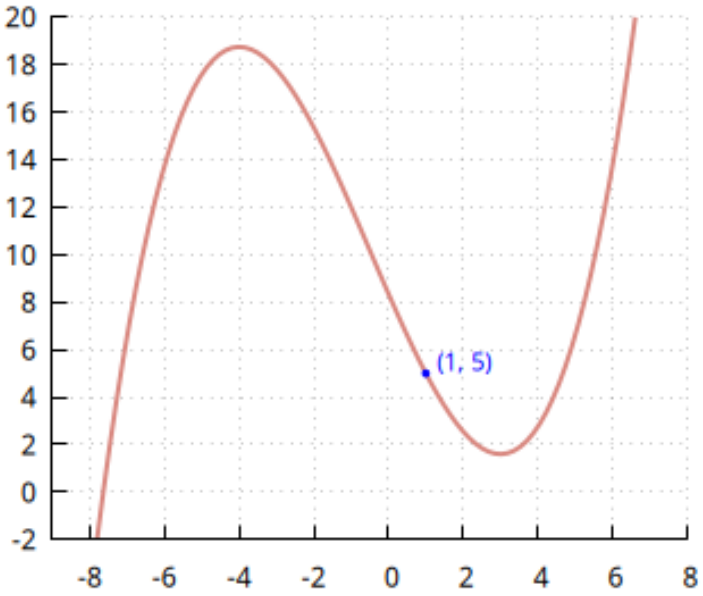


Increasing: $-2 < x < 0,\ x > 2$

Decreasing: $x < -2,\ 0 < x < 2$

Section 3: 2.1 – Sketching From Conditions

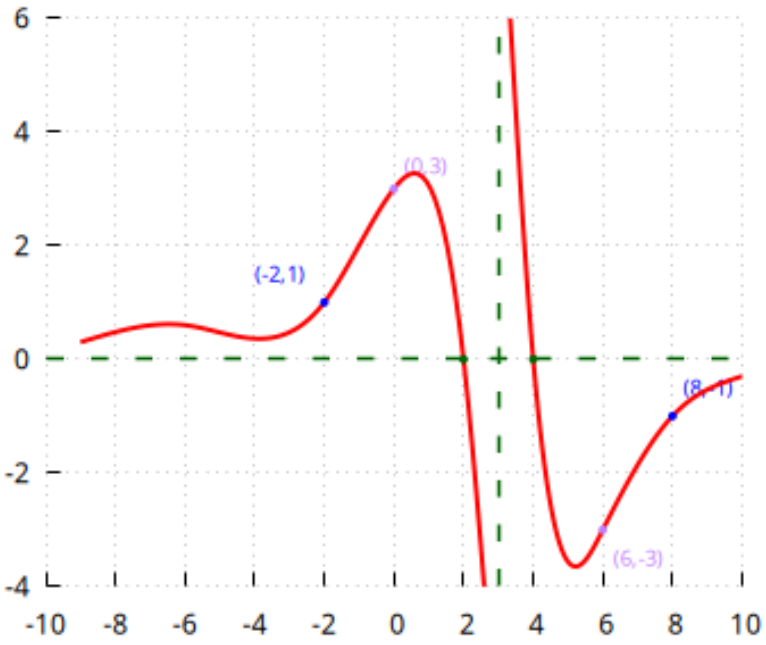
3) Sketch a continuous graph that satisfies the following set of conditions: $f'(x) > 0$ when $x < -4$ and $x > 3$, $f'(x) < 0$ when $-4 < x < 3$ and $f(1) = 5$.



4) Given the following information about $y = f(x)$, sketch a graph for the function on the axes provided below. Label an appropriate scale for the sketch.

Local minimum $(6, -3)$. Local maximum $(0, 3)$. Points of inflection at $(-2, 1)$ and $(8, -1)$. Increasing when $x < 0$ and $x > 6$; decreasing when $0 < x < 3$ and $3 < x < 6$. Concave up when $x < -2$ and $3 < x < 8$; concave down when $-2 < x < 3$ and $x > 8$. HA at $y = 0$. VA at $x = 3$. y-intercept at $(0, 3)$. x-intercepts at $(2, 0)$ and $(4, 0)$.

