1.6 Applications of Rates of Change Worksheet with solutions

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1) The demand function for a DVD player is $p(x) = \frac{575}{\sqrt{x}} - 3$, where x is the number of DVD players sold and p is the price, in dollars. Determine…

a) the revenue function

$$R(x) = x \cdot p(x)$$
$$R(x) = x\left(\frac{575}{\sqrt{x}} - 3\right)$$
$$R(x) = 575\sqrt{x} - 3x$$

b) the marginal revenue function

$$R'(x) = \frac{575}{2\sqrt{x}} - 3$$

c) the marginal revenue when 200 DVD players are sold

$$R'(200) = \frac{575}{2\sqrt{200}} - 3$$
$$R'(200) \cong 17.33 \text{ dollars per DVD player}$$

2) Refer to question 1. If the cost, C, in dollars, of producing x DVD players is $C(x) = 2000 + 150x - 0.002x^2$, determine…

a) the profit function

$$P(x) = R(x) - C(x)$$
$$P(x) = 575\sqrt{x} - 3x - (2000 + 150x - 0.002x^2)$$
$$P(x) = 0.002x^2 - 153x + 575\sqrt{x} - 2000$$

b) the marginal profit function

$$P'(x) = 0.004x - 153 + \frac{575}{2\sqrt{x}}$$

c) the marginal profit for the sale of 500 DVD players

$$P'(500) = 0.004(500) - 153 + \frac{575}{2\sqrt{500}}$$
$$P'(500) \cong -138.14 \text{ dollars per DVD player}$$

3) A paint store sells 270 cans of paint per month at a price of $32 each. A customer survey indicates that for each $1.20 decrease in price, sales will increase by six cans of paint.

a) Determine the demand, or price, function.

Let n be the number of $1.20 price decreases.

$$x = 270 + 6n \quad \Rightarrow \quad n = \frac{x - 270}{6}$$
$$p = 32 - 1.20n$$
$$p(x) = 32 - 1.2\left(\frac{x - 270}{6}\right)$$
$$p(x) = 32 - \frac{x - 270}{5} = 32 - \frac{x}{5} + \frac{270}{5}$$
$$p(x) = -0.2x + 86$$

b) Determine the revenue function.

$$R(x) = x \cdot p(x)$$
$$R(x) = x(-0.2x + 86)$$
$$R(x) = -0.2x^2 + 86x$$

c) Determine the marginal revenue function.

$$R'(x) = -0.4x + 86$$

d) Solve $R'(x) = 0$. Interpret this value for this situation.

$$0 = -0.4x + 86$$
$$x = 215$$

Selling 215 cans per month maximizes the revenue.

e) What price corresponds to the value found in part d)? How can the paint store use this information.

$$p(215) = -0.2(215) + 86$$
$$p(215) = 43 \text{ dollars per can}$$

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Charging \$43 per can will maximize the revenue.

4) A yogurt company estimates that the revenue from selling x containers of yogurt is $4.5x$. Its cost, C, in dollars, for producing this number of containers of yogurt is $C(x) = 0.0001x^2 + 2x + 3200$.

a) Determine the marginal cost of producing 4000 containers of yogurt.

$$C'(x) = 0.0002x + 2$$
$$C'(4000) = 0.0002(4000) + 2$$
$$C'(4000) = 2.80 \text{ dollars per yogurt container}$$

b) Determine the marginal profit from selling 4000 containers of yogurt.

$$P(x) = R(x) - C(x)$$
$$P(x) = 4.5x - (0.0001x^2 + 2x + 3200)$$
$$P(x) = -0.0001x^2 + 2.5x - 3200$$
$$P'(x) = -0.0002x + 2.5$$
$$P'(4000) = -0.0002(4000) + 2.5$$
$$P'(4000) = 1.70 \text{ dollars per yogurt container}$$

c) What is the selling price of a container of yogurt?

Since the revenue is $4.5x$, the selling price is \$4.50 per container.

