1.5 The Chain Rule Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Differentiate using the chain rule.

a) $f(x) = (-4x^2)^2$

$$f'(x) = 2(-4x^2)(-8x)$$
$$f'(x) = 64x^3$$

b) $f(x) = (16x^2)^{\frac{3}{4}}$

$$f'(x) = \frac{3}{4}(16x^2)^{-\frac{1}{4}}(32x)$$
$$f'(x) = \frac{96x}{4(16x^2)^{\frac{1}{4}}}$$
$$f'(x) = \frac{96x}{8x^{\frac{1}{2}}}$$
$$f'(x) = 12\sqrt{x}$$

c) $y = (4x+1)^2$

$$y' = 2(4x+1)(4)$$
$$y' = 8(4x+1)$$

d) $y = (x^3 - x)^{-3}$

$$y' = -3(x^3 - x)^{-4}(3x^2 - 1)$$
$$y' = \frac{-3(3x^2-1)}{(x^3-x)^4}$$
$$y' = \frac{-3(3x^2-1)}{x^4(x^2-1)^4}$$

e) $y = \sqrt{2x - 3x^5}$

$$y = (2x - 3x^5)^{\frac{1}{2}}$$
$$y' = \frac{1}{2}(2x - 3x^5)^{-\frac{1}{2}}(2 - 15x^4)$$
$$y' = \frac{2 - 15x^4}{2(2x - 3x^5)^{\frac{1}{2}}}$$

f) $y = \sqrt[5]{2 + 3x^2 - x^3}$

$$y = (2 + 3x^2 - x^3)^{\frac{1}{5}}$$
$$y' = \frac{1}{5}(2 + 3x^2 - x^3)^{-\frac{4}{5}}(6x - 3x^2)$$
$$y' = \frac{3x(2-x)}{5(2 + 3x^2 - x^3)^{\frac{4}{5}}}$$

2) Determine $f'(1)$.

a) $f(x) = (4x^2 - x + 1)^2$

$$f'(x) = 2(4x^2 - x + 1)(8x - 1)$$
$$f'(1) = 2\left[4(1)^2 - 1 + 1\right][8(1) - 1]$$
$$f'(1) = 2(4)(7)$$
$$f'(1) = 56$$

b) $f(x) = \dfrac{5}{\sqrt[3]{2x - x^2}}$

$$f'(x) = \frac{0\left(\sqrt[3]{2x - x^2}\right) - \frac{1}{3}(2x - x^2)^{-\frac{2}{3}}(2 - 2x)(5)}{\left[(2x - x^2)^{\frac{1}{3}}\right]^2}$$
$$f'(x) = \frac{-5(2-2x)}{3(2x - x^2)^{\frac{4}{3}}}$$
$$f'(1) = \frac{-5[2 - 2(1)]}{3\left[2(1) - (1)^2\right]^{\frac{4}{3}}}$$
$$f'(1) = 0$$

3) Determine an equation for the tangent to the curve $y = (x^3 - 4x^2)^3$ at $x = 3$

$$y' = 3(x^3 - 4x^2)^2(3x^2 - 8x)$$

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Slope:

$$y'(3) = 3\left[(3)^3 - 4(3)^2\right]^2\left[3(3)^2 - 8(3)\right]$$
$$y'(3) = 3(-9)^2(3)$$
$$y'(3) = 729$$

Point:

$$y(3) = \left[(3)^3 - 4(3)^2\right]^3$$
$$y(3) = (-9)^3$$
$$y(3) = -729$$

Equation:

$$y = mx + b$$
$$-729 = 729(3) + b$$
$$b = -2916$$
$$y = 729x - 2916$$

4) Determine the point(s) on the curve $y = x^2(x^3 - x)^2$ where the tangent line is horizontal.

$$y' = 2x(x^3 - x)^2 + 2(x^3 - x)(3x^2 - 1)(x^2)$$
$$y' = 2x(x^3 - x)\left[(x^3 - x) + x(3x^2 - 1)\right]$$
$$y' = 2x(x^3 - x)(4x^3 - 2x)$$
$$y' = 2x(x)(x^2 - 1)(2x)(2x^2 - 1)$$
$$y' = 4x^3(x - 1)(x + 1)(2x^2 - 1)$$
$$0 = 4x^3(x - 1)(x + 1)(2x^2 - 1)$$
$$x_1 = 0 \qquad x_2 = 1 \qquad x_3 = -1 \qquad x_{4,5} = \pm\frac{1}{\sqrt{2}}$$

Finding the corresponding y-values:

$$y(0) = 0 \qquad y(1) = 0 \qquad y(-1) = 0$$
$$y\left(\pm\frac{1}{\sqrt{2}}\right) = \frac{1}{16}$$

$(-1, 0)$, $(1, 0)$, $(0, 0)$, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$, and $\left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$

5) Differentiate each of the following.

a) $f(x) = (x + 4)^3(x - 3)^6$

$$f'(x) = 3(x + 4)^2(1)(x - 3)^6 + 6(x - 3)^5(1)(x + 4)^3$$
$$f'(x) = 3(x + 4)^2(x - 3)^5[(x - 3) + 2(x + 4)]$$
$$f'(x) = 3(x + 4)^2(x - 3)^5(3x + 5)$$

b) $y = \left(\frac{x^2 - 3}{x^2 + 3}\right)^4$

$$y' = 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3\left[\frac{2x(x^2 + 3) - 2x(x^2 - 3)}{(x^2 + 3)^2}\right]$$
$$y' = 4\frac{(x^2 - 3)^3}{(x^2 + 3)^3}(2x)\left[\frac{(x^2 + 3) - (x^2 - 3)}{(x^2 + 3)^2}\right]$$
$$y' = \frac{8x(x^2 - 3)^3(6)}{(x^2 + 3)^5}$$
$$y' = \frac{48x(x^2 - 3)^3}{(x^2 + 3)^5}$$

Answer Key

1) a) $f'(x) = 64x^3$ b) $f'(x) = 12\sqrt{x}$ c) $y' = 8(4x + 1)$ d) $y' = \frac{-3(3x^2 - 1)}{x^4(x^2 - 1)^4}$ e) $\frac{dy}{dx} = \frac{2 - 15x^4}{2(2x - 3x^5)^{1/2}}$ f) $\frac{dy}{dx} = \frac{6x - 3x^2}{5(2 + 3x^2 - x^3)^{4/5}}$

2) a) 56 b) 0

3) $y = 729x - 2916$

4) $(-1, 0)$, $(1, 0)$, $(0, 0)$, $\left(-\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$, and $\left(\frac{1}{\sqrt{2}}, \frac{1}{16}\right)$

5) a) $3(x + 4)^2(x - 3)^5(3x + 5)$ b) $\frac{48x(x^2 - 3)^3}{(x^2 + 3)^5}$

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