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Section 4: 2.2 – Maxima and Minima

5) Find the local extrema for each function and classify them as local max or local min.

a) $f(x) = 16 - x^4$

$$f'(x) = -4x^3$$
$$0 = -4x^3$$
$$x = 0$$
$$f(0) = 16$$

Test	-1	1
$f'(x)$	+	−
$f(x)$	inc.	dec.

Local max at $(0,\ 16)$

b) $g(x) = x^3 + 9x^2 - 21x - 12$

$$g'(x) = 3x^2 + 18x - 21$$
$$0 = x^2 + 6x - 7$$
$$0 = (x+7)(x-1)$$
$$x_1 = -7 \qquad x_2 = 1$$
$$g(-7) = 233 \qquad g(1) = -23$$

Test	-8	0	2
$g'(x)$	+	−	+

Local max: $(-7,\ 233)$

Local min: $(1,\ -23)$

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Section 5: 2.6 – Optimization: Max/Min of a Function

6) The speed, in km/h, of a certain car t seconds after passing a police radar location is given by the function $v(t) = 3t^2 - 24t + 88$.

a) Find the min speed of the car.

$$v'(t) = 6t - 24$$
$$0 = 6(t - 4)$$
$$t = 4$$

Critical point: $v(4) = 40$

Second derivative test:

$$v''(t) = 6$$

$v''(4) = 6$; concave up, so a local min.

The min speed is 40 km/h.

b) The radar tracks the car on the interval $2 \le t \le 5$. Find the max speed of the car on this interval.

Test endpoints and critical #'s.

$$v(2) = 52 \text{ km/h}$$
$$v(4) = 40 \text{ km/h}$$
$$v(5) = 43 \text{ km/h}$$

Max speed is 52 km/h.

Section 6: 2.2 – Absolute Extrema on an Interval

7) Determine the absolute extreme values of each function on the given interval.

a) $y = x^2 - 3x + 2, \ -4 \le x \le 4$

$$y' = 2x - 3$$
$$0 = 2x - 3$$
$$x = \frac{3}{2} = 1.5$$

Test endpoints and Critical #'s:

$$y(-4) = 30$$
$$y(1.5) = -0.25$$
$$y(4) = 6$$

Absolute max: $(-4, 30)$

Absolute min: $(1.5, -0.25)$

b) $g(x) = 2x^3 - 24x + 3, \ -4 \le x \le 2$

$$g'(x) = 6x^2 - 24$$
$$0 = 6(x^2 - 4)$$
$$0 = 6(x - 2)(x + 2)$$
$$x_1 = 2 \qquad x_2 = -2$$

Tests:

$$g(-4) = -29$$
$$g(-2) = 35$$
$$g(2) = -29$$

Absolute max: $(-2, 35)$

Absolute min: $(-4, -29)$ and $(2, -29)$

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Section 7: 2.3 – Points of Inflection and Concavity

8) For the function $f(x) = x^4 - 2x^3 - 12x^2 + 3$, determine the points of inflection and the intervals of concavity.

$$f'(x) = 4x^3 - 6x^2 - 24x$$
$$f''(x) = 12x^2 - 12x - 24$$

Possible POI:

$$0 = 12(x^2 - x - 2)$$
$$0 = 12(x - 2)(x + 1)$$
$$x_1 = 2 \qquad x_2 = -1$$
$$f(2) = -45 \qquad f(-1) = -6$$

Test	-2	0	3
$f''(x)$	+	−	+
$f(x)$	C.U.	C.D.	C.U.

