1.1 Derivatives of Polynomial Functions Lesson with solutions

This document was composed and formatted by Annie Bernatchez.

The course material originated from www.jensenmath.ca

Newton's Quotient and the Derivative

In advanced functions, you should have been introduced to the idea that the instantaneous rate of change is represented by the slope of the tangent at a point on the curve. You also learned that you can determine this value by taking the derivative of the function using the Newton Quotient.

Newton's Quotient:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Newton Quotient Example:

a) Find the equation of the derivative of $f(x) = 3x^2 + 2x + 4$

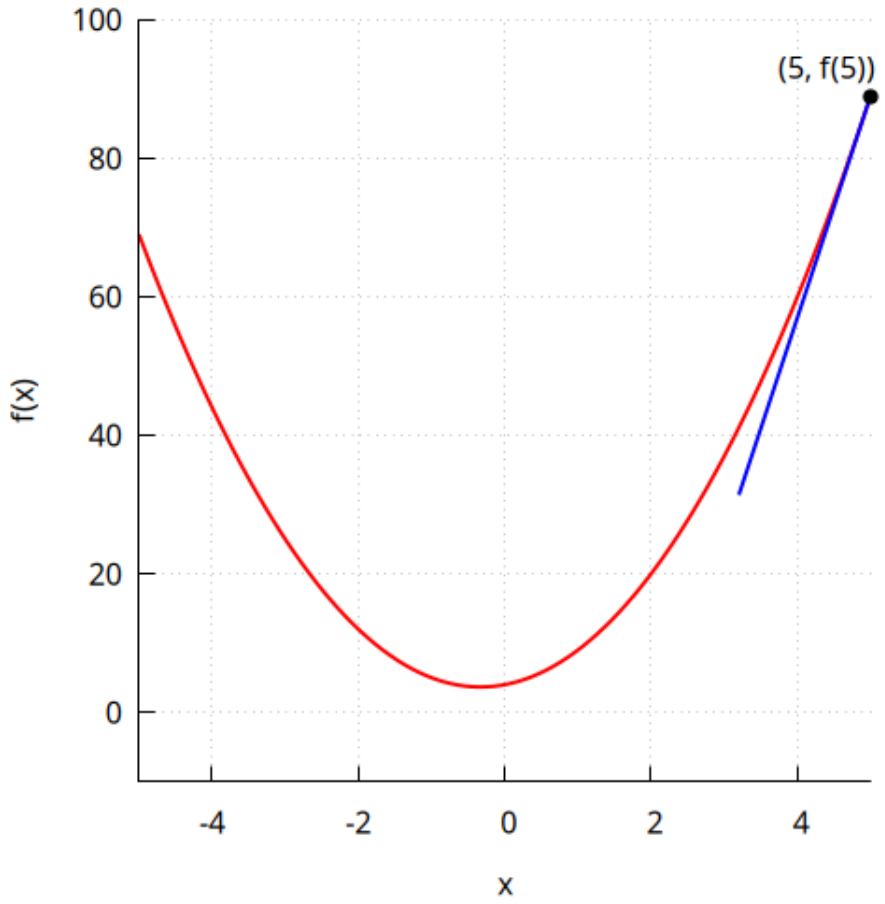
$$f'(x) = \lim_{h \to 0} \frac{3(x+h)^2 + 2(x+h) + 4 - (3x^2 + 2x + 4)}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) + 2x + 2h + 4 - 3x^2 - 2x - 4}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h + 4 - 3x^2 - 2x - 4}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{6xh + 3h^2 + 2h}{h}$$
$$f'(x) = \lim_{h \to 0} \frac{h(6x + 3h + 2)}{h}$$
$$f'(x) = 6x + 3(0) + 2$$
$$f'(x) = 6x + 2$$

b) Calculate $f'(5)$. What does it represent?

$$f'(5) = 6(5) + 2$$
$$f'(5) = 32$$

This tells us that the instantaneous rate of change of the original function when $x = 5$ is 32. Graphically speaking, this means the slope of the tangent line drawn on the original function at $(5,\ f(5))$ is 32.

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Rules for Finding Derivatives

Mathematicians have derived a set of rules for calculating derivatives that make this process more efficient.

