1.7 Unit 1 Pre-Test Review – Derivative Rules with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

This review covers all of Unit 1: Derivative Rules, and is meant to be used to prepare for the unit test.

Section 1: 1.1 – Basic Differentiation Rules

1) Differentiate each function.

a) $h(t) = t^3 - 2t^2 + \frac{1}{t^2}$

$$h(t) = t^3 - 2t^2 + t^{-2}$$
$$h'(t) = 3t^2 - 4t - 2t^{-3}$$
$$h'(t) = 3t^2 - 4t - \frac{2}{t^3}$$

b) $p(n) = -n^5 + 5n^3 + \sqrt[3]{n^2}$

$$p(n) = -n^5 + 5n^3 + n^{\frac{2}{3}}$$
$$p'(n) = -5n^4 + 15n^2 + \frac{2}{3}n^{-\frac{1}{3}}$$
$$p'(n) = -5n^4 + 15n^2 + \frac{2}{3\sqrt[3]{n}}$$

c) $p(r) = r^6 - \frac{2}{5\sqrt{r}} + r - 1$

$$p(r) = r^6 - \frac{2}{5}r^{-\frac{1}{2}} + r - 1$$
$$p'(r) = 6r^5 + \frac{1}{5}r^{-\frac{3}{2}} + 1$$
$$p'(r) = 6r^5 + \frac{1}{5\sqrt{r^3}} + 1$$

Section 2: 1.2 – The Product Rule

2) Differentiate using the product rule.

a) $f(x) = (5x + 3)(2x - 11)$

$$f'(x) = 5(2x - 11) + 2(5x + 3)$$
$$f'(x) = 10x - 55 + 10x + 6$$
$$f'(x) = 20x - 49$$

b) $h(t) = \left(2t^2 + \sqrt[3]{t}\right)(4t - 5)$

$$h'(t) = \left(4t + \frac{1}{3}t^{-\frac{2}{3}}\right)(4t - 5) + 4\left(2t^2 + t^{\frac{1}{3}}\right)$$
$$h'(t) = 16t^2 - 20t + \frac{4t}{3t^{\frac{2}{3}}} - \frac{5}{3t^{\frac{2}{3}}} + 8t^2 + 4t^{\frac{1}{3}}$$
$$h'(t) = 24t^2 - 20t + \frac{4t^{\frac{1}{3}}}{3} + 4t^{\frac{1}{3}} - \frac{5}{3t^{\frac{2}{3}}}$$
$$h'(t) = 24t^2 - 20t + \frac{16t^{\frac{1}{3}}}{3} - \frac{5}{3t^{\frac{2}{3}}}$$

c) $g(x) = (-1.5x^6 + 1)(3 - 8x)$

$$g'(x) = -9x^5(3 - 8x) + (-8)(-1.5x^6 + 1)$$
$$g'(x) = -27x^5 + 72x^6 + 12x^6 - 8$$
$$g'(x) = 84x^6 - 27x^5 - 8$$

d) $p(n) = (11n + 2)(-5 + 3n^2)$

$$p'(n) = 11(-5 + 3n^2) + 6n(11n + 2)$$
$$p'(n) = -55 + 33n^2 + 66n^2 + 12n$$
$$p'(n) = 99n^2 + 12n - 55$$

3) Determine an equation for the tangent to the graph of $y = (-3x + 8)(x^3 - 7)$ at $x = 2$.

Slope:

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$$\frac{dy}{dx} = -3(x^3-7)+3x^2(-3x+8)$$
$$\left.\frac{dy}{dx}\right|_{x=2} = -3\left[(2)^3-7\right]+3(2)^2[-3(2)+8]$$
$$= -3(1)+12(2)$$
$$= 21$$

Point:

$$y(2) = [-3(2)+8]\left[(2)^3-7\right] = (2)(1) = 2 \quad \Rightarrow \quad (2,\ 2)$$

Equation:

$$y = mx+b$$
$$2 = 21(2)+b$$
$$b = -40$$
$$y = 21x-40$$

4) Determine $f''(-2)$ for $f(x) = (4-x^2)(3x+1)$

$$f'(x) = -2x(3x+1)+3(4-x^2)$$
$$f'(x) = -6x^2-2x+12-3x^2$$
$$f'(x) = -9x^2-2x+12$$
$$f''(x) = -18x-2$$
$$f''(-2) = -18(-2)-2$$
$$f''(-2) = 34$$

5) Determine the first and second derivative of each function.