Concave up: $x < -1,\ x > 2$

Concave down: $-1 < x < 2$

Points of inflection: $(-1, -6)$ and $(2, -45)$

Section 8: 2.3 – Second Derivative Test (Edge Case)

9) For the function $f(x) = 2x^3 - x^4$, determine the critical points and classify them using the second derivative test.

$$f'(x) = 6x^2 - 4x^3$$
$$0 = 2x^2(3 - 2x)$$
$$x_1 = 0 \qquad x_2 = \frac{3}{2} = 1.5$$

Critical Points:

$$f(0) = 0 \qquad (0,\ 0)$$
$$f(1.5) = \frac{27}{16} = 1.6875 \qquad (1.5,\ 1.6875)$$

Second Derivative Test:

$$f''(x) = 12x - 12x^2$$

$f''(0) = 0$; 2nd derivative test fails; would need to do 1st derivative test to classify critical point.

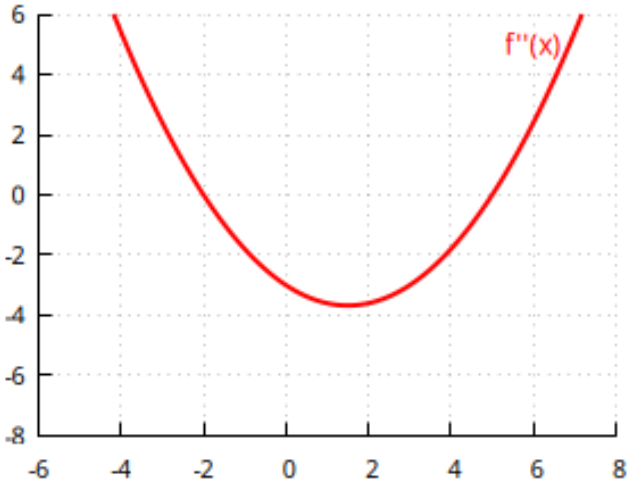
$f''(1.5) = -9$; concave down, local max at $(1.5,\ 1.6875)$

$(0,\ 0)$ is a point of inflection NOT a local min or max; $(1.5,\ 1.7)$ is a local max

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Section 9: 2.3 – Reading a Second Derivative Graph

10) Given the graph of $f''(x)$, state the intervals of concavity for $f(x)$.



Concave up: $x < -2,\ x > 5$

Concave down: $-2 < x < 5$

Section 10: 2.4 – Asymptotes of Rational Functions

11) For each function, state equations for any asymptotes.

a) $f(x) = \dfrac{x^2 - 4}{x}$

SA: $y = x$

VA: $x = 0$

b) $g(x) = \dfrac{2x - 3}{2x - 4}$

HA: $y = 1$

VA: $x = 2$

c) $y = \dfrac{x^2 + 1}{x^2 - 3x - 10} = \dfrac{x^2 + 1}{(x - 5)(x + 2)}$

HA: $y = 1$

VA: $x = 5$ and $x = -2$

d) $\dfrac{x - 1}{x^2 + 2x + 1} = \dfrac{x - 1}{(x + 1)^2}$

HA: $y = 0$

VA: $x = -1$

Section 11: 2.4 – Tangent to a Rational Function

12) State the equation of the tangent to the graph of $f(x) = \dfrac{x + 1}{x^2 + 1}$ at the point where $x = -1$.

Point:

$$f(-1) = \frac{(-1) + 1}{(-1)^2 + 1}$$

$$f(-1) = 0 \qquad (-1,\ 0)$$

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Slope:

$$f'(x) = \frac{1(x^2+1) - 2x(x+1)}{(x^2+1)^2}$$
$$f'(x) = \frac{-x^2 - 2x + 1}{(x^2+1)^2}$$
$$f'(-1) = \frac{-1(-1)^2 - 2(-1) + 1}{\left[(-1)^2+1\right]^2}$$
$$f'(-1) = \frac{1}{2}$$

Equation:

$$y = mx + b$$
$$0 = \left(\frac{1}{2}\right)(-1) + b$$
$$b = \frac{1}{2}$$
$$y = \frac{1}{2}x + \frac{1}{2}$$

Section 12: 2.5 – Full Curve Sketching Algorithm

13) Analyze and sketch each function using the algorithm for curve sketching

a) $k(x) = \frac{1}{4}x^4 - \frac{9}{2}x^2$

1. No domain restrictions; no asymptotes

2.