Rule	Derivative	Example
Constant Rule If $f(x) = c$ where c is a constant	$f'(x) = 0$	$f(x) = 87$ $f'(x) = 0$
Power Rule If $f(x) = x^n$	$f'(x) = nx^{n-1}$	$f(x) = x^5$ $f'(x) = 5x^4$
Constant Multiple Rule If $f(x) = c \cdot g(x)$ where c is a constant	$f'(x) = c \cdot g'(x)$	$f(x) = 3x^5$ $f'(x) = 3 \cdot 5x^4$ $f'(x) = 15x^4$
Sum Rule If $h(x) = f(x) + g(x)$	$h'(x) = f'(x) + g'(x)$	$h(x) = x^5 + x^4$ $h'(x) = 5x^4 + 4x^3$
Difference Rule If $h(x) = f(x) - g(x)$	$h'(x) = f'(x) - g'(x)$	$h(x) = x^5 - x^4$ $h'(x) = 5x^4 - 4x^3$

Proof of Power Rule:

Use Binomial Theorem: $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$$\frac{d}{dx} x^n = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$\frac{d}{dx} x^n = \lim_{h \to 0} \frac{\binom{n}{0} x^n h^0 + \binom{n}{1} x^{n-1} h^1 + \binom{n}{2} x^{n-2} h^2 + \cdots + \binom{n}{n} x^0 h^n - x^n}{h}$$

$$\frac{d}{dx} x^n = \lim_{h \to 0} \frac{\binom{n}{1} x^{n-1} h^1 + \binom{n}{2} x^{n-2} h^2 + \cdots + \binom{n}{n} x^0 h^n}{h}$$

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$$\frac{d}{dx}x^n = \lim_{h\to 0}\frac{h\left[\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}h^1 + \cdots + \binom{n}{n}x^0h^{n-1}\right]}{h}$$
$$\frac{d}{dx}x^n = \lim_{h\to 0}\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}h^1 + \cdots + \binom{n}{n}x^0h^{n-1}$$
$$\frac{d}{dx}x^n = \binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}0^1 + \cdots + \binom{n}{n}x^00^{n-1}$$
$$\frac{d}{dx}x^n = nx^{n-1}$$

Example 1: Determine the equation of the derivative of each of the following functions:

a) $f(x) = 3x^5$

$$f'(x) = 15x^4$$

b) $f(x) = 71$

$$f'(x) = 0$$

c) $f(x) = \sqrt{x}$

$$f(x) = x^{\frac{1}{2}}$$
$$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1}$$
$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$$
$$f'(x) = \frac{1}{2\sqrt{x}}$$

d) $y = \sqrt[3]{x}$

$$y = x^{\frac{1}{3}}$$
$$y' = \frac{1}{3}x^{\frac{1}{3}-1}$$
$$y' = \frac{1}{3}x^{-\frac{2}{3}}$$
$$y' = \frac{1}{3x^{\frac{2}{3}}}$$

e) $y = \frac{1}{x}$

$$y = x^{-1}$$
$$y' = -1x^{-2}$$
$$y' = \frac{-1}{x^2}$$

f) $y = -\frac{1}{x^5}$

$$y = -1x^{-5}$$
$$\frac{dy}{dx} = 5x^{-6}$$
$$\frac{dy}{dx} = \frac{5}{x^6}$$

Example 2: Differentiate each function

a) $y = 5x^6 - 4x^3 + 6$

$$y' = 30x^5 - 12x^2 + 0$$
$$y' = 30x^5 - 12x^2$$

b) $f(x) = -3x^5 + 8\sqrt{x} - 9.3$

$$f(x) = -3x^5 + 8x^{\frac{1}{2}} - 9.3$$
$$f'(x) = -15x^4 + 4x^{-\frac{1}{2}} - 0$$
$$f'(x) = -15x^4 + \frac{4}{\sqrt{x}}$$

c) $g(x) = (2x - 3)(x + 1)$

$$g(x) = 2x^2 - x - 3$$
$$g'(x) = 4x - 1$$

d) $h(x) = \frac{-8x^6 + 8x^2}{4x^5}$

$$h(x) = \frac{-8x^6}{4x^5} + \frac{8x^2}{4x^5}$$
$$h(x) = -2x + 2x^{-3}$$
$$h'(x) = -2 - 6x^{-4}$$
$$h'(x) = -2 - \frac{6}{x^4}$$

Example 3: Determine an equation for the tangent to the curve $f(x) = 4x^3 + 3x^2 - 5$ at $x = -1$.

