1.5 The Chain Rule Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

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A composite function consists of an outer function, $g(x)$, and an inner function, $h(x)$. The chain rule says to differentiate the outer function with respect to the inner function, and then multiply by the derivative of the inner function.

Given two differentiable functions $g(x)$ and $h(x)$, the derivative of the composite function $f(x) = g[h(x)]$ is

$$f'(x) = g'[h(x)] \times h'(x)$$

A special case of the chain rule is the 'Power of a Function Rule'. This occurs when the outer function is a power function:

$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1} g'(x)$$

Summary of Derivative Rules:

Rule	Derivative
Power Rule If $f(x) = x^n$	$f'(x) = nx^{n-1}$
Constant Multiple Rule If $f(x) = c \cdot g(x)$ where c is a constant	$f'(x) = c \cdot g'(x)$
Sum Rule If $h(x) = f(x) + g(x)$	$h'(x) = f'(x) + g'(x)$
Difference Rule If $h(x) = f(x) - g(x)$	$h'(x) = f'(x) - g'(x)$
Product Rule If $h(x) = f(x)g(x)$	$h'(x) = f'(x)g(x) + f(x)g'(x)$
Quotient Rule If $h(x) = f(x) \div g(x)$	$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$
Power of a Function Rule If $h(x) = [f(x)]^n$	$h'(x) = n[f(x)]^{n-1} \times f'(x)$
Chain Rule If $h(x) = f[g(x)]$	$h'(x) = f'[g(x)] \times g'(x)$

Example 1: Differentiate each function using the chain rule. Express in simplified factored form.

a) $f(x) = (3x - 5)^4$

$$f'(x) = 4(3x - 5)^3(3)$$
$$f'(x) = 12(3x - 5)^3$$

b) $g(x) = \sqrt{4 - x^2}$

$$g'(x) = \frac{1}{2}(4 - x^2)^{-\frac{1}{2}}(-2x)$$
$$g'(x) = \frac{-x}{\sqrt{4 - x^2}}$$

c) $y = (2\sqrt{x})^3 - 4\sqrt{x} + 1$

$$\frac{dy}{dx} = 3(2\sqrt{x})^2(x)^{-\frac{1}{2}} - 2x^{-\frac{1}{2}}$$
$$\frac{dy}{dx} = 3(4x)(x)^{-\frac{1}{2}} - 2x^{-\frac{1}{2}}$$
$$\frac{dy}{dx} = (12x)(x)^{-\frac{1}{2}} - 2x^{-\frac{1}{2}}$$
$$\frac{dy}{dx} = \frac{2(6x - 1)}{\sqrt{x}}$$

d) $h(x) = (x^2 + 3)^4(4x - 5)^3$

$$h'(x) = 4(x^2 + 3)^3(2x)(4x - 5)^3 + 3(4x - 5)^2(4)(x^2 + 3)^4$$
$$h'(x) = 4(x^2 + 3)^3(4x - 5)^2\left[2x(4x - 5) + 3(x^2 + 3)\right]$$
$$h'(x) = 4(x^2 + 3)^3(4x - 5)^2\left[11x^2 - 10x + 9\right]$$

Example 2: Determine an equation for the tangent to $f(x) = 3x(1 - x)^2$ at $x = 0.5$

Point on Tangent Line:

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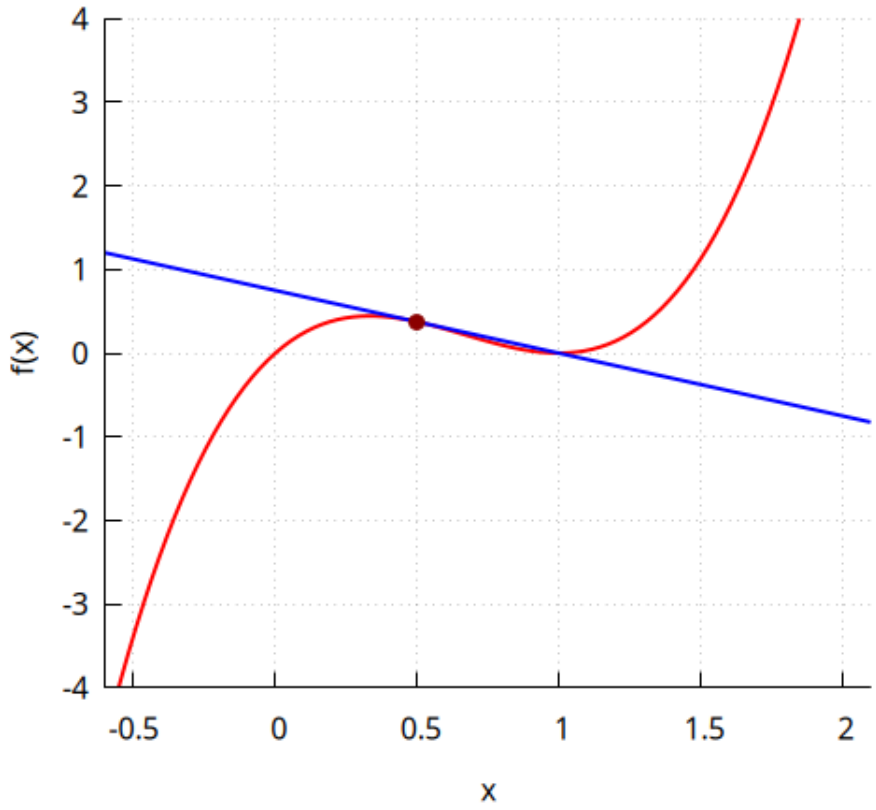
$$f(0.5) = 3(0.5)(1-0.5)^2$$
$$f(0.5) = \frac{3}{8}$$

Slope of tangent line:

$$f'(x) = 3(1-x)^2 + 2(1-x)(-1)(3x)$$
$$f'(x) = 3(1-x)^2 - 6x(1-x)$$
$$f'(x) = 3(1-x)[(1-x) - 2x]$$
$$f'(x) = 3(1-x)(1-3x)$$
$$f'(0.5) = 3(1-0.5)[1-3(0.5)]$$
$$f'(0.5) = -\frac{3}{4}$$

Equation of line:

$$y = mx + b$$
$$\frac{3}{8} = \left(-\frac{3}{4}\right)\left(\frac{1}{2}\right) + b$$
$$b = \frac{3}{4}$$
$$y = -\frac{3}{4}x + \frac{3}{4}$$



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