1.2 The Product Rule Worksheet with solutions

This document was composed and formatted by Annie Bernatchez.

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1) Use the product rule to differentiate each function

a) $f(x) = (5x + 2)(8x - 6)$

$$f'(x) = 5(8x - 6) + 8(5x + 2)$$
$$f'(x) = 40x - 30 + 40x + 16$$
$$f'(x) = 80x - 14$$

b) $h(t) = (-t + 4)(2t + 1)$

$$h'(t) = -1(2t + 1) + 2(-t + 4)$$
$$h'(t) = -2t - 1 - 2t + 8$$
$$h'(t) = -4t + 7$$

c) $p(x) = (-2x + 3)(x - 9)$

$$p'(x) = -2(x - 9) + 1(-2x + 3)$$
$$p'(x) = -2x + 18 - 2x + 3$$
$$p'(x) = -4x + 21$$

d) $g(x) = (x^2 + 2)(4x - 5)$

$$g'(x) = 2x(4x - 5) + 4(x^2 + 2)$$
$$g'(x) = 8x^2 - 10x + 4x^2 + 8$$
$$g'(x) = 12x^2 - 10x + 8$$

e) $f(x) = (1 - x)(x^2 - 5)$

$$f'(x) = -1(x^2 - 5) + 2x(1 - x)$$
$$f'(x) = -x^2 + 5 + 2x - 2x^2$$
$$f'(x) = -3x^2 + 2x + 5$$

f) $h(t) = (t^2 + 3)(3t^2 - 7)$

$$h'(t) = 2t(3t^2 - 7) + 6t(t^2 + 3)$$
$$h'(t) = 6t^3 - 14t + 6t^3 + 18t$$
$$h'(t) = 12t^3 + 4t$$

2) Determine $f'(-2)$ for each function.

a) $f(x) = (x^2 - 2x)(3x + 1)$

$$f'(x) = (2x - 2)(3x + 1) + 3(x^2 - 2x)$$
$$f'(x) = 6x^2 - 4x - 2 + 3x^2 - 6x$$
$$f'(x) = 9x^2 - 10x - 2$$
$$f'(-2) = 9(-2)^2 - 10(-2) - 2$$
$$f'(-2) = 54$$

b) $f(x) = (1 - x^3)(-x^2 + 2)$

$$f'(x) = -3x^2(-x^2 + 2) + (-2x)(1 - x^3)$$
$$f'(x) = 3x^4 - 6x^2 - 2x + 2x^4$$
$$f'(x) = 5x^4 - 6x^2 - 2x$$
$$f'(-2) = 5(-2)^4 - 6(-2)^2 - 2(-2)$$
$$f'(-2) = 60$$

3) Determine an equation for the tangent to each curve at the indicated value.

a) $f(x) = (x^2 - 3)(x^2 + 1)$ at $x = -4$

Slope:

$$f'(x) = 2x(x^2 + 1) + 2x(x^2 - 3)$$
$$f'(x) = 2x^3 + 2x + 2x^3 - 6x$$
$$f'(x) = 4x^3 - 4x$$
$$f'(-4) = 4(-4)^3 - 4(-4)$$
$$f'(-4) = -240$$

$m = -240$

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Point:

$$f(-4) = [(-4)^2 - 3][(-4)^2 + 1]$$
$$f(-4) = 221$$

Equation:

$$y = mx + b$$
$$221 = (-240)(-4) + b$$
$$b = -739$$
$$y = -240x - 739$$

b) $h(x) = (x^4 + 4)(2x^2 - 6)$ at $x = -1$

Slope:

$$h'(x) = 4x^3(2x^2 - 6) + 4x(x^4 + 4)$$
$$h'(x) = 8x^5 - 24x^3 + 4x^5 + 16x$$
$$h'(x) = 12x^5 - 24x^3 + 16x$$
$$h'(-1) = 12(-1)^5 - 24(-1)^3 + 16(-1)$$
$$h'(-1) = -4$$

Point:

$$h(-1) = [(-1)^4 + 4][2(-1)^2 - 6]$$
$$h(-1) = -20$$

Equation:

$$y = mx + b$$
$$-20 = -4(-1) + b$$
$$b = -24$$
$$y = -4x - 24$$

4) Determine the point(s) on each curve that correspond to the given slope of the tangent.

