Grade 12 Calculus and Vectors Exam Review Lesson (Unit 6) with solutions

This document was composed and formatted by Annie Bernatchez.

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Vectors Exam Review – Unit 6: Lines and Planes

1) Write the vector and parametric equations of each line.

a) $\vec{m} = [3, 5]$, $P(4, -5)$ $[x, y] = [4, -5] + t[3, 5]$ $\ell : \begin{cases} x = 4 + 3t \\ y = -5 + 5t \end{cases}$	**b)** $\vec{m} = [3, 7, -2]$, $P(-6, 2, 1)$ $[x, y, z] = [-6, 2, 1] + t[3, 7, -2]$ $\ell : \begin{cases} x = -6 + 3t \\ y = 2 + 7t \\ z = 1 - 2t \end{cases}$
c) perpendicular to the yz-plane and through $(0, 1, 2)$ A vector perpendicular to the yz-plane is parallel to the x-axis: $\vec{m} = [1, 0, 0]$ $[x, y, z] = [0, 1, 2] + t[1, 0, 0]$ $\ell : \begin{cases} x = t \\ y = 1 \\ z = 2 \end{cases}$	**d)** through the points $A(4, -5, 3)$ and $B(3, -7, 1)$ $\vec{m} = \overrightarrow{AB} = [-1, -2, -2] = -1[1, 2, 2]$ $[x, y, z] = [4, -5, 3] + t[1, 2, 2]$ $\ell : \begin{cases} x = 4 + t \\ y = -5 + 2t \\ z = 3 + 2t \end{cases}$
e) through the points $A(1, -3, 2)$ and $B(9, 2, 0)$ $\vec{m} = \overrightarrow{AB} = [8, 5, -2]$ $[x, y, z] = [1, -3, 2] + t[8, 5, -2]$ $\ell : \begin{cases} x = 1 + 8t \\ y = -3 + 5t \\ z = 2 - 2t \end{cases}$	**f)** parallel to the y-axis and containing the point $(1, 3, 5)$ $[x, y, z] = [1, 3, 5] + t[0, 1, 0]$ $\ell : \begin{cases} x = 1 \\ y = 3 + t \\ z = 5 \end{cases}$

2) A line is defined by the equation $[x, y, z] = [-2, 3, 7] + t[3, -2, 5]$. Write the parametric equations for the line and determine if it contains the point $(10, -5, 22)$.

$\ell : \begin{cases} x = -2 + 3t \\ y = 3 - 2t \\ z = 7 + 5t \end{cases}$

$10 = -2 + 3t \Rightarrow t = 4, \quad -5 = 3 - 2t \Rightarrow t = 4, \quad 22 = 7 + 5t \Rightarrow t = 3$

The three values of t don't agree $(4, 4, 3)$, so the point is **NOT** on the line.

3) Which pair of lines represented by vector equations are coincident?

A $[x, y] = [-2, 5] + s[2, -1]$ $[x, y] = [12, -30] + t[5, -7]$	**B** $[x, y] = [4, -1] + s[-3, 5]$ $[x, y] = [-2, 9] + t[-3, 5]$
C $[x, y] = [0, 0] + s[1, 1]$ $[x, y] = [-1, 1] + t[1, -1]$	**D** $[x, y] = [5, 4] + s[2, -4]$ $[x, y] = [9, -8] + t[2, -4]$

Options **B** and **D** both have identical direction vectors in their two lines, so both are at least parallel.

Checking **B**: is $(4, -1)$ on the second line, $[x, y] = [-2, 9] + t[-3, 5]$?

$4 = -2 - 3t \Rightarrow t = -2, \quad -1 = 9 + 5t \Rightarrow t = -2$

Both equations give $t = -2$, so $(4, -1)$ is on the second line too, and the two lines in **B** are coincident.

(Checking **D** the same way would show the two lines are parallel but distinct, not coincident.)

The answer is **B**.