$$0 = \frac{1}{4}x^2(x^2 - 18)$$

x-int: $(0,\ 0)$, $(4.24,\ 0)$, $(-4.24,\ 0)$

y-int: $(0,\ 0)$

3.

$$k'(x) = x^3 - 9x$$
$$0 = x(x^2 - 9)$$
$$0 = x(x-3)(x+3)$$
$$x_1 = 0 \qquad x_2 = 3 \qquad x_3 = -3$$
$$k(0) = 0 \qquad k(3) = -20.25 \qquad k(-3) = -20.25$$

Critical points: $(0,\ 0)$, $(3,\ -20.25)$, $(-3,\ -20.25)$

4.

$$k''(x) = 3x^2 - 9$$
$$0 = 3(x^2 - 3)$$
$$x = \pm\sqrt{3} \cong \pm 1.73$$

Possible POI: $(\sqrt{3},\ -11.25)$ and $(-\sqrt{3},\ -11.25)$

5/6/7.

Test	-4	-2	-1	1	2	4	
$k'(x)$	$-$	$+$	$+$	$-$	$-$	$+$	
$k''(x)$	$+$	$+$	$-$	$-$	$+$	$+$	
$k(x)$	dec. CU	inc. CU	inc. CD	dec. CD	dec. CU	inc. CU	

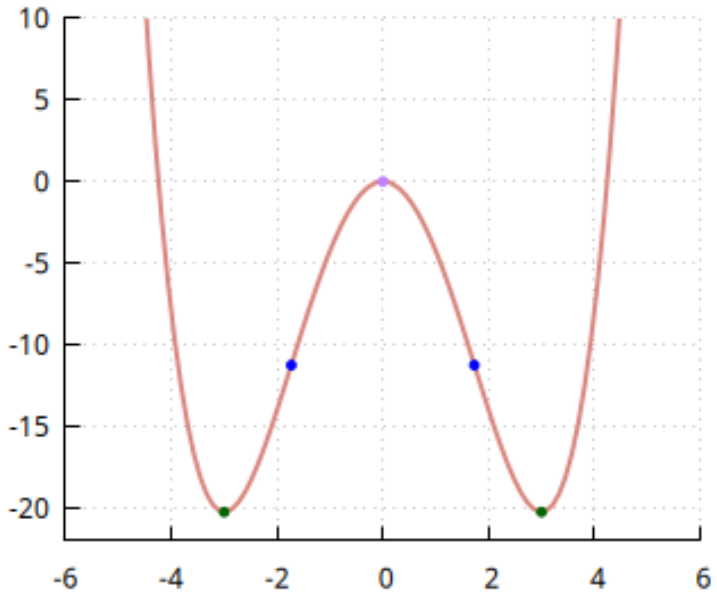
Local min: $(-3,\ -20.25)$ and $(3,\ -20.25)$

Local max: $(0,\ 0)$

POI: $(-\sqrt{3},\ -11.25)$ and $(\sqrt{3},\ -11.25)$

8.

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b) $h(x) = 2x^3 - 3x^2 - 3x + 2$

1. No domain restrictions; no asymptotes.

2.

$h(-1) = 0$; so $x + 1$ is a factor

$$\begin{array}{rrrr} -1 & 2 & -3 & -3 & 2 \\ & & -2 & 5 & -2 \\ & 2 & -5 & 2 & 0 \end{array}$$

$$0 = (x+1)(2x^2 - 5x + 2)$$

$$0 = (x+1)[2x(x-2) - 1(x-2)]$$

$$0 = (x+1)(x-2)(2x-1)$$

$$x_1 = -1 \qquad x_2 = 2 \qquad x_3 = \frac{1}{2}$$

x-int: $(-1, 0)$, $(2, 0)$, and $(0.5, 0)$

y-int: $h(0) = 2$ $\quad (0, 2)$

3.

$$h'(x) = 6x^2 - 6x - 3$$

$$0 = 3(2x^2 - 2x - 1)$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{2 \pm 2\sqrt{3}}{4}$$

$$x = \frac{1 \pm \sqrt{3}}{2}$$

$x_1 \cong 1.37 \qquad x_2 \cong -0.37$

Critical points: $(1.37, -2.6)$, $(-0.37, 2.6)$

4.

$$h''(x) = 12x - 6$$

$$0 = 6(2x - 1)$$

$$x = \frac{1}{2}$$

Possible POI: $(0.5, 0)$

5/6/7.