a) $g(x) = \frac{2}{3}x^3+\frac{1}{2}x^4-3$

$$g'(x) = 2x^2+2x^3$$
$$g''(x) = 4x+6x^2$$

b) $h(x) = (2x-3)(3x+1)$

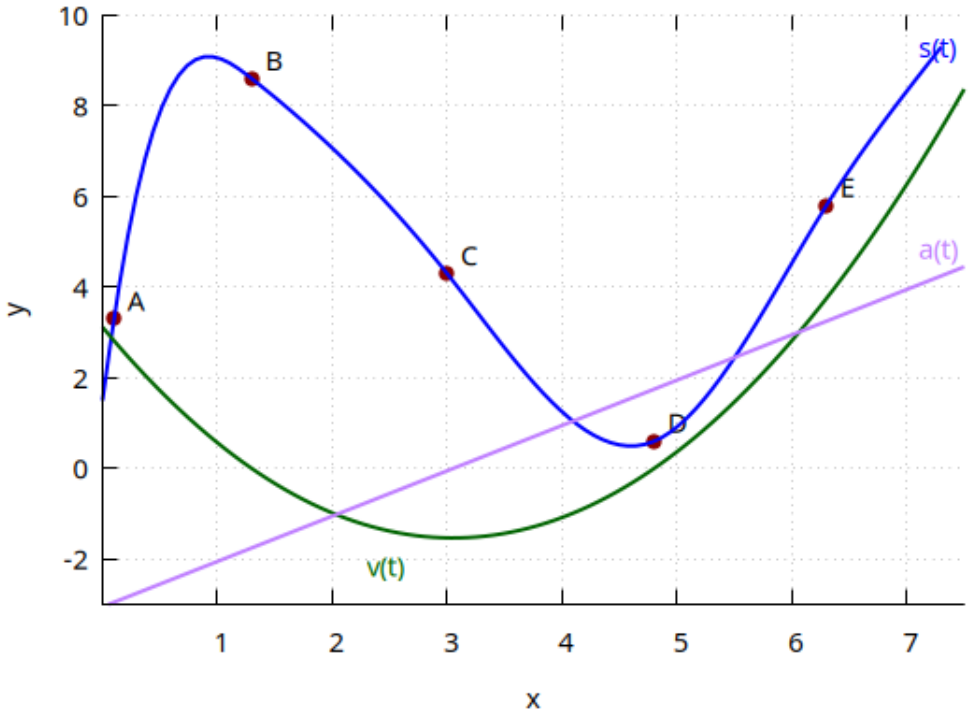
$$h'(x) = 2(3x+1)+3(2x-3)$$
$$h'(x) = 6x+2+6x-9$$
$$h'(x) = 12x-7$$
$$h''(x) = 12$$

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Section 3: 1.3 – Velocity, Acceleration, and Second Derivatives

6) For the distance time graph given,

a) sketch the velocity and acceleration function.



b) Complete the table to determine the motion of the object.

Interval	$v(t)$	$a(t)$	$v(t) \times a(t)$	**Slope of** $s(t)$	**Motion of particle**
$(A,\ B)$	+	−	−	Positive and decreasing	Slowing down and moving forward
$(B,\ C)$	−	−	+	Negative and decreasing	Speeding up and moving in reverse
$(C,\ D)$	−	+	−	Negative and increasing	Slowing down and moving in reverse
$(D,\ E)$	+	+	+	Positive and increasing	Speeding up and moving forward

7) A toy missile is shot into the air. Its height, h, in meters, after t seconds can be modelled by the function $h(t) = -4.9t^2 + 15t + 0.4,\ t \geq 0$.

a) Determine the height of the toy missile at 2 seconds.

$$h(2) = -4.9(2)^2 + 15(2) + 0.4$$
$$h(2) = 10.8 \text{ m}$$

b) Determine the rate of change of the height of the toy missile at 1 s and 4 s.

$$h'(t) = -9.8t + 15$$
$$h'(1) = -9.8(1) + 15 = 5.2 \text{ m/s}$$
$$h'(4) = -9.8(4) + 15 = -24.2 \text{ m/s}$$

c) How long does it take the toy missile to return to the ground?

$$0 = -4.9t^2 + 15t + 0.4$$
$$t = \frac{-15 \pm \sqrt{(15)^2 - 4(-4.9)(0.4)}}{2(-4.9)}$$
$$t \cong 3.088 \text{ s}$$

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d) How fast was the toy missile travelling when it hit the ground?

$$h'(3.088) = -9.8(3.088) + 15$$
$$h'(3.088) \cong -15.26 \text{ m/s}$$

Section 4: 1.4 – The Quotient Rule

8) Differentiate using the quotient rule.