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4) A line passes through the point $(4, -3)$ with direction vector $\vec{m} = [1, 5]$

a) Determine the parametric equations of the line.	**b)** What point on the line corresponds to the parameter value $t = 2$?	**c)** Does the line contain the point $P(3, -7)$?
$\ell : \begin{cases} x = 4 + t \\ y = -3 + 5t \end{cases}$	$x = 4 + 2 = 6, \quad y = -3 + 5(2) = 7$ $(6, 7)$	$3 = 4 + t \Rightarrow t = -1, \quad -7 = -3 + 5t \Rightarrow t = -\frac{4}{5}$ The two values of t disagree, so **no**.

5) Consider the lines

$\ell_1 : [x, y] = [1, 4] + t[-7, 2]$

$\ell_2 : [x, y] = [-2, 6] + t[a, 9]$

a) For what value of a are the lines parallel?	**b)** For what value of a are the lines perpendicular?
Parallel if $\vec{m_1} = k\vec{m_2}$ for some scalar k: $2 = 9k \Rightarrow k = \frac{2}{9}$ $-7 = ka = \frac{2}{9}a$ $a = -\frac{63}{2}$	Perpendicular if $\vec{m_1} \cdot \vec{m_2} = 0$: $-7a + 2(9) = 0$ $a = \frac{18}{7}$

6) Write a vector equation of the line through $P(9, -1)$ and perpendicular to $[x, y] = [1, -2] + t[6, -7]$

A vector perpendicular to $[6, -7]$ (swap the coordinates, flip one sign) is $\vec{n} = [7, 6]$

$[x, y] = [9, -1] + t[7, 6]$

7) Write the vector, parametric, and scalar equations of each plane.

a) contains the points $D(1, 7, 2)$, $E(4, 0, -1)$, and $F(1, 2, 3)$

$\vec{m_1} = \overrightarrow{DE} = [3, -7, -3], \quad \vec{m_2} = \overrightarrow{DF} = [0, -5, 1]$

$[x, y, z] = [1, 7, 2] + s[3, -7, -3] + t[0, -5, 1]$

$\pi : \begin{cases} x = 1 + 3s \\ y = 7 - 7s - 5t \\ z = 2 - 3s + t \end{cases}$

$\vec{n} = \vec{m_1} \times \vec{m_2} = [-22, -3, -15] = -1[22, 3, 15]$

$22x + 3y + 15z + D = 0, \quad 22(1) + 3(7) + 15(2) + D = 0 \Rightarrow D = -73$

$22x + 3y + 15z - 73 = 0$

b) parallel to the xz-plane and through the point $Q(2, -3, 4)$

Two direction vectors in the xz-plane: $\vec{m_1} = [1, 0, 0]$, $\vec{m_2} = [0, 0, 1]$

$[x, y, z] = [2, -3, 4] + s[1, 0, 0] + t[0, 0, 1]$

$\pi : \begin{cases} x = 2 + s \\ y = -3 \\ z = 4 + t \end{cases}$

$y = -3$

c) contains the point $(3, -5, 1)$ and is parallel to $[x, y, z] = [-5, 2, -5] + t[3, -1, 1] + s[1, 1, 1]$

$[x, y, z] = [3, -5, 1] + s[3, -1, 1] + t[1, 1, 1]$

$\pi : \begin{cases} x = 3 + 3s + t \\ y = -5 - s + t \\ z = 1 + s + t \end{cases}$

$\vec{n} = [3, -1, 1] \times [1, 1, 1] = [-2, -2, 4] = -2[1, 1, -2]$

$x + y - 2z + D = 0, \quad 3 - 5 - 2 + D = 0 \Rightarrow D = 4$

$x + y - 2z + 4 = 0$

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8) Determine the x, y, and z-intercepts of each plane

a) $2x + 4y - 6z = 24$

x-int: $2x = 24 \Rightarrow x = 12$

y-int: $4y = 24 \Rightarrow y = 6$

z-int: $-6z = 24 \Rightarrow z = -4$

$(12, 0, 0)$, $(0, 6, 0)$, $(0, 0, -4)$

b) $[x, y, z] = [12, -9, 4] + s[2, -1, -2] + t[3, -3, 12]$

$\vec{n} = [2, -1, -2] \times [3, -3, 12] = [-18, -30, -3] = -3[6, 10, 1]$

$6x + 10y + z + D = 0$, $6(12) + 10(-9) + 4 + D = 0 \Rightarrow D = 14$

$6x + 10y + z + 14 = 0$

x-int: $x = -\frac{7}{3}$; y-int: $y = -\frac{7}{5}$; z-int: $z = -14$

$\left(-\frac{7}{3}, 0, 0\right)$, $\left(0, -\frac{7}{5}, 0\right)$, $(0, 0, -14)$