a) $y = (-4x + 3)(x + 3)$, $m = 0$

$$y' = -4(x + 3) + 1(-4x + 3)$$
$$y' = -4x - 12 - 4x + 3$$
$$y' = -8x - 9$$
$$0 = -8x - 9$$
$$x = -\frac{9}{8}$$
$$y\left(-\frac{9}{8}\right) = \left[-4\left(-\frac{9}{8}\right) + 3\right]\left[-\frac{9}{8} + 3\right]$$
$$y\left(-\frac{9}{8}\right) = \frac{225}{16}$$
$$\left(-\frac{9}{8}, \frac{225}{16}\right)$$

b) $y = (x^2 - 2)(2x + 1)$, $m = -2$

$$y' = 2x(2x + 1) + 2(x^2 - 2)$$
$$y' = 4x^2 + 2x + 2x^2 - 4$$
$$y' = 6x^2 + 2x - 4$$
$$-2 = 6x^2 + 2x - 4$$
$$0 = 6x^2 + 2x - 2$$
$$0 = 3x^2 + x - 1$$
$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(3)(-1)}}{2(3)}$$
$$x = \frac{-1 \pm \sqrt{13}}{6}$$
$$x_1 \approx 0.43 \qquad x_2 \approx -0.77$$

Point 1: $y(0.43) \approx -3.38$, giving $(0.43, -3.38)$

Point 2: $y(-0.77) \approx 0.76$, giving $(-0.77, 0.76)$

5) Differentiate using the product rule.

a) $y = (5x^2 - x + 1)(x + 2)$

$$\frac{dy}{dx} = (10x - 1)(x + 2) + 1(5x^2 - x + 1)$$
$$\frac{dy}{dx} = 10x^2 + 19x - 2 + 5x^2 - x + 1$$
$$\frac{dy}{dx} = 15x^2 + 18x - 1$$

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b) $y = -x^2(4x-1)(x^3+2x+3)$

Consider $-x^2(4x-1)$ as the 1st function and x^3+2x+3 as the 2nd function.

$$\frac{d}{dx}\left[-x^2(4x-1)\right] = -2x(4x-1)+4(-x^2)$$
$$\frac{d}{dx}\left[-x^2(4x-1)\right] = -8x^2+2x-4x^2$$
$$\frac{d}{dx}\left[-x^2(4x-1)\right] = -12x^2+2x$$
$$\frac{dy}{dx} = (-12x^2+2x)(x^3+2x+3)+(3x^2+2)(-x^2)(4x-1)$$
$$\frac{dy}{dx} = -12x^5-24x^3-36x^2+2x^4+4x^2+6x+(3x^2+2)(-4x^3+x^2)$$
$$\frac{dy}{dx} = -12x^5+2x^4-24x^3-32x^2+6x-12x^5+3x^4-8x^3+2x^2$$
$$\frac{dy}{dx} = -24x^5+5x^4-32x^3-30x^2+6x$$

6) The owner of a local hair salon is planning to raise the price for a haircut and blow dry. The current rate is $30 for this service, with the salon averaging 550 clients a month. A survey indicates that the salon will lose 5 clients for every incremental price increase of $2.50.

a) Write an equation to model the salon's monthly revenue, R, in dollars, as a function of x, where x represents the number of $2.50 increases in the price.

$$R(x) = (30+2.5x)(550-5x)$$

b) Use the product rule to determine $R'(x)$

$$R'(x) = 2.5(550-5x)+(-5)(30+2.5x)$$
$$R'(x) = 1375-12.5x-150-12.5x$$
$$R'(x) = -25x+1225$$

c) Evaluate $R'(3)$ and interpret it for this situation.

$$R'(3) = -25(3)+1225$$
$$R'(3) = 1150$$

At an increase of $7.50, the rate of change of revenue is $1150.

d) Solve $R'(x) = 0$.

$$0 = -25x+1225$$
$$x = 49$$

e) Explain how the owner can use the result of part d).

$$R(49) = [30+2.5(49)][550-5(49)]$$
$$R(49) = (152.50)(305)$$
$$R(49) = 46\,512.50$$

Increasing the price by $2.50, 49 times, results in a max revenue of $46 512.50.

Answer Key

1)

a) $f'(x) = 80x-14$
b) $h'(t) = -4t+7$
c) $p'(x) = -4x+21$
d) $g'(x) = 12x^2-10x+8$
e) $f'(x) = -3x^2+2x+5$
f) $h'(t) = 12t^3+4t$

2) a) 54 b) 60

3) a) $y = -240x-739$ b) $y = -4x-24$

4) a) $\left(-\frac{9}{8}, \frac{225}{16}\right)$ b) $(0.43, -3.38)$ and $(-0.77, 0.76)$

5) a) $15x^2+18x-1$ b) $-24x^5+5x^4-32x^3-30x^2+6x$

6) a) $R(x) = (30+2.50x)(550-5x)$ b) $R'(x) = 1225-25x$ c) 1150; this is the rate of change of revenue at a $7.50 increase d) $x = 49$ e) The owner could maximize the revenue by making 49 increases of $2.50. A visit to the hair salon would cost $152.50 and would generate a max revenue of $46 512.50.

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