a) $y = \dfrac{x-2}{2x+5}$

$$y' = \frac{1(2x+5) - 2(x-2)}{(2x+5)^2}$$
$$y' = \frac{2x+5-2x+4}{(2x+5)^2}$$
$$y' = \frac{9}{(2x+5)^2}$$

b) $y = \dfrac{x^2-4}{2x+5}$

$$y' = \frac{2x(2x+5) - 2(x^2-4)}{(2x+5)^2}$$
$$y' = \frac{4x^2+10x-2x^2+8}{(2x+5)^2}$$
$$y' = \frac{2x^2+10x+8}{(2x+5)^2}$$

9) Determine the slope of the tangent to $y = \dfrac{3x}{x^2-4x+3}$ at $x = 4$.

$$y' = \frac{3(x^2-4x+3) - (2x-4)(3x)}{(x^2-4x+3)^2}$$
$$y' = \frac{3x^2-12x+9-6x^2+12x}{(x^2-4x+3)^2}$$
$$y' = \frac{-3x^2+9}{(x^2-4x+3)^2}$$
$$y'(4) = \frac{-3(4)^2+9}{\left[(4)^2-4(4)+3\right]^2}$$
$$y'(4) = \frac{-39}{9}$$
$$y'(4) = -\frac{13}{3}$$

Section 5: 1.5 – The Chain Rule

10) Differentiate each of the following.

a) $f(x) = (3x-2)^2$

$$f'(x) = 2(3x-2)(3)$$
$$f'(x) = 6(3x-2)$$
$$f'(x) = 18x-12$$

b) $y = (3x^2-x)^3$

$$y' = 3(3x^2-x)^2(6x-1)$$
$$y' = 3(6x-1)(9x^4-6x^3+x^2)$$
$$y' = 162x^5-135x^4+36x^3-3x^2$$

c) $h(x) = \sqrt[3]{3x+5x^4}$

$$h(x) = (3x+5x^4)^{\frac{1}{3}}$$
$$h'(x) = \frac{1}{3}(3x+5x^4)^{-\frac{2}{3}}(3+20x^3)$$
$$h'(x) = \frac{20x^3+3}{3\left(\sqrt[3]{3x+5x^4}\right)^2}$$

d) $f(x) = (2x-3)^3(3x-1)^2$

$$f'(x) = 3(2x-3)^2(2)(3x-1)^2 + 2(3x-1)(3)(2x-3)^3$$
$$f'(x) = 6(2x-3)^2(3x-1)^2 + 6(3x-1)(2x-3)^3$$
$$f'(x) = 6(2x-3)^2(3x-1)[(3x-1)+(2x-3)]$$

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$$f'(x) = 6(2x-3)^2(3x-1)(5x-4)$$

e) $y = \dfrac{(2x-5)^4}{(x+1)^3}$

$$y' = \frac{4(2x-5)^3(2)(x+1)^3 - 3(x+1)^2(2x-5)^4}{(x+1)^6}$$
$$y' = \frac{(2x-5)^3(x+1)^2[8(x+1)-3(2x-5)]}{(x+1)^6}$$
$$y' = \frac{(2x-5)^3(2x+23)}{(x+1)^4}$$

f) $y = \dfrac{8x^3}{\sqrt{3x-2}}$

$$y = 8x^3(3x-2)^{-\frac{1}{2}}$$
$$y' = 24x^2(3x-2)^{-\frac{1}{2}} + 8x^3\left(-\frac{1}{2}\right)(3x-2)^{-\frac{3}{2}}(3)$$
$$y' = \frac{24x^2(3x-2) - 12x^3}{(3x-2)^{\frac{3}{2}}}$$
$$y' = \frac{60x^3 - 48x^2}{(3x-2)^{\frac{3}{2}}}$$
$$\frac{dy}{dx} = \frac{12x^2(5x-4)}{(3x-2)^{\frac{3}{2}}}$$

11) Find an equation for the tangent at $x = 1$ to the curve $y = \left(\dfrac{2x}{x+1}\right)^6$.

Point:

$$y(1) = \left[\frac{2(1)}{1+1}\right]^6 = (1)^6 = 1 \quad \Rightarrow \quad (1,\ 1)$$

Slope:

$$y' = 6\left(\frac{2x}{x+1}\right)^5\left[\frac{2(x+1)-1(2x)}{(x+1)^2}\right]$$
$$y' = 6\left[\frac{32x^5}{(x+1)^5}\right]\left[\frac{2}{(x+1)^2}\right]$$
$$y' = \frac{384x^5}{(x+1)^7}$$
$$y'(1) = \frac{384(1)^5}{(1+1)^7} = 3$$

Equation:

$$y = mx + b$$
$$1 = 3(1) + b$$
$$b = -2$$
$$y = 3x - 2$$

12) Find all tangents to the curve $y = 4x^3$ that have slope of 3.