c) $7x - 3z + 42 = 0$

x-int: $7x + 42 = 0 \Rightarrow x = -6$

y-int: none (the equation has no y term)

z-int: $-3z + 42 = 0 \Rightarrow z = 14$

$(-6, 0, 0)$, none, $(0, 0, 14)$

9) Determine if the point $P(3, -6, -1)$ is contained in the plane $[x, y, z] = [2, 1, -3] + s[5, 3, 1] + t[6, -4, 3]$

$\vec{n} = [5, 3, 1] \times [6, -4, 3] = [13, -9, -38]$

$13x - 9y - 38z + D = 0$, $13(2) - 9(1) - 38(-3) + D = 0 \Rightarrow D = -131$

Scalar equation: $13x - 9y - 38z - 131 = 0$

Check $P(3, -6, -1)$: $13(3) - 9(-6) - 38(-1) - 131 = 39 + 54 + 38 - 131 = 0$

Yes, P is on the plane.

10) Determine if the point $A(1, 3, 7)$ lies on the plane $3x - 2y + z = 4$.

$3(1) - 2(3) + 7 = 3 - 6 + 7 = 4$

LS = RS = 4, so **yes**, A is on the plane.

11) Write the scalar equation of the plane with $\vec{n} = [2, -4, 3]$ that contains the point $R(3, -5, 1)$.

$2x - 4y + 3z + D = 0$, $2(3) - 4(-5) + 3(1) + D = 0 \Rightarrow D = -29$

$2x - 4y + 3z - 29 = 0$

12) Write the scalar equation of this plane $[x, y, z] = [2, 1, 4] + t[-2, 5, 3] + s[1, 0, -5]$

$\vec{n} = [-2, 5, 3] \times [1, 0, -5] = [-25, -7, -5] = -1[25, 7, 5]$

$25x + 7y + 5z + D = 0$, $25(2) + 7(1) + 5(4) + D = 0 \Rightarrow D = -77$

$25x + 7y + 5z - 77 = 0$

13) Write the scalar equation of the plane that contains the points $A(1, 2, 3)$, $B(2, 3, 4)$, and $C(4, 5, 5)$

$\overrightarrow{m_1} = \overrightarrow{AB} = [1, 1, 1]$, $\overrightarrow{m_2} = \overrightarrow{AC} = [3, 3, 2]$

$\vec{n} = \overrightarrow{m_1} \times \overrightarrow{m_2} = [-1, 1, 0]$

$x - y + D = 0$, $1 - 2 + D = 0 \Rightarrow D = 1$

$x - y + 1 = 0$

14) Determine if the lines in each pair intersect. If they intersect, find the intersection point. If they don't, explain why.

a) $\ell_1 : [x, y] = [-1, -4] + t[1, -1]$, $\ell_2 : \begin{cases} x = 3 - 2s \\ y = -1 + 3s \end{cases}$

Not parallel ($\overrightarrow{m_1} \neq k\overrightarrow{m_2}$), so they intersect.

$-1 + t = 3 - 2s \Rightarrow t + 2s = 4$

$-4 - t = -1 + 3s \Rightarrow -t - 3s = 3$

Adding the two equations: $-s = 7 \Rightarrow s = -7$, then $t = 4 - 2(-7) = 18$

$x = 3 - 2(-7) = 17$, $y = -1 + 3(-7) = -22$

Intersection point: $(17, -22)$

b) $\ell_1 : [x, y, z] = [3, -2, 3] + t[-1, 1, 2]$,
$\ell_2 : [x, y, z] = [1, -1, 4] + s[1, 1, 4]$

Not parallel, so they intersect at a point or are skew.

$3 - t = 1 + s$, $-2 + t = -1 + s$, $3 + 2t = 4 + 4s$

Solving the first two equations gives $t = \frac{3}{2}$, $s = \frac{1}{2}$, and these also satisfy the third equation.

