1.3 Velocity, Acceleration, and Second Derivatives Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

1) Determine the second derivative of each function.

a) $y = 2x^3 + 21$

$$\frac{dy}{dx} = 6x^2$$
$$\frac{d^2y}{dx^2} = 12x$$

b) $s(t) = -t^4 + 5t^3 - 2t^2 + t$

$$s'(t) = -4t^3 + 15t^2 - 4t + 1$$
$$s''(t) = -12t^2 + 30t - 4$$

c) $h(x) = \frac{1}{6}x^6 - \frac{1}{5}x^5$

$$h'(x) = x^5 - x^4$$
$$h''(x) = 5x^4 - 4x^3$$

2) Determine $f''(3)$ for each function:

a) $f(x) = 4x^3 - 5x + 6$

$$f'(x) = 12x^2 - 5$$
$$f''(x) = 24x$$
$$f''(3) = 24(3)$$
$$f''(3) = 72$$

b) $f(x) = (3x^2 + 2)(1 - x)$

$$f'(x) = 6x(1 - x) + (-1)(3x^2 + 2)$$
$$f'(x) = 6x - 6x^2 - 3x^2 - 2$$
$$f'(x) = -9x^2 + 6x - 2$$
$$f''(x) = -18x + 6$$
$$f''(3) = -18(3) + 6$$
$$f''(3) = -48$$

3) Determine the velocity and acceleration functions for each position function $s(t)$.

a) $s(t) = 5 + 7t - 8t^3$

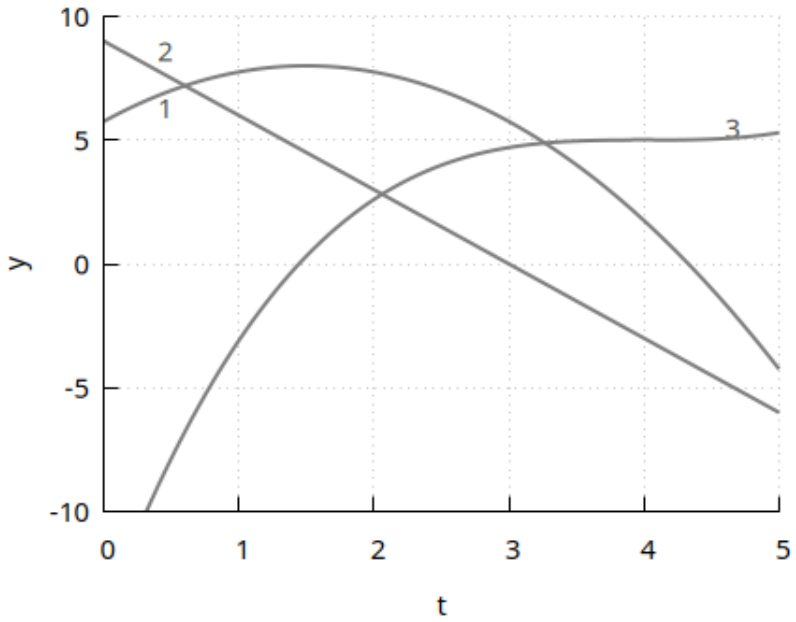
$$v(t) = s'(t) = 7 - 24t^2$$
$$a(t) = s''(t) = -48t$$

b) $s(t) = (2t + 3)(4 - 5t)$

$$v(t) = s'(t) = 2(4 - 5t) + (-5)(2t + 3)$$
$$v(t) = 8 - 10t - 10t - 15$$
$$v(t) = -20t - 7$$
$$a(t) = s''(t) = -20$$

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4) For the following graph,



a) Identify which curve or line represents $s(t)$, $v(t)$, and $a(t)$.

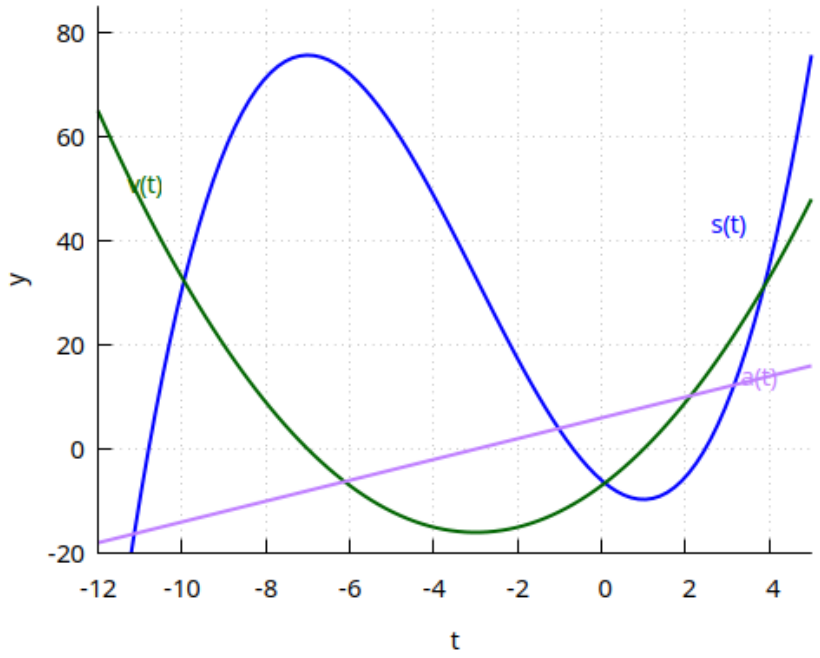
$s(t)$ is curve 3; highest degree (3). $v(t)$ is curve 1; degree 2. $a(t)$ is line 2; degree 1.

