1.4 The Quotient Rule Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

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The Quotient Rule:

If $h(x) = \frac{f(x)}{g(x)}$, then $h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$

Proof:

$$h(x) = \frac{f(x)}{g(x)}$$
$$h(x)g(x) = f(x)$$
$$h'(x)g(x) + g'(x)h(x) = f'(x)$$
$$h'(x) = \frac{f'(x) - g'(x)h(x)}{g(x)}$$
$$h'(x) = \frac{f'(x) - g'(x)\frac{f(x)}{g(x)}}{g(x)}$$
$$h'(x) = \left[\frac{f'(x) - g'(x)\frac{f(x)}{g(x)}}{g(x)}\right]\left[\frac{g(x)}{g(x)}\right]$$
$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

Example 1: Find the derivative of each of the following:

a) $f(x) = \frac{3x-4}{x^2+5}$

$$f'(x) = \frac{3(x^2+5) - 2x(3x-4)}{(x^2+5)^2}$$
$$f'(x) = \frac{3x^2 + 15 - 6x^2 + 8x}{(x^2+5)^2}$$
$$f'(x) = \frac{-3x^2 + 8x + 15}{(x^2+5)^2}$$

b) $g(x) = \frac{6x-5}{x^3+4}$

$$g'(x) = \frac{6(x^3+4) - 3x^2(6x-5)}{(x^3+4)^2}$$
$$g'(x) = \frac{6x^3 + 24 - 18x^3 + 15x^2}{(x^3+4)^2}$$
$$g'(x) = \frac{-12x^3 + 15x^2 + 24}{(x^3+4)^2}$$
$$g'(x) = \frac{-3(4x^3 - 5x^2 - 8)}{(x^3+4)^2}$$

c) $h(x) = \frac{2x+8}{\sqrt{x}}$

$$h'(x) = \frac{2(\sqrt{x}) - \frac{1}{2}x^{-\frac{1}{2}}(2x+8)}{(\sqrt{x})^2}$$
$$h'(x) = \frac{2\sqrt{x} - \frac{1}{2\sqrt{x}}(2)(x+4)}{x}$$
$$h'(x) = \frac{2\sqrt{x} - \frac{1}{\sqrt{x}}(x+4)}{x} \times \frac{\sqrt{x}}{\sqrt{x}}$$
$$h'(x) = \frac{2x - 1(x+4)}{x\sqrt{x}}$$
$$h'(x) = \frac{x-4}{x^{\frac{3}{2}}}$$

d) $r(x) = \frac{x+3}{\sqrt{x^2-1}}$

$$r'(x) = \frac{1(x^2-1)^{\frac{1}{2}} - \frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x)(x+3)}{(\sqrt{x^2-1})^2}$$
$$r'(x) = \frac{(x^2-1)^{-\frac{1}{2}}\left[1(x^2-1)^1 - \frac{1}{2}(2x)(x+3)\right]}{x^2-1}$$

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$$r'(x) = \frac{1(x^2-1) - 1(x)(x+3)}{(x^2-1)^{\frac{1}{2}}(x^2-1)^1}$$
$$r'(x) = \frac{x^2 - 1 - x^2 - 3x}{(x^2-1)^{\frac{3}{2}}}$$
$$r'(x) = \frac{-3x-1}{(x^2-1)^{\frac{3}{2}}}$$

Note: $\frac{d}{dx}\sqrt{x^2-1}$ *from part d) uses the 'chain rule'. We will learn this in depth in the next lesson.*

Example 2: Determine an equation for the tangent to the curve $y = \frac{x^2-3}{5-x}$ at $x = 2$.

Point on Tangent Line:

