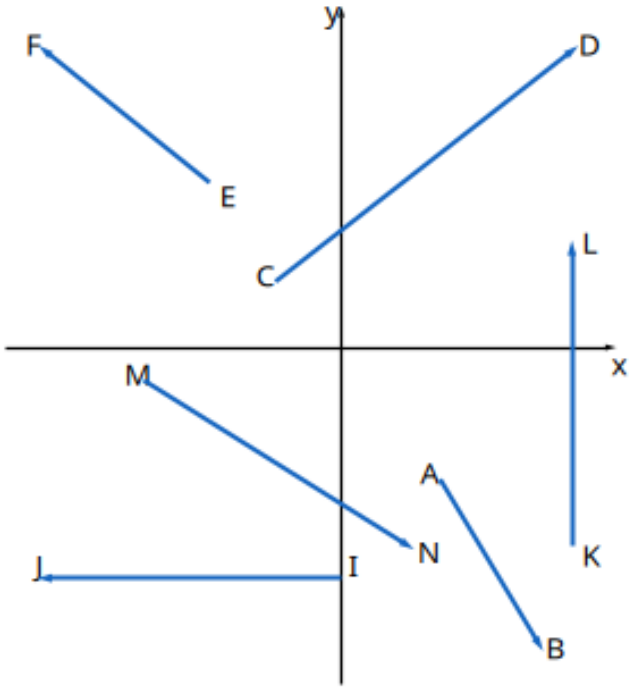
Grade 12 Calculus and Vectors Exam Review Lesson (Unit 5) with solutions

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Vectors Exam Review – Unit 5: Cartesian Vectors

1) Write the coordinates of each Cartesian vector. Then find its magnitude.



$\overrightarrow{AB} = [3, -5]$, $|\overrightarrow{AB}| = \sqrt{34}$

$\overrightarrow{CD} = [9, 7]$, $|\overrightarrow{CD}| = \sqrt{130}$

$\overrightarrow{EF} = [-5, 4]$, $|\overrightarrow{EF}| = \sqrt{41}$

$\overrightarrow{IJ} = [-9, 0]$, $|\overrightarrow{IJ}| = 9$

$\overrightarrow{KL} = [0, 9]$, $|\overrightarrow{KL}| = 9$

$\overrightarrow{MN} = [8, -5]$, $|\overrightarrow{MN}| = \sqrt{89}$

2) Given $\vec{u} = [2, -1]$ and $\vec{v} = [5, -7]$, evaluate each expression.

a) $2\vec{v}$ $2\vec{v} = 2[5, -7]$ $2\vec{v} = [10, -14]$	**b)** $\vec{u} - \vec{v}$ $\vec{u} - \vec{v} = [2, -1] - [5, -7]$ $\vec{u} - \vec{v} = [-3, 6]$
c) $3\vec{u} + 5\vec{v}$ $3\vec{u} + 5\vec{v} = 3[2, -1] + 5[5, -7]$ $3\vec{u} + 5\vec{v} = [6, -3] + [25, -35]$ $3\vec{u} + 5\vec{v} = [31, -38]$	**d)** $4\vec{u} - 2\vec{v}$ $4\vec{u} - 2\vec{v} = 4[2, -1] - 2[5, -7]$ $4\vec{u} - 2\vec{v} = [8, -4] - [10, -14]$ $4\vec{u} - 2\vec{v} = [-2, 10]$

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3) Consider the points $A(2, -7)$, $B(-4, 5)$ and $C(6, 8)$.