$x = 1 + \frac{1}{2} = \frac{3}{2}$, $y = -1 + \frac{1}{2} = -\frac{1}{2}$, $z = 4 + 4\left(\frac{1}{2}\right) = 6$

Intersection point: $\left(\frac{3}{2}, -\frac{1}{2}, 6\right)$

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c) $\ell_1 : [x, y, z] = [3, -3, 0] + t[3, -1, 1]$,
$\ell_2 : [x, y, z] = [4, 0, 4] + s[-1, 1, 1]$

Not parallel. Set equal: $3 + 3t = 4 - s$, $-3 - t = s$, $t = 4 + s$

$3t + s = 1$, $-t - s = 3$

Adding: $2t = 4 \Rightarrow t = 2$, then $s = -5$ from the second equation

Check: $t - s = 2 - (-5) = 7$, but the third equation needs $t - s = 4$

$7 \neq 4$, so the lines are **skewed**; there is no intersection.

d) $\ell_1 : x = 1 + 3t$, $y = 5t$, $z = 4t - 3$,
$\ell_2 : [x, y, z] = [0, -9, -1] + s[-1, 2, -3]$

Not parallel. Set equal: $3t + s = -1$, $5t - 2s = -9$, $4t + 3s = 2$

$2 \times (3t + s = -1) \Rightarrow 6t + 2s = -2$; add to $5t - 2s = -9 \Rightarrow 11t = -11 \Rightarrow t = -1$

Then $s = 2$ from the first equation; checking the third equation, $4(-1) + 3(2) = 2$, consistent.

$x = -s = -2$, $y = -9 + 2s = -5$, $z = -1 - 3s = -7$

Intersection point: $(-2, -5, -7)$

e) $\ell_1 : [x, y, z] = [3, -3, 0] + t[3, -1, 1]$,
$\ell_2 : [x, y, z] = [6, -4, 1] + s[6, -2, 2]$

$\vec{m_2} = 2\vec{m_1}$, so the lines are parallel. Check if $(3, -3, 0)$ is on ℓ_2:

$6 + 6s = 3$, $-4 - 2s = -3$, $1 + 2s = 0 \Rightarrow s = -\frac{1}{2}$ in all three equations

Since the same s works in all three equations, the lines are **parallel and coincident** (infinite solutions).

15) Determine if each line intersects the plane. If so, state the solution.

a) $\ell : [x, y, z] = [8, -1, 4] + t[3, 0, -1]$, $\pi : x + y - z = 6$

$(8 + 3t) + (-1) - (4 - t) = 6 \Rightarrow 4t + 3 = 6 \Rightarrow t = \frac{3}{4}$

$x = 8 + 3\left(\frac{3}{4}\right) = \frac{41}{4}$, $y = -1$, $z = 4 - \frac{3}{4} = \frac{13}{4}$

Intersection point: $\left(\frac{41}{4}, -1, \frac{13}{4}\right)$

b) $\ell : [x, y, z] = [4, 2, 5] + t[-1, -2, -2]$, $\pi : 2x + 3y - 4z = -7$

$2(4 - t) + 3(2 - 2t) - 4(5 - 2t) = -7 \Rightarrow -6 + 0t = -7$

$0 = -1$ is never true, so the line does **not** intersect the plane; it is parallel and distinct.

c) $\ell : [x, y, z] = [1, 13, 2] + t[1, 1, -1]$, $\pi : 3x - y + 2z + 6 = 0$

$3(1 + t) - (13 + t) + 2(2 - t) + 6 = 0 \Rightarrow 0 + 0t = 0$

This is always true, so there are **infinite intersections**; the line lies in the plane (parallel and coincident).

d) $\ell : x = 3 + 2t$, $y = -3 + 3t$, $z = -3 + 2t$, $\pi : 4x - 2y + z = 19$

$4(3 + 2t) - 2(-3 + 3t) + (-3 + 2t) = 19 \Rightarrow 15 + 4t = 19 \Rightarrow t = 1$

$x = 3 + 2 = 5$, $y = -3 + 3 = 0$, $z = -3 + 2 = -1$

Intersection point: $(5, 0, -1)$

16) State the intersections of each of the following systems of equations. If there are no solutions, make sure you clearly indicate how you know and describe the scenario that is resulting in no solutions.

a) $\pi_1 : 3x + 2y + 5z = 4$, $\pi_2 : 4x - 3y + z = -1$

The normals $[3, 2, 5]$ and $[4, -3, 1]$ aren't parallel, so the planes intersect in a line.