Test	-1	-0.37	0	0.5	1	1.37
$h'(x)$	$+$	0	$-$	$-$	$-$	0
$h''(x)$	$-$	$-$	$-$	0	$+$	$+$
$h(x)$	inc. CD	local max	dec. CD	POI	dec. CU	local min

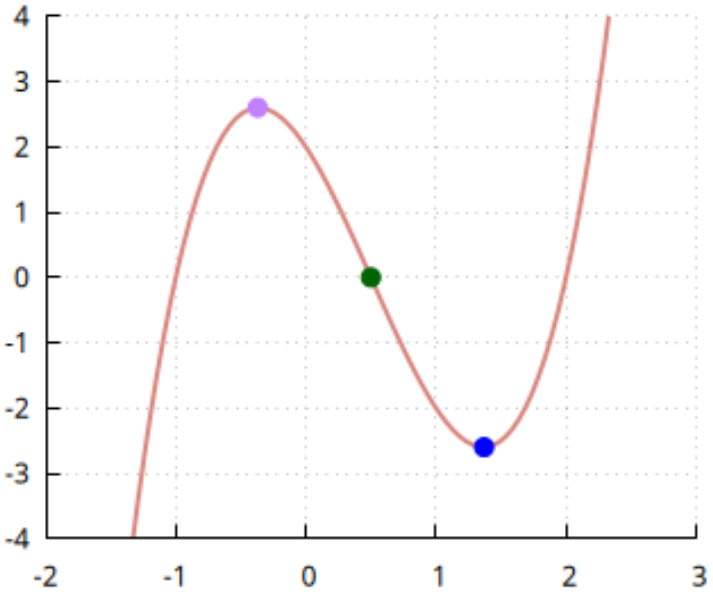
Local min: $(1.37, -2.6)$

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Local max: $(-0.37,\ 2.6)$

POI: $(0.5,\ 0)$

8.



c) $f(x) = \dfrac{x^2 + 2x - 4}{x^2}$

1. HA: $y = 1$

VA: $x = 0$

2.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$
$$x = -1 \pm \sqrt{5}$$

x-int: $(1.24,\ 0)$ and $(-3.24,\ 0)$

y-int: $f(0) = -\dfrac{4}{0}$ is undefined, so no y-intercept.

3.

$$f'(x) = \frac{(2x+2)(x^2) - 2x(x^2 + 2x - 4)}{x^4}$$
$$f'(x) = \frac{x\left[(2x+2)x - 2(x^2 + 2x - 4)\right]}{x^4}$$
$$f'(x) = \frac{-2x + 8}{x^3}$$
$$0 = -2x + 8$$
$$x = 4$$

Critical point: $(4,\ 1.25)$

4.

$$f''(x) = \frac{-2(x^3) - 3x^2(-2x + 8)}{x^6}$$
$$f''(x) = \frac{4x - 24}{x^4}$$
$$0 = 4x - 24$$
$$x = 6$$