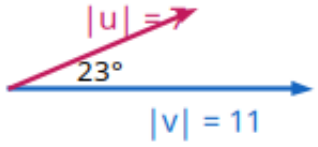
a) Determine $\overrightarrow{AC}$	**b)** Determine $\lvert\overrightarrow{AB}\rvert$	**c)** Determine the perimeter of $\triangle ABC$
$\overrightarrow{AC} = [6-2, 8-(-7)]$ $\overrightarrow{AC} = [4, 15]$	$\overrightarrow{AB} = [-4-2, 5-(-7)] = [-6, 12]$ $\lvert\overrightarrow{AB}\rvert = \sqrt{(-6)^2+12^2} = 6\sqrt{5}$ units	$\lvert\overrightarrow{AC}\rvert = \sqrt{4^2+15^2} = \sqrt{241}$ $\overrightarrow{BC} = [10, 3]$, $\lvert\overrightarrow{BC}\rvert = \sqrt{109}$ Perimeter $= 6\sqrt{5}+\sqrt{241}+\sqrt{109} \approx 39.4$ units

4) Determine the value of k so that the vectors in each pair are collinear.

a) $\vec{u} = [2, k]$, $\vec{v} = [-12, 30]$	**b)** $\vec{u} = [-4, 6]$, $\vec{v} = [30, k]$
If collinear, $\vec{u} = a\vec{v}$ for some scalar a: $2 = -12a \Rightarrow a = -\frac{1}{6}$ $k = 30a = 30\left(-\frac{1}{6}\right)$ $k = -5$	If collinear, $\vec{u} = a\vec{v}$ for some scalar a: $-4 = 30a \Rightarrow a = -\frac{2}{15}$ $6 = ka \Rightarrow k = \frac{6}{a} = 6\left(-\frac{15}{2}\right)$ $k = -45$

5) Calculate the dot product of each pair of vectors.

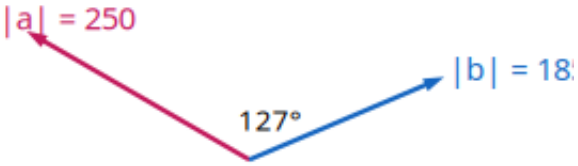
a) $\lvert\vec{u}\rvert = 7$, $\lvert\vec{v}\rvert = 11$, angle between them $23°$



$\vec{u}\cdot\vec{v} = \lvert\vec{u}\rvert\lvert\vec{v}\rvert\cos\theta = 7(11)\cos(23°)$

$\vec{u}\cdot\vec{v} \approx 70.88$

b) $\lvert\vec{a}\rvert = 250$, $\lvert\vec{b}\rvert = 185$, angle between them $127°$



$\vec{a}\cdot\vec{b} = 250(185)\cos(127°)$

$\vec{a}\cdot\vec{b} \approx -27\,833.94$

c) $\vec{u} = [-8, 9]$ and $\vec{v} = [1, -7]$

$\vec{u}\cdot\vec{v} = (-8)(1)+9(-7)$

$\vec{u}\cdot\vec{v} = -71$

d) $\vec{u} = -5\hat{i}+11\hat{j}$ and $\vec{v} = 7\hat{i}-2\hat{j}$

$\vec{u}\cdot\vec{v} = (-5)(7)+11(-2)$

$\vec{u}\cdot\vec{v} = -57$

6) Write a 250 N force applied at $60°$ to the horizontal as a Cartesian vector.

$F_x = 250\cos(60°) = 125$

$F_y = 250\sin(60°) = 125\sqrt{3} \approx 216.5$

$\vec{F} = [125, 125\sqrt{3}] \approx [125, 216.5]$

7) Evaluate, given vectors $\vec{u} = [2, -3]$ and $\vec{v} = [3, 1]$.

a) $3\vec{u}+2\vec{v}$	**b)** $\lvert\vec{u}\rvert$	**c)** $\vec{u}\cdot\vec{v}$	**d)** $\lvert\vec{u}+\vec{v}\rvert$
$3\vec{u}+2\vec{v} = [6, -9]+[6, 2]$ $3\vec{u}+2\vec{v} = [12, -7]$	$\lvert\vec{u}\rvert = \sqrt{2^2+(-3)^2} = \sqrt{13}$	$\vec{u}\cdot\vec{v} = 2(3)+(-3)(1)$ $\vec{u}\cdot\vec{v} = 3$	$\vec{u}+\vec{v} = [5, -2]$ $\lvert\vec{u}+\vec{v}\rvert = \sqrt{29}$