$5 \times \pi_2 : 20x - 15y + 5z = -5$; subtract $\pi_1 \Rightarrow -17x + 17y = 9$

Let $x = t$: $y = \frac{9}{17} + t$, and substituting back into π_1 gives $z = \frac{10}{17} - t$.

Line of intersection: $[x, y, z] = \left[\frac{10}{17}, \frac{19}{17}, 0\right] + t[-1, -1, 1]$

b) $\pi_1 : 3x + 2y + 5z = 4$, $\pi_2 : 6x + 4y + 10z = 3$

π_2's normal is exactly $2\times$ π_1's normal, so the planes are parallel.

$2 \times \pi_1 \Rightarrow 6x + 4y + 10z = 8$, but π_2's RHS is $3 \neq 8$

The planes are **parallel and distinct**; there are **no solutions**.

c) $\pi_1 : 4x + 2y + 3z = 5$, $\pi_2 : x - 6y + z = 14$,
$\pi_3 : -5x - 14y + 2z = 11$

$3 \times \pi_2 : 3x - 18y + 3z = 42$; $\pi_1 - (3 \times \pi_2) \Rightarrow x + 20y = -37$

$2 \times \pi_2 : 2x - 12y + 2z = 28$; $(2 \times \pi_2) - \pi_3 \Rightarrow 7x + 2y = 17$

$7 \times (x + 20y = -37) \Rightarrow 7x + 140y = -259$; subtracting $7x + 2y = 17$ gives $138y = -276 \Rightarrow y = -2$

$7x + 2(-2) = 17 \Rightarrow x = 3$

$4(3) + 2(-2) + 3z = 5 \Rightarrow z = -1$

Point of intersection: $(3, -2, -1)$

d) $\pi_1 : 2x + y - z = 3$, $\pi_2 : x + y = 2$, $\pi_3 : x - z = 1$

From π_2: $y = 2 - x$; from π_3: $z = x - 1$

$2x + (2 - x) - (x - 1) = 3 \Rightarrow 0x + 3 = 3$, always true

The three planes intersect in a line. Let $x = 1 + t$: $y = 1 - t$, $z = t$.

Line of intersection: $[x, y, z] = [1, 1, 0] + t[-1, -1, 1]$

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e) $\pi_1 : x + 3y + 3z = 8$, $\pi_2 : x - y + 3z = 4$, $\pi_3 : 2x + 6y + 6z = 16$ $\pi_3 = 2\times\pi_1$, so π_1 and π_3 are coincident; only π_1 and π_2 need to be solved. $\pi_1 - \pi_2 \Rightarrow 4y = 4 \Rightarrow y = 1$ Let $z = t$: from π_1, $x + 3 + 3t = 8 \Rightarrow x = 5 - 3t$ Line of intersection: $[x, y, z] = [5, 1, 0] + t[-3, 0, 1]$	**f)** $\pi_1 : 3x + 5y - 2z = 8$, $\pi_2 : -2x - 3y + 4z = 2$, $\pi_3 : 7x + 11y - 10z = 21$ $2\times\pi_1 + \pi_2 \Rightarrow 4x + 7y = 18$ $5\times\pi_1 - \pi_3 \Rightarrow 8x + 14y = 19$ $2\times(4x + 7y = 18) \Rightarrow 8x + 14y = 36$, which contradicts $8x + 14y = 19$ $(0 = 17)$. **Inconsistent system**; the three planes intersect pairwise (in three separate lines) with no common point.
g) $\pi_1 : x + 3y - 5z = -1$, $\pi_2 : 3x + 9y - 15z = -3$, $\pi_3 : 5x + 15y - 25z = 5$ $\pi_2 = 3\times\pi_1$, so π_1 and π_2 are coincident. π_3's normal is also parallel to π_1's ($[5, 15, -25] = 5[1, 3, -5]$), but $5\times(-1) = -5 \neq 5$, so π_3 is parallel and distinct from π_1 / π_2. **Inconsistent system**; planes 1 and 2 are coincident, but plane 3 is parallel and distinct.	