b) Complete the table to determine the motion of the object.

Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	**Slope of** $s(t)$	**Motion of particle**
$(0,\ 1)$	−	+	−	Negative slope that is increasing	Slowing down and moving in reverse
$(1,\ 2)$	+	+	+	Positive slope that is increasing	Speeding up and moving forward
$(2,\ 3)$	+	−	−	Positive slope that is decreasing	Slowing down and moving forward
$(3,\ \infty)$	−	−	+	Negative slope that is decreasing	Speeding up and moving in reverse

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5) For the graph of $s(t)$ given, sketch possible graphs of $v(t)$ and $a(t)$.

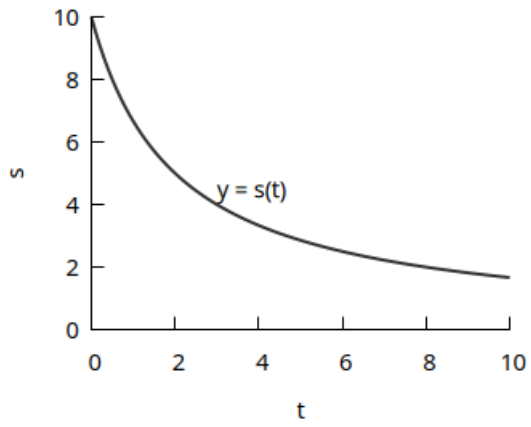


6) For each of the following graphs of $s(t)$,

i) Is the velocity increasing, decreasing, or constant?

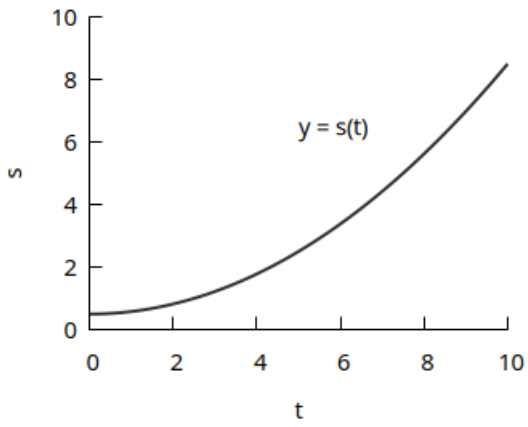
ii) Is the acceleration positive, negative, or zero?

a)



i) increasing ii) positive

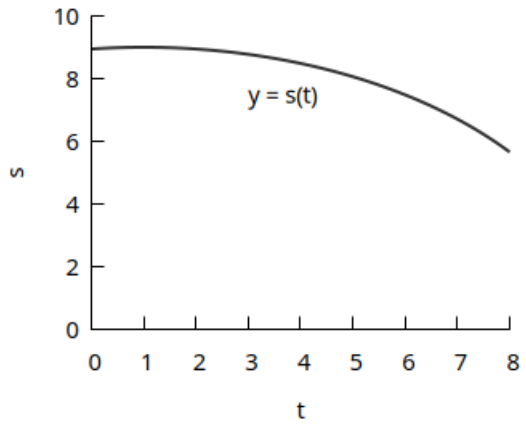
b)



i) increasing ii) positive

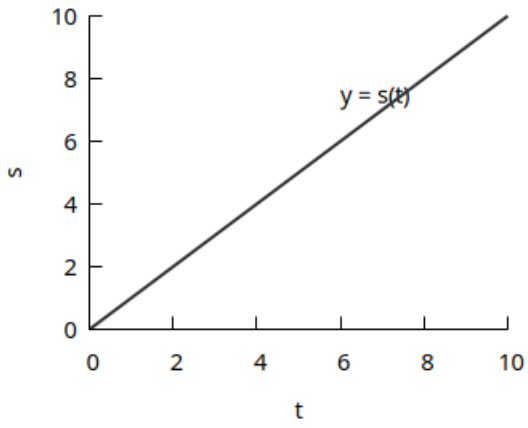
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c)