8) Calculate the angle θ between the vectors.

a) $\vec{u} = [-1, 8]$ and $\vec{v} = [3, -5]$	**b)** $\vec{u} = [3, -7]$ and $\vec{v} = [6, -1]$
$\cos\theta = \frac{(-1)(3)+8(-5)}{\sqrt{65}\sqrt{34}}$ $\theta \approx 156.2°$	$\cos\theta = \frac{3(6)+(-7)(-1)}{\sqrt{58}\sqrt{37}}$ $\theta \approx 57.3°$

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9) Determine the projection of $\vec{u}$ on $\vec{v}$.

a) $|\vec{u}| = 95$ and $|\vec{v}| = 275$, and the angle between $\vec{u}$ and $\vec{v}$ is $13°$.

$\text{proj}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta\,\hat{v} = 95\cos(13°)\,\hat{v}$

$\text{proj}_{\vec{v}}\vec{u} \approx 92.57\,\hat{v}$

b) $\vec{u} = [6, -5]$ and $\vec{v} = [7, -11]$

$\text{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v} = \dfrac{6(7)+(-5)(-11)}{7^2+(-11)^2}[7, -11]$

$\text{proj}_{\vec{v}}\vec{u} = \dfrac{97}{170}[7, -11]$

10) For vectors $\vec{c} = [2, -7]$ and $\vec{d} = [9, 12]$

a) determine the magnitude of the projection of $\vec{c}$ on $\vec{d}$

$|\text{proj}_{\vec{d}}\vec{c}| = \dfrac{|\vec{c}\cdot\vec{d}|}{|\vec{d}|} = \dfrac{|2(9)+(-7)(12)|}{15}$

$|\text{proj}_{\vec{d}}\vec{c}| = \dfrac{66}{15} = 4.4$

b) determine the vector projection of $\vec{c}$ on $\vec{d}$

$\text{proj}_{\vec{d}}\vec{c} = \dfrac{\vec{c}\cdot\vec{d}}{\vec{d}\cdot\vec{d}}\vec{d} = \dfrac{-66}{225}[9, 12]$

$\text{proj}_{\vec{d}}\vec{c} = \left[-\dfrac{66}{25}, -\dfrac{88}{25}\right]$

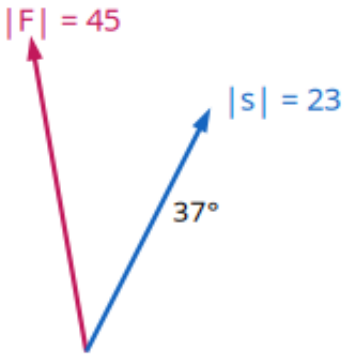
11) Determine the work done by each $\vec{F}$, in Newtons, for an object moved along the vector, $\vec{d}$, in metres.

a) $\vec{F} = [18, 23]$ and $\vec{d} = [9, 12]$

$W = \vec{F}\cdot\vec{d} = 18(9)+23(12)$

$W = 438$ J

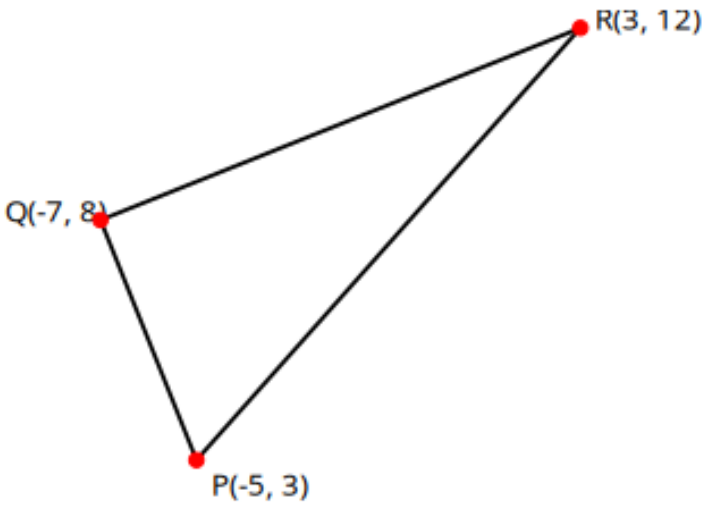
b) $|\vec{F}| = 45$, $|\vec{s}| = 23$, angle between them $37°$



