1.4 The Quotient Rule Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

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1) Use the quotient rule to differentiate each function

a) $h(x) = \frac{x}{x+1}$

$$h'(x) = \frac{1(x+1) - 1(x)}{(x+1)^2}$$
$$h'(x) = \frac{x+1-x}{(x+1)^2}$$
$$h'(x) = \frac{1}{(x+1)^2}$$

b) $h(t) = \frac{2t-3}{t+5}$

$$h'(t) = \frac{2(t+5) - 1(2t-3)}{(t+5)^2}$$
$$h'(t) = \frac{2t+10-2t+3}{(t+5)^2}$$
$$h'(t) = \frac{13}{(t+5)^2}$$

c) $h(x) = \frac{x^3}{2x^2-1}$

$$h'(x) = \frac{3x^2(2x^2-1) - 4x(x^3)}{(2x^2-1)^2}$$
$$h'(x) = \frac{6x^4 - 3x^2 - 4x^4}{(2x^2-1)^2}$$
$$h'(x) = \frac{2x^4 - 3x^2}{(2x^2-1)^2}$$

d) $h(x) = \frac{1}{x^2+3}$

$$h'(x) = \frac{0(x^2+3) - 2x(1)}{(x^2+3)^2}$$
$$h'(x) = \frac{-2x}{(x^2+3)^2}$$

e) $y = \frac{x(3x+5)}{1-x^2}$

$y = \frac{3x^2+5x}{1-x^2}$

$$y' = \frac{(6x+5)(1-x^2) - (-2x)(3x^2+5x)}{(1-x^2)^2}$$
$$y' = \frac{6x - 6x^3 + 5 - 5x^2 + 6x^3 + 10x^2}{(1-x^2)^2}$$
$$y' = \frac{5x^2 + 6x + 5}{(1-x^2)^2}$$

f) $y = \frac{x^2-x+1}{x^2+3}$

$$y' = \frac{(2x-1)(x^2+3) - (2x)(x^2-x+1)}{(x^2+3)^2}$$
$$y' = \frac{2x^3 + 6x - x^2 - 3 - 2x^3 + 2x^2 - 2x}{(x^2+3)^2}$$
$$y' = \frac{x^2 + 4x - 3}{(x^2+3)^2}$$

2) Determine $\frac{dy}{dx}$ at each given value of x.

a) $y = \frac{3x+2}{x+5}$ at $x = -3$

$$\frac{dy}{dx} = \frac{3(x+5) - 1(3x+2)}{(x+5)^2}$$
$$\frac{dy}{dx} = \frac{3x+15-3x-2}{(x+5)^2}$$
$$\frac{dy}{dx} = \frac{13}{(x+5)^2}$$

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$$\left.\frac{dy}{dx}\right|_{x=-3} = \frac{13}{(-3+5)^2} = \frac{13}{4}$$

b) $y = \dfrac{x^3}{x^2+9}$ at $x = 1$

$$\frac{dy}{dx} = \frac{3x^2(x^2+9) - 2x(x^3)}{(x^2+9)^2}$$
$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{3(1)^2[(1)^2+9] - 2(1)[(1)^3]}{[(1)^2+9]^2}$$
$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{30-2}{100}$$
$$\left.\frac{dy}{dx}\right|_{x=1} = \frac{7}{25}$$

3) Find the point(s) at which the tangent to the curve is horizontal.

a) $y = \dfrac{2x^2}{x-4}$

$$y' = \frac{4x(x-4) - 1(2x^2)}{(x-4)^2}$$
$$y' = \frac{4x^2 - 16x - 2x^2}{(x-4)^2}$$
$$y' = \frac{2x^2 - 16x}{(x-4)^2}$$
$$0 = 2x^2 - 16x$$
$$0 = 2x(x-8)$$
$$x_1 = 0 \qquad x_2 = 8$$
$$y_1 = \frac{2(0)^2}{0-4} = 0 \qquad y_2 = \frac{2(8)^2}{8-4} = 32$$

$(0,\ 0)$ and $(8,\ 32)$

b) $y = \dfrac{x^2-1}{x^2+x-2}$

Note: $y = \dfrac{(x-1)(x+1)}{(x-1)(x+2)} = \dfrac{x+1}{x+2};\ x \neq -2,\ 1$ (a hole in the graph when $x = 1$)

$$y' = \frac{2x(x^2+x-2) - (2x+1)(x^2-1)}{(x^2+x-2)^2}$$
$$y' = \frac{2x^3 + 2x^2 - 4x - (2x^3 - 2x + x^2 - 1)}{(x^2+x-2)^2}$$
$$y' = \frac{x^2 - 2x + 1}{(x^2+x-2)^2}$$
$$y' = \frac{(x-1)^2}{(x+2)^2(x-1)^2}$$

For $x \neq 1$:

$$y' = \frac{1}{(x+2)^2}$$
$$0 = \frac{1}{(x+2)^2}$$

No solutions – there are no horizontal tangents.

4) Determine the equation of the tangent to the curve $y = \dfrac{x^2-1}{3x}$ at $x = 2$.

$$y' = \frac{2x(3x) - 3(x^2-1)}{(3x)^2}$$
$$y' = \frac{6x^2 - 3x^2 + 3}{9x^2}$$
$$y' = \frac{3x^2+3}{9x^2}$$
$$y' = \frac{x^2+1}{3x^2}$$

Slope:

$$y'(2) = \frac{(2)^2+1}{3(2)^2}$$
$$y'(2) = \frac{5}{12}$$

Point:

$$y(2) = \frac{(2)^2-1}{3(2)}$$
$$y(2) = \frac{1}{2}$$

Equation:

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$$y = mx + b$$
$$\frac{1}{2} = \frac{5}{12}(2) + b$$
$$\frac{3}{6} - \frac{5}{6} = b$$
$$b = -\frac{1}{3}$$
$$y = \frac{5}{12}x - \frac{1}{3}$$

Answer Key

1) a) $h'(x) = \frac{1}{(x+1)^2}$ b) $h'(t) = \frac{13}{(t+5)^2}$ c) $h'(x) = \frac{2x^4 - 3x^2}{(2x^2-1)^2}$ d) $h'(x) = \frac{-2x}{(x^2+3)^2}$ e) $y' = \frac{5x^2+6x+5}{(1-x^2)^2}$ f) $\frac{dy}{dx} = \frac{x^2+4x-3}{(x^2+3)^2}$

2) a) $\frac{13}{4}$ b) $\frac{7}{25}$

3) a) $(0,\ 0)$ and $(8,\ 32)$ b) no horizontal tangents

4) $y = \frac{5}{12}x - \frac{1}{3}$

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