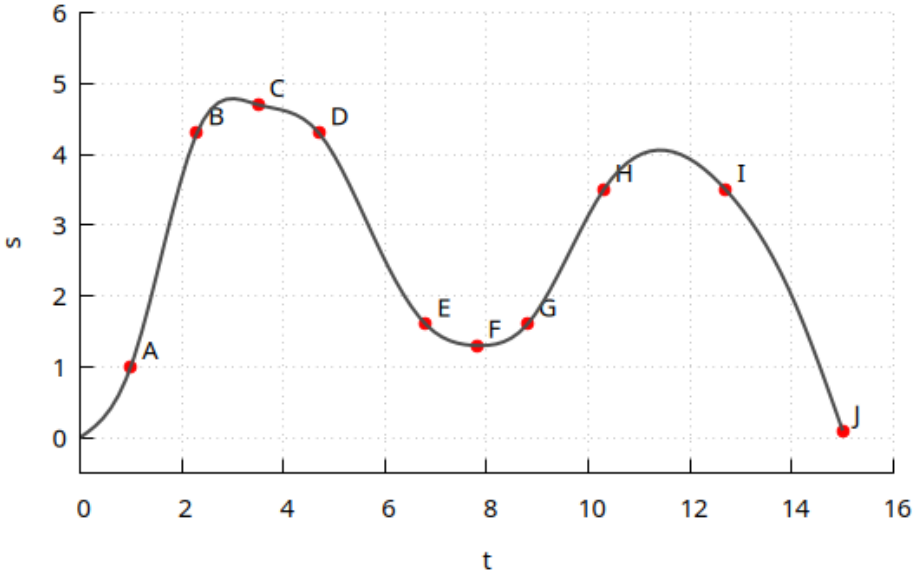
i) decreasing ii) negative

d)



i) constant ii) zero

7) The graph shows the position function of a bus during a 15-minute trip.



a) What is the initial velocity of the bus?

Zero

b) What is the bus's velocity at C and at F?

Zero

c) Is the bus going faster at A or at B? Explain.

A; the slope is steeper.

d) What happens to the motion of the bus between H and I?

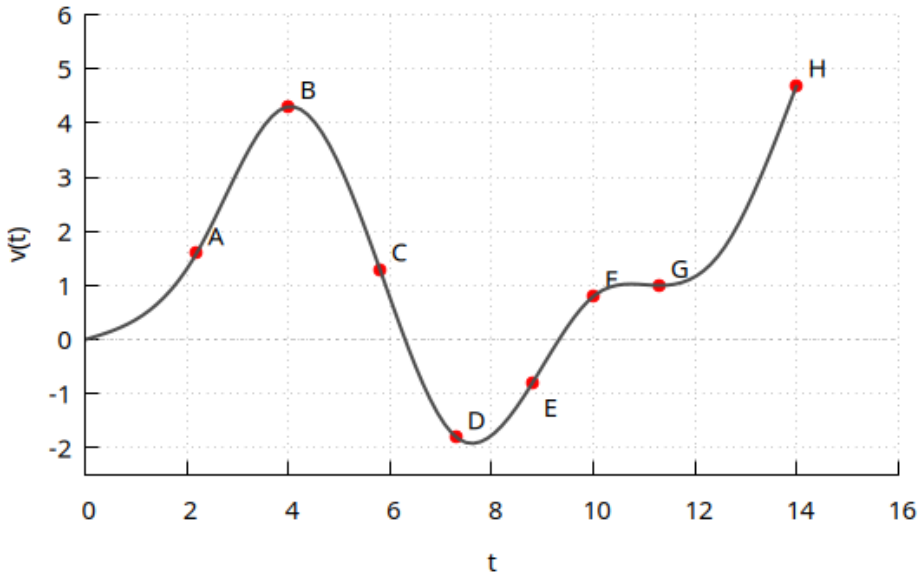
The bus is stopped (it doesn't move).

e) Is the bus speeding up or slowing down at B and D?

B – slowing down. D – speeding up.

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8) The graph shows a velocity function. State whether the acceleration is positive or negative for the following intervals:



a) 0 to B

Positive

b) B to D

Negative

c) D to F

Positive

d) F to G

Zero

e) G to H

Positive

f) at B and at D

Zero

Answer Key

1) a) $\frac{d^2y}{dx^2} = 12x$ b) $s''(t) = -12t^2 + 30t - 4$ c) $h''(x) = 5x^4 - 4x^3$

2) a) 72 b) -48

3) a) $v(t) = 7 - 24t^2$, $a(t) = -48t$ b) $v(t) = -20t - 7$, $a(t) = -20$

4) a) Curve 3 is the position function since it is a cubic with the highest exponent. Curve 1 is the velocity function since it is a quadratic with an exponent one less than the position function. Line 2 is the acceleration function since it is linear and its exponent is one less than the velocity function.

b) See table above.

5) See graph above.

6) a)i) increasing ii) positive b)i) increasing ii) positive c)i) decreasing ii) negative d)i) constant ii) zero

7) a) 0 b) 0 c) A; slope is steeper d) the bus is stopped e) B – slowing down, D – speeding up

8) a) + b) - c) + d) 0 e) + f) 0

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