$W = \vec{F}\cdot\vec{s} = |\vec{F}||\vec{s}|\cos\theta = 45(23)\cos(37°)$

$W \approx 826.59$ J

12) Determine if $\triangle PQR$ with vertices $P(-5, 3)$, $Q(-7, 8)$ and $R(3, 12)$ is a right triangle. If so, identify the right angle.



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$\overrightarrow{PQ} = [-2, 5]$, $|\overrightarrow{PQ}| = \sqrt{29}$

$\overrightarrow{PR} = [8, 9]$, $|\overrightarrow{PR}| = \sqrt{145}$

$\overrightarrow{RQ} = [-10, -4]$, $|\overrightarrow{RQ}| = \sqrt{116}$

Check the Pythagorean theorem against the longest side, $\overrightarrow{PR}$:

$\left(\sqrt{29}\right)^2 + \left(\sqrt{116}\right)^2 \stackrel{?}{=} \left(\sqrt{145}\right)^2$

$29 + 116 = 145$

$145 = 145$ so $\triangle PQR$ is a right triangle, with the right angle at Q.

13) Determine the exact magnitude of the vector $\overrightarrow{AB}$ joining $A(3, 8, -11)$ and $B(7, -3, 15)$.

$\overrightarrow{AB} = [7-3, -3-8, 15-(-11)] = [4, -11, 26]$

$|\overrightarrow{AB}| = \sqrt{4^2 + (-11)^2 + 26^2} = \sqrt{813}$

14) For each of the following calculate the angle between the two vectors.

a) $[0, 1, -2]$ and $[-3, 1, 4]$ $\cos\theta = \frac{0(-3) + 1(1) + (-2)(4)}{\sqrt{5}\sqrt{26}}$ $\theta \approx 127.9°$	**b)** $[1.5, 20, 0]$ and $[-20, 1, 10]$ $\cos\theta = \frac{1.5(-20) + 20(1) + 0(10)}{\sqrt{402.25}\sqrt{501}}$ $\theta \approx 91.3°$

15) Given $\vec{c} = [-6, 4, 0]$, $\vec{d} = [0, -5, -7]$, and $\vec{e} = [3, 1, 2]$.

a) $10\vec{c} - 10\vec{d}$ $10\vec{c} - 10\vec{d} = [-60, 40, 0] - [0, -50, -70]$ $10\vec{c} - 10\vec{d} = [-60, 90, 70]$	**b)** $2\vec{c} \cdot 3\vec{d} - 2\vec{c} \cdot 4\vec{e}$ $= 6(\vec{c} \cdot \vec{d}) - 8(\vec{c} \cdot \vec{e}) = 6(-20) - 8(-14)$ $= -8$
c) $(3\vec{d} - 4\vec{e}) \cdot 2\vec{c}$ by the distributive property, $= 3\vec{d} \cdot 2\vec{c} - 4\vec{e} \cdot 2\vec{c}$ $= -8$ (same value as part b, dot product is commutative)	**d)** $\vec{c} \times \vec{d}$ $\vec{c} \times \vec{d} = [4(-7) - 0(-5), 0(0) - (-6)(-7), (-6)(-5) - 4(0)]$ $\vec{c} \times \vec{d} = [-28, -42, 30]$
e) $\vec{d} \times \vec{e}$ $\vec{d} \times \vec{e} = [(-5)(2) - (-7)(1), (-7)(3) - 0(2), 0(1) - (-5)(3)]$ $\vec{d} \times \vec{e} = [-3, -21, 15]$	**f)** $\vec{c} \times \vec{c}$ All collinear vectors have a cross product of $\vec{0}$. Since $\vec{c}$ is collinear with itself, $\vec{c} \times \vec{c} = \vec{0}$.