5) The cost, C, in dollars, of producing x hot tubs can be modelled by the function $C(x) = 3450x - 1.02x^2,\ 0 \le x \le 1500$.

a) Determine the marginal cost at a production level of 750 hot tubs. Explain what this means to the manufacturer.

$$C'(x) = 3450 - 2.04x$$
$$C'(750) = 3450 - 2.04(750)$$
$$C'(750) = 1920 \text{ dollars per hot tub}$$

The negative slope of the linear equation shows that the rate of change of cost will decrease as you produce more hot tubs.

b) Find the cost of producing the 751st hot tub.

$$C(750) = 3450(750) - 1.02(750)^2 = 2\,013\,750$$
$$C(751) = 3450(751) - 1.02(751)^2 = 2\,015\,668.98$$
$$\text{Cost of 751st hot tub} = C(751) - C(750) = 1918.98 \text{ dollars}$$

c) Compare and comment on the values you found in parts a) and b).

The marginal cost of producing x items is approximately equal to the cost of producing 1 more item.

d) Each hot tub is sold for \$9200. Write an expression to model the total revenue from the sale of x hot tubs.

$$R(x) = 9200x$$

e) Determine the rate of change of profit for the sale of 750 hot tubs.

$$P(x) = 9200x - (3450x - 1.02x^2)$$
$$P(x) = 5750x + 1.02x^2$$
$$P'(x) = 5750 + 2.04x$$
$$P'(750) = 5750 + 2.04(750)$$
$$P'(750) = 7280 \text{ dollars per hot tub}$$

6) The mass, in grams, of the first x meters of a wire can be modelled by the function $f(x) = \sqrt{2x - 1}$.

a) Determine the average linear density of the part of the wire from $x = 1$ to $x = 8$.

$$\text{average linear density} = \frac{f(8) - f(1)}{8 - 1} = \frac{\sqrt{2(8) - 1} - \sqrt{2(1) - 1}}{7} = \frac{\sqrt{15} - 1}{7}$$

or about 0.41 g/m.

b) Determine the linear density at $x = 5$ and at $x = 8$, and compare the densities at the two points. What do these values confirm about the wire?

$$f'(x) = \frac{1}{2}(2x - 1)^{-\frac{1}{2}}(2)$$
$$f'(x) = \frac{1}{\sqrt{2x - 1}}$$
$$f'(5) = \frac{1}{\sqrt{2(5) - 1}} = \frac{1}{3} \text{ g/m}$$
$$f'(8) = \frac{1}{\sqrt{2(8) - 1}} = \frac{1}{\sqrt{15}} \cong 0.26 \text{ g/m}$$

The density is decreasing as the length of the wire increases. The material is non-homogeneous.

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Answer Key

1) a) $R(x) = 575\sqrt{x} - 3x$ b) $R'(x) = \frac{575}{2\sqrt{x}} - 3$ c) $17.33 per DVD player

2) a) $P(x) = 0.002x^2 - 153x + 575\sqrt{x} - 2000$ b) $P'(x) = 0.004x + \frac{575}{2\sqrt{x}} - 153$ c) -$138.14 per DVD player

3) a) $p(x) = 86 - 0.2x$ b) $R(x) = 86x - 0.2x^2$ c) $R'(x) = 86 - 0.4x$ d) if 215 cans per month are sold, revenue is at a maximum e) charging $43 per can will maximize the revenue

4) a) $2.80 per container b) $1.70 per container c) $4.50

5) a) $1920 per hot tub. The equation shows that the rate of change in cost of producing x hot tubs reduces for greater values of x. b) $1918.98 c) the marginal cost when producing x items is approximately equal to the cost of producing one more item d) $R(x) = 9200x$ e) $7280 per hot tub

6) a) 0.41 g/m b) $\frac{1}{3}$ g/m at $x = 5$ and $\frac{1}{\sqrt{15}}$ at $x = 8$. The density of the wire decreases as the distance increases.

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