Possible POI: $(6,\ 1.22)$

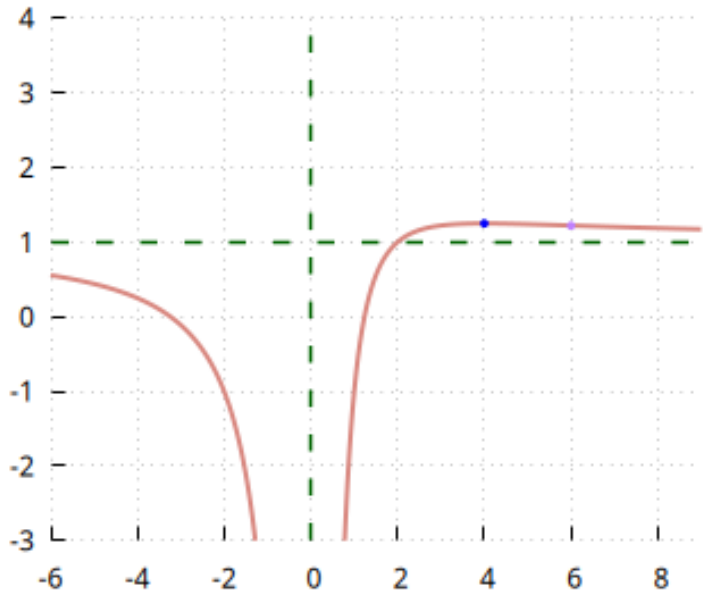
5/6/7.

Test	-1	1	5	7
$f'(x)$	$-$	$+$	$-$	$-$
$f''(x)$	$-$	$-$	$-$	$+$

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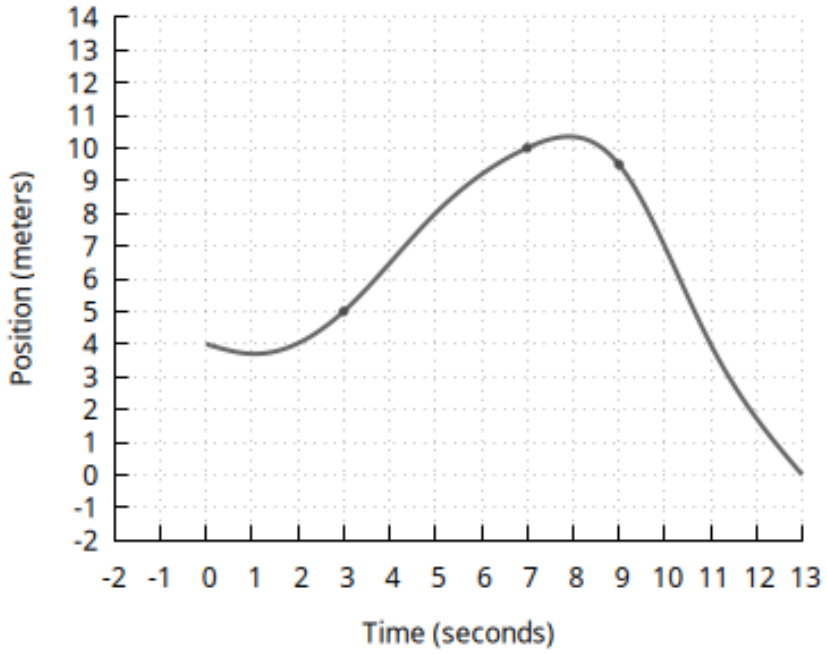
VA, local max, POI

8.



Section 13: 2.1 – Reading a Graph (Derivative Signs at Points)

14) Consider the graph to the right.



a) How many times does the function have a derivative $= 0$? How do you know?

2 times; 2 turning points.

b) State the sign of the first and second derivatives at the following times.

i) $t = 3$ s

$y'(3) = +$, $y''(3) = +$

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ii) $t = 7$ s

$y'(7) = +$, $y''(7) = -$

iii) $t = 10$ s

$y'(10) = -$, $y''(10) = -$

Section 14: 2.6 – Optimization: Containers

15) A garbage can is in the shape of a cylinder with no lid. It needs to have a volume of 5000 cm^3. What will be the radius and height of the can that uses the least amount of material to construct it? (Hint: Surface Area)

$$V = \pi r^2 h$$
$$5000 = \pi r^2 h$$
$$h = \frac{5000}{\pi r^2}$$
$$SA = \pi r^2 + 2\pi r h$$
$$SA(r) = \pi r^2 + 2\pi r\left(\frac{5000}{\pi r^2}\right)$$
$$SA(r) = \pi r^2 + 10\,000 r^{-1}$$
$$SA'(r) = 2\pi r - 10\,000 r^{-2}$$
$$0 = 2\pi r - \frac{10\,000}{r^2}$$
$$\frac{10\,000}{r^2} = 2\pi r$$
$$\frac{10\,000}{2\pi} = r^3$$
$$r = \left(\frac{5000}{\pi}\right)^{\frac{1}{3}}$$
$$r \cong 11.7 \text{ cm}$$