$$y' = 12x^2$$
$$3 = 12x^2$$
$$x = \pm\frac{1}{2}$$

Point 1: $x = \dfrac{1}{2}$

$$y\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^3 = \frac{1}{2} \quad \Rightarrow \quad \left(\frac{1}{2},\ \frac{1}{2}\right)$$
$$\frac{1}{2} = 3\left(\frac{1}{2}\right) + b \quad \Rightarrow \quad b = -1$$
$$y = 3x - 1$$

Point 2: $x = -\dfrac{1}{2}$

$$y\left(-\frac{1}{2}\right) = 4\left(-\frac{1}{2}\right)^3 = -\frac{1}{2} \quad \Rightarrow \quad \left(-\frac{1}{2},\ -\frac{1}{2}\right)$$
$$-\frac{1}{2} = 3\left(-\frac{1}{2}\right) + b \quad \Rightarrow \quad b = 1$$
$$y = 3x + 1$$

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Section 6: 1.3/1.6 – Motion and Applications of Rates of Change

13) Suppose a particle travels according to the position function in meters $s(t) = \frac{t^3}{3} - 2t^2 + 3t - 4$.

a) At what two times is the particle stationary (stopped)? That is, when is the velocity zero.

$$v(t) = s'(t) = t^2 - 4t + 3$$
$$0 = (t-3)(t-1)$$
$$t_1 = 3\text{ s} \qquad t_2 = 1\text{ s}$$

b) How far does the particle travel between the two stationary times?

$$\text{distance} = s(3) - s(1)$$
$$= \left[\frac{3^3}{3} - 2(3)^2 + 3(3) - 4\right] - \left[\frac{1^3}{3} - 2(1)^2 + 3(1) - 4\right]$$
$$= -4 - \left(-\frac{8}{3}\right)$$
$$= -\frac{4}{3}$$

The particle travels $\frac{4}{3}$ meters backwards.

14) When the price is \$1.75 each, 3000 fruit bars will be sold. If the price of a fruit bar is raised to \$2.00, sales will drop to 2500.

a) Determine the demand, or price, function.

Let x be the number sold, p be the price, and n be the number of \$0.25 price increases from \$1.75.

$$x = 3000 - 500n \quad \Rightarrow \quad n = \frac{3000 - x}{500} = 6 - 0.002x$$
$$p = 1.75 + 0.25n$$
$$p(x) = 1.75 + 0.25(6 - 0.002x)$$
$$p(x) = 1.75 + 1.5 - 0.0005x$$
$$p(x) = 3.25 - 0.0005x$$

b) Determine the marginal revenue from the sale of 2700 bars.

$$R(x) = x(3.25 - 0.0005x)$$
$$R(x) = 3.25x - 0.0005x^2$$
$$R'(x) = 3.25 - 0.001x$$
$$R'(2700) = 3.25 - 0.001(2700)$$
$$R'(2700) = 0.55 \text{ dollars per bar sold}$$

c) The cost for the bars is given by the function $C(x) = 30 + 0.25x$. Determine the marginal cost of purchasing 3000 bars.

$$C'(x) = 0.25$$
$$C'(3000) = 0.25 \text{ dollars per bar}$$

d) Determine the marginal profit function for the sale of the fruit bars.

$$P(x) = R(x) - C(x)$$
$$P(x) = 3.25x - 0.0005x^2 - (30 + 0.25x)$$
$$P(x) = -0.0005x^2 + 3x - 30$$
$$P'(x) = -0.001x + 3$$

e) Determine the marginal profit from the sale of 3000 bars.

$$P'(3000) = -0.001(3000) + 3$$
$$P'(3000) = 0 \text{ dollars per bar}$$

15) The mass, in grams, of the first x meters of a wire is represented by the function $f(x) = \sqrt{4x-1}$.

a) Determine the average linear density of a segment of the wire from $x = 3$ to $x = 7$.

$$\text{average linear density} = \frac{f(7) - f(3)}{7 - 3} = \frac{\sqrt{4(7)-1} - \sqrt{4(3)-1}}{4} = \frac{\sqrt{27} - \sqrt{11}}{4}$$
$$\cong 0.470 \text{ g/m}$$

b) Determine the linear density at $x = 4$ and $x = 10$. What does these values confirm about the wire?

$$f'(x) = \frac{1}{2}(4x-1)^{-\frac{1}{2}}(4)$$
$$f'(x) = \frac{2}{\sqrt{4x-1}}$$
$$f'(4) = \frac{2}{\sqrt{4(4)-1}} \cong 0.516 \text{ g/m}$$

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$$f'(10) = \frac{2}{\sqrt{4(10)-1}} \cong 0.320 \text{ g/m}$$

These confirm that the material of which the wire is composed is not homogeneous.

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