16) If $\vec{u} = [5, 7, 9]$ is orthogonal to $\vec{v} = [1, t, 1]$, determine t.

$\vec{u} \cdot \vec{v} = 0$

$5(1) + 7t + 9(1) = 0$

$7t = -14$

$t = -2$

17) Determine the area of the parallelogram bound by the vectors $\vec{a} = [7, 11, 4]$ and $\vec{b} = [3, -2, 5]$.

$\vec{a} \times \vec{b} = [11(5) - 4(-2), 4(3) - 7(5), 7(-2) - 11(3)]$

$\vec{a} \times \vec{b} = [63, -23, -47]$

Area $= |\vec{a} \times \vec{b}| = \sqrt{63^2 + 23^2 + 47^2} = \sqrt{6707}$ units^2

18) A triangle has vertices $A(2, 5, -1)$, $B(4, 1, -3)$, and $C(-5, -6, 4)$. Find the area of the triangle.

$\overrightarrow{AB} = [2, -4, -2]$, $\overrightarrow{AC} = [-7, -11, 5]$

$\overrightarrow{AB} \times \overrightarrow{AC} = [(-4)(5) - (-2)(-11), (-2)(-7) - 2(5), 2(-11) - (-4)(-7)]$

$\overrightarrow{AB} \times \overrightarrow{AC} = [-42, 4, -50]$

Area $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{4280} = \sqrt{1070}$ units^2

19) Calculate the area of the parallelogram $ABCD$ where $A(4, 6)$, $B(5, 6)$, $C(11, 15)$, and $D(-2, -3)$

$\overrightarrow{AB} = [1, 0]$, $\overrightarrow{AD} = [-6, -9]$

$\overrightarrow{AB} \times \overrightarrow{AD} = [0, 0, 1(-9) - 0(-6)] = [0, 0, -9]$

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Area $= |\overrightarrow{AB} \times \overrightarrow{AD}| = 9$ units^2

20) Find the volume of the parallelepiped defined by $\vec{a} = [1,\ 0,\ -4]$, $\vec{b} = [8,\ -7,\ 9]$, and $\vec{c} = [2,\ 3,\ -4]$

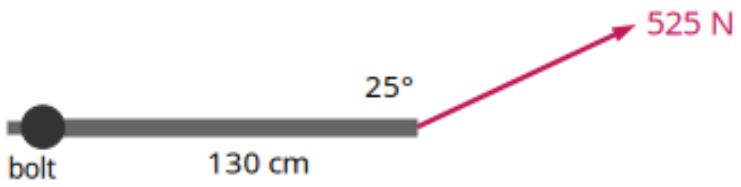
$\vec{b} \times \vec{c} = [(-7)(-4) - 9(3),\ 9(2) - 8(-4),\ 8(3) - (-7)(2)]$

$\vec{b} \times \vec{c} = [1,\ 50,\ 38]$

$V = |\vec{a} \cdot (\vec{b} \times \vec{c})| = |1(1) + 0(50) + (-4)(38)|$

$V = |-151| = 151$ units^3

21) A 525 N force is applied to the end of a 130 cm wrench and makes an angle of $25°$ with the handle of the wrench. What is the torque on the bolt at the other end of the wrench if the force is applied in a clockwise direction?