Verify $r \cong 11.7$ is a max:

Test	10	12
$SA'(r)$	$-$	$+$
$SA(r)$	dec.	inc.

MIN

$$h = \frac{5000}{\pi\left(\frac{5000}{\pi}\right)^{\frac{2}{3}}}$$
$$h \cong 11.7 \text{ cm}$$

Min SA when: $r \cong 11.7$ cm, $h \cong 11.7$ cm

Section 15: 2.6 – Optimization: Open Box

16) A carpenter builds an open box with a square base. She has 8 m^2 of wood available. Find the volume of the largest box she can build.

$$V = b^2 h$$
$$SA = b^2 + 4bh$$
$$8 = b^2 + 4bh$$
$$h = \frac{8 - b^2}{4b}$$
$$V(b) = b^2\left(\frac{8 - b^2}{4b}\right)$$
$$V(b) = \frac{b(8 - b^2)}{4}$$
$$V(b) = \frac{8b - b^3}{4}$$
$$V(b) = 2b - \frac{1}{4}b^3$$
$$V'(b) = 2 - \frac{3}{4}b^2$$
$$0 = 2 - \frac{3}{4}b^2$$
$$\frac{3}{4}b^2 = 2$$

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$$b^2 = \frac{8}{3}$$
$$b = \pm\sqrt{\frac{8}{3}}$$
$$b \cong 1.63$$

2nd derivative test:

$$V''(b) = -\frac{3}{2}b$$
$$V''(1.63) \cong -2.445$$

So $V(1.63)$ is a MAX.

$$V(1.63) \cong 2.18 \text{ m}^3$$

The max volume is 2.18 m^3.

Section 16: 2.6 – Optimization: Two-Material Box

17) Sandy will make a closed rectangular jewellery box with a square base from two different woods. The wood from the top and bottom costs \$0.0020/cm^2. The wood for the sides costs \$0.0030/cm^2. Find the dimensions that minimize the cost for a box with volume 4000 cm^3.

$$SA = 2x^2 + 4xh$$

Constraint:

$$V = x^2h$$
$$4000 = x^2h$$
$$h = \frac{4000}{x^2}$$
$$SA(x) = 2x^2 + 4x\left(\frac{4000}{x^2}\right)$$
$$SA(x) = 2x^2 + 16\,000x^{-1}$$
$$C(x) = 0.002(2x^2) + 0.003(16\,000)x^{-1}$$
$$C(x) = 0.004x^2 + 48x^{-1}$$
$$C'(x) = 0.008x - 48x^{-2}$$
$$0 = 0.008x - \frac{48}{x^2}$$
$$\frac{48}{x^2} = 0.008x$$
$$6000 = x^3$$
$$x \cong 18.17$$

2nd derivative test:

$$C''(x) = 0.008 + 96x^{-3}$$
$$C''(18.17) \cong 0.024$$

Since $C''(18.17) > 0$, $C(18.17)$ is concave UP and $(18.17,\ C(18.17))$ is a MIN point.

Answer: The min cost of $C(18.17) = 3.96$ dollars will occur when the length and width of the box are 18.17 cm and the height is 12.12 cm.

Section 17: 2.6 – Optimization: Profit

18) A music store sells an average of 120 CDs per week at \$24 each. A market survey indicates that for each \$0.75 decrease in price, five additional CDs will be sold per week. The cost of producing x CDs is $C(x) = -0.003x^2 + 3x + 2000$. What price and quantity of CDs maximizes profit?