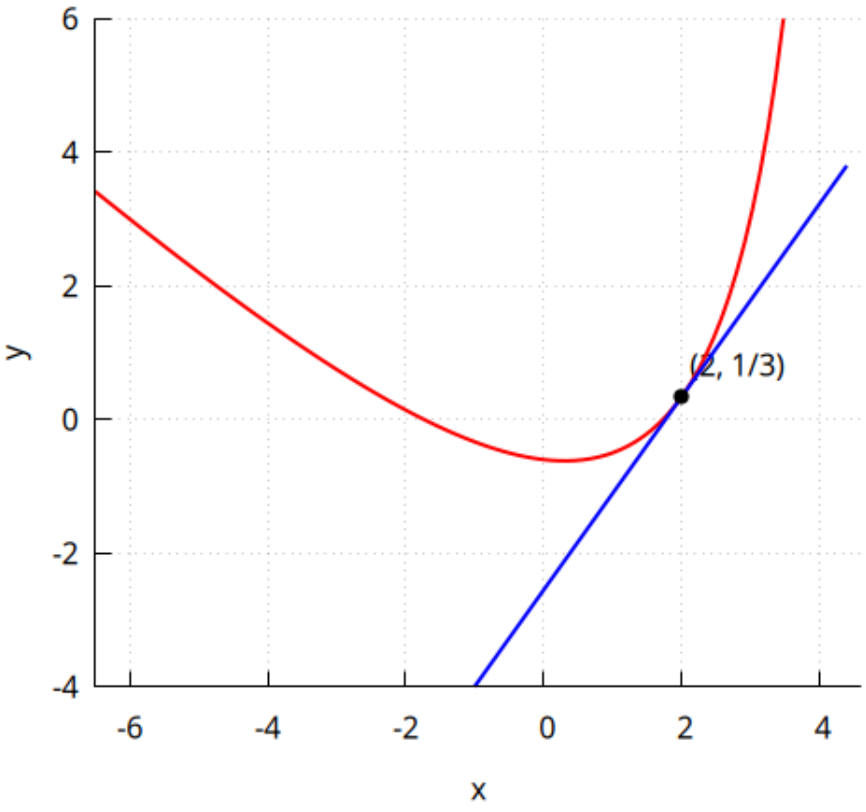
$$y = \frac{2^2-3}{5-2}$$
$$y = \frac{1}{3}$$
$$\left(2, \frac{1}{3}\right)$$

Slope of tangent line:

$$\frac{dy}{dx} = \frac{(2x)(5-x) - (-1)(x^2-3)}{(5-x)^2}$$
$$\frac{dy}{dx} = \frac{-x^2+10x-3}{(5-x)^2}$$
$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{-(2)^2+10(2)-3}{(5-2)^2}$$
$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{13}{9}$$

Equation of tangent line:

$$y = mx + b$$
$$\frac{1}{3} = \frac{13}{9}(2) + b$$
$$\frac{3}{9} - \frac{26}{9} = b$$
$$b = -\frac{23}{9}$$
$$y = \frac{13}{9}x - \frac{23}{9}$$



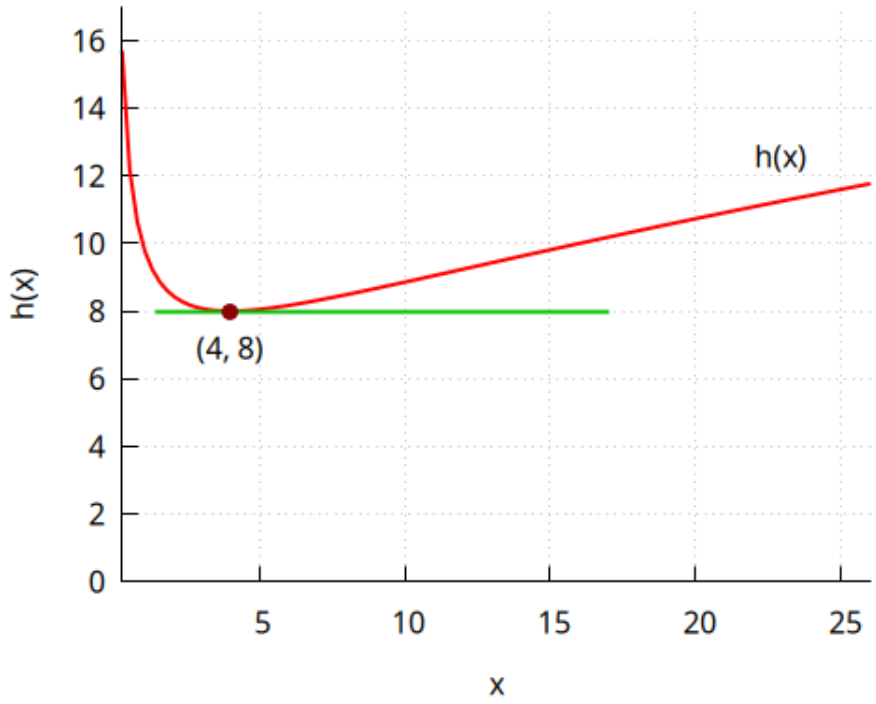
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Example 3: Determine the coordinates of each point on the graph of $h(x) = \frac{2x+8}{\sqrt{x}}$ where the tangent is horizontal.

The tangent will be horizontal when $h'(x) = 0$

$$h'(x) = \frac{x-4}{x^{\frac{3}{2}}}$$
$$0 = \frac{x-4}{x^{\frac{3}{2}}}$$
$$0 = x - 4$$
$$x = 4$$
$$h(4) = \frac{2(4)+8}{\sqrt{4}}$$
$$h(4) = 8$$

There will be a horizontal tangent to the point $(4,\ 8)$ on the graph of $h(x)$.



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