Point on the tangent line:

$$f(-1) = 4(-1)^3 + 3(-1)^2 - 5$$

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$$f(-1) = -6$$
$$(-1, -6)$$

Slope of tangent line:

$$f'(x) = 12x^2 + 6x$$
$$f'(-1) = 12(-1)^2 + 6(-1)$$
$$f'(-1) = 6$$

Slope of the tangent is 6

Remember: the equation of the derivative tells you the slope of the tangent to the original function.

Equation of tangent line:

$$y = mx + b$$
$$-6 = 6(-1) + b$$
$$b = 0$$
$$y = 6x$$

Example 4: Determine the point(s) on the graph of $y = x^2(x + 3)$ where the slope of the tangent is 24.

$$y = x^3 + 3x^2$$
$$\frac{dy}{dx} = 3x^2 + 6x$$
$$24 = 3x^2 + 6x$$
$$0 = 3x^2 + 6x - 24$$
$$0 = 3(x^2 + 2x - 8)$$
$$0 = (x + 4)(x - 2)$$
$$x_1 = -4 \qquad x_2 = 2$$

Now find the y-coordinates of the points:

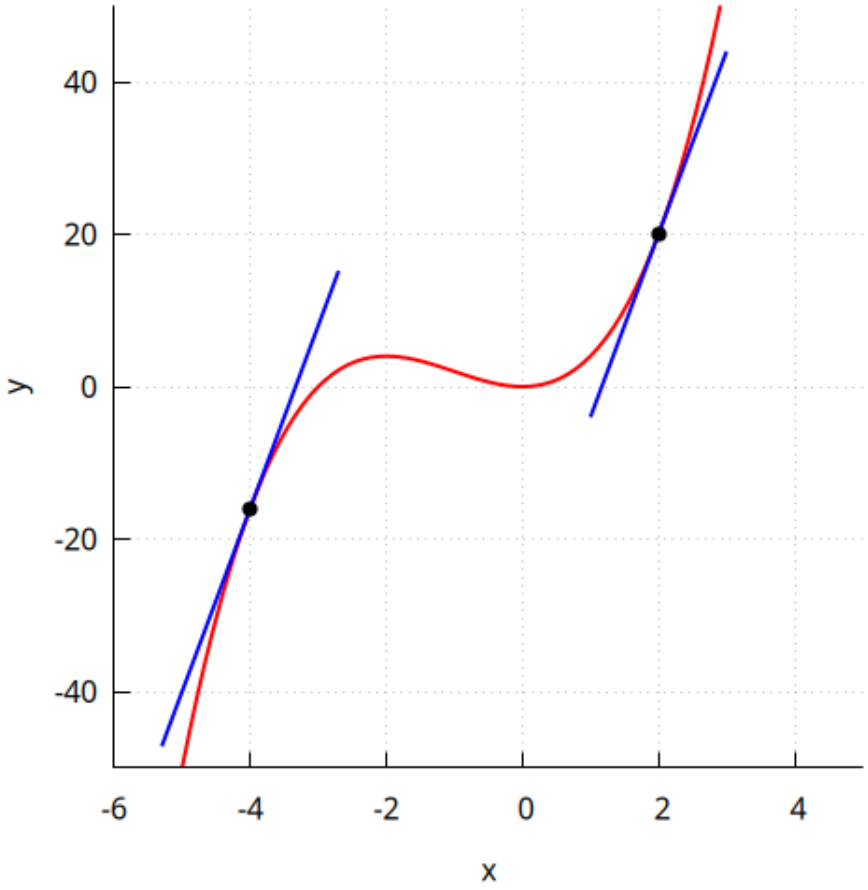
Point 1:

$$y = (-4)^3 + 3(-4)^2$$
$$y = -16$$
$$(-4, -16)$$

Point 2:

$$y = (2)^3 + 3(2)^2$$
$$y = 20$$
$$(2, 20)$$

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