Number sold: $x = 120 + 5n \Rightarrow n = \frac{x - 120}{5} = 0.2x - 24$

Price: $p = 24 - 0.75n$

$$p(x) = 24 - 0.75(0.2x - 24)$$
$$p(x) = 24 - 0.15x + 18$$
$$p(x) = 42 - 0.15x$$
$$R(x) = x(42 - 0.15x)$$
$$R(x) = 42x - 0.15x^2$$
$$P(x) = (42x - 0.15x^2) - (-0.003x^2 + 3x + 2000)$$
$$P(x) = -0.147x^2 + 39x - 2000$$
$$P'(x) = -0.294x + 39$$

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$$0 = -0.294x + 39$$
$$x \cong 132.65 \cong 133 \text{ CD's}$$
$$p(133) = 42 - 0.15(133) = 22.05 \text{ dollars}$$

$P''(133) < 0$; a max

Selling 133 CDs for $22.05 each maximizes the profit.

Section 18: 2.6 – Optimization: Closest Distance

19) A boat leaves a dock at 2:00 p.m., heading west at 15 km/h. Another boat heads south at 12 km/h and reaches the same dock at 3:00 p.m. When were the boats closest to each other?

Let t = hours after 2 pm

$$s(t) = \sqrt{(15t)^2 + (12 - 12t)^2}$$
$$s(t) = \sqrt{225t^2 + 144 - 288t + 144t^2}$$
$$s(t) = \left(369t^2 - 288t + 144\right)^{\frac{1}{2}}$$
$$s'(t) = \frac{1}{2}\left(369t^2 - 288t + 144\right)^{-\frac{1}{2}}(738t - 288)$$
$$s'(t) = \frac{369t - 144}{\left(369t^2 - 288t + 144\right)^{\frac{1}{2}}}$$
$$0 = 369t - 144$$
$$t \cong 0.39 \text{ hours}$$

1st derivative test:

0.39	
−	+
dec.	inc.

min

The boats were closest together 0.39 hours after 2 pm. (2:23 pm).

Answer Key

1) a) increasing: $(-\infty, 3)$; decreasing: $(3, \infty)$ b) increasing: $(-\infty, -4) \cup (4, \infty)$; decreasing: $(-4, 4)$ c) increasing: $(-3, 0) \cup (3, \infty)$; decreasing: $(-\infty, -3) \cup (0, 3)$ d) increasing: $(-\infty, \infty)$; decreasing: never

2) increasing: $(-2, 0) \cup (2, \infty)$; decreasing: $(-\infty, -2) \cup (0, 2)$

5) a) $(0, 16)$ is a local max b) local max at $(-7, 233)$; local min at $(1, -23)$

6) a) 40 km/h b) 52 km/h

7) a) absolute min: $(1.5, -0.25)$; absolute max: $(-4, 30)$ b) absolute min: $(-4, -29)$ and $(2, -29)$; absolute max: $(-2, 35)$

8) concave up: $x < -1$ and $x > 2$; concave down: $-1 < x < 2$; points of inflection: $(-1, -6)$ and $(2, -45)$

9) $(0, 0)$ is a point of inflection NOT a local min or max; $(1.5, 1.7)$ is a local max

10) concave up: $x < -2$, $x > 5$; concave down: $-2 < x < 5$

11) a) VA: $x = 0$; SA: $y = x$ b) VA: $x = 2$; HA: $y = 1$ c) VA: $x = 5$ and $x = -2$; HA: $y = 1$ d) VA: $x = -1$; HA: $y = 0$

12) $y = \frac{1}{2}x + \frac{1}{2}$

14) a) twice—there are two turning points for the graph, so two points with $y' = 0$ b) i) $y' > 0, y'' > 0$ ii) $y' > 0, y'' < 0$ iii) $y' < 0, y'' < 0$

15) $r = h = 11.7$ cm

16) $V \cong 2.18 \text{ m}^3$

17) 12.1 cm X 18.2 cm X 18.2 cm

18) approximately 133 CDs at a price of $22.05

19) 2:23 pm

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