17) Find the angle between

$\pi_1 : 3x - 6y - 2z = 15$

$\pi_2 : 2x + y - 2z = 5$

$\cos\theta = \dfrac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = \dfrac{3(2) + (-6)(1) + (-2)(-2)}{\sqrt{49}\sqrt{9}} = \dfrac{4}{21}$

$\theta \approx 79°$

18) Determine the shortest distance between the point $P(3, -4, 6)$ and the plane $2x + 3y - 4z = 8$

$\vec{n} = [2, 3, -4]$. Pick a point on the plane, e.g. $Q(0, 0, -2)$.

$\overrightarrow{PQ} = [0 - 3, 0 - (-4), -2 - 6] = [-3, 4, -8]$

$d = \dfrac{|\overrightarrow{PQ}\cdot\vec{n}|}{|\vec{n}|} = \dfrac{|-3(2) + 4(3) + (-8)(-4)|}{\sqrt{29}}$

$d = \dfrac{38}{\sqrt{29}}$ units

Answer Key

1)a) $\ell : \begin{cases} x = 4 + 3t \\ y = -5 + 5t \end{cases}$ **b)** $\ell : \begin{cases} x = -6 + 3t \\ y = 2 + 7t \\ z = 1 - 2t \end{cases}$ **c)** $\ell : \begin{cases} x = t \\ y = 1 \\ z = 2 \end{cases}$ **d)** $\ell : \begin{cases} x = 4 + t \\ y = -5 + 2t \\ z = 3 + 2t \end{cases}$ **e)** $\ell : \begin{cases} x = 1 + 8t \\ y = -3 + 5t \\ z = 2 - 2t \end{cases}$ **f)** $\ell : \begin{cases} x = 1 \\ y = 3 + t \\ z = 5 \end{cases}$ **2)** $\ell : \begin{cases} x = -2 + 3t \\ y = 3 - 2t \\ z = 7 + 5t \end{cases}$; the point is **NOT** on the line **3) B 4)a)** $\ell : \begin{cases} x = 4 + t \\ y = -3 + 5t \end{cases}$ **b)** (6, 7) **c)** no **5)a)** $a = -\frac{63}{2}$ **b)** $a = \frac{18}{7}$ **6)** $[x, y] = [9, -1] + t[7, 6]$ **7)a)** $22x + 3y + 15z - 73 = 0$ **b)** $y = -3$ **c)** $x + y - 2z + 4 = 0$ **8)a)** (12, 0, 0), (0, 6, 0), (0, 0, −4) **b)** $\left(-\frac{7}{3}, 0, 0\right)$, $\left(0, -\frac{7}{5}, 0\right)$, (0, 0, −14) **c)** (−6, 0, 0), none, (0, 0, 14) **9)** Yes **10)** Yes **11)** $2x - 4y + 3z - 29 = 0$ **12)** $25x + 7y + 5z - 77 = 0$ **13)** $x - y + 1 = 0$ **14)a)** (17, −22) **b)** $\left(\frac{3}{2}, -\frac{1}{2}, 6\right)$ **c)** skewed lines, no intersection **d)** (−2, −5, −7) **e)** infinite solutions; parallel and coincident **15)a)** $\left(\frac{41}{4}, -1, \frac{13}{4}\right)$ **b)** do not intersect; parallel and distinct **c)** infinite intersections; parallel and coincident **d)** (5, 0, −1) **16)a)** $[x, y, z] = \left[\frac{10}{17}, \frac{19}{17}, 0\right] + t[-1, -1, 1]$ **b)** no solutions; parallel and distinct **c)** (3, −2, −1) **d)** $[x, y, z] = [1, 1, 0] + t[1, -1, 1]$ **e)** $[x, y, z] = [5, 1, 0] + t[-3, 0, 1]$ **f)** inconsistent system; intersects in pairs **g)** inconsistent system; planes 1 & 2 are coincident but plane 3 is parallel and distinct **17)** 79° **18)** $\frac{38}{\sqrt{29}}$ units

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