$|\vec{r}| = 1.3$ m (converting 130 cm to metres)

$T = |\vec{r}||\vec{F}|\sin(25°) = 1.3(525)\sin(25°)$

$T \approx 288.44$ N·m, into the material (the bolt is being tightened)

22) Find a unit vector parallel to $\vec{p} = [-8,\ 2,\ -3]$

$|\vec{p}| = \sqrt{(-8)^2 + 2^2 + (-3)^2} = \sqrt{77}$

$\hat{p} = \frac{1}{\sqrt{77}}[-8,\ 2,\ -3] = \left[-\frac{8}{\sqrt{77}},\ \frac{2}{\sqrt{77}},\ -\frac{3}{\sqrt{77}}\right]$

23) Given $\vec{a} = [3,\ 4,\ -2]$, $\vec{b} = [2,\ -7,\ 1]$, and $\vec{c} = [-6,\ 5,\ 4]$, evaluate each expression

a) $\vec{a} + \vec{c} \times \vec{b}$ $\vec{c} \times \vec{b} = [5(1) - 4(-7),\ 4(2) - (-6)(1),\ (-6)(-7) - 5(2)]$ $\vec{c} \times \vec{b} = [33,\ 14,\ 32]$ $\vec{a} + \vec{c} \times \vec{b} = [3 + 33,\ 4 + 14,\ -2 + 32] = [36,\ 18,\ 30]$	**b)** $(\vec{a} + \vec{c}) \times \vec{b}$ $\vec{a} + \vec{c} = [-3,\ 9,\ 2]$ $(\vec{a} + \vec{c}) \times \vec{b} = [9(1) - 2(-7),\ 2(2) - (-3)(1),\ (-3)(-7) - 9(2)]$ $(\vec{a} + \vec{c}) \times \vec{b} = [23,\ 7,\ 3]$
c) $\lvert -2\vec{b} \times \vec{c} \rvert$ $-2\vec{b} = [-4,\ 14,\ -2]$ $-2\vec{b} \times \vec{c} = [14(4) - (-2)(5),\ (-2)(-6) - (-4)(4),\ (-4)(5) - 14(-6)]$ $-2\vec{b} \times \vec{c} = [66,\ 28,\ 64]$ $\lvert -2\vec{b} \times \vec{c} \rvert = \sqrt{66^2 + 28^2 + 64^2} = \sqrt{9236} = 2\sqrt{2309}$	

24) A storage box is dragged 7 m along a smooth, level floor by a 43-N force, applied at $38°$ to the floor. Then, it is pulled 3.5 m up a ramp inclined at $24°$ to the horizontal, using the same force. The storage box is then dragged a further 6 m along a level platform using the same force again. Determine the total work done in moving the storage box.

On the level floor and the level platform, the angle between the force and the direction of motion stays $38°$. On the ramp, the angle between the force and the direction of motion (along the ramp) is $38° - 24° = 14°$:

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$W = 7(43)\cos(38°) + 3.5(43)\cos(14°) + 6(43)\cos(38°)$

$W \approx 586.5$ J

25) A rock climber pulls himself 21 m up the side of a cliff with a force of 5.2 N, at an angle of 11.2° to the vertical. What is the work done?

$W = \vec{F} \cdot \vec{d} = 5.2(21)\cos(11.2°)$

$W \approx 107.1$ J

Answer Key

1) $\overrightarrow{AB} = [3, -5]$, $|\overrightarrow{AB}| = \sqrt{34}$; $\overrightarrow{CD} = [9, 7]$, $|\overrightarrow{CD}| = \sqrt{130}$; $\overrightarrow{EF} = [-5, 4]$, $|\overrightarrow{EF}| = \sqrt{41}$; $\overrightarrow{IJ} = [-9, 0]$, $|\overrightarrow{IJ}| = 9$; $\overrightarrow{KL} = [0, 9]$, $|\overrightarrow{KL}| = 9$; $\overrightarrow{MN} = [8, -5]$, $|\overrightarrow{MN}| = \sqrt{89}$ **2)a)** $[10, -14]$ **b)** $[-3, 6]$ **c)** $[31, -38]$ **d)** $[-2, 10]$ **3)a)** $[4, 15]$ **b)** $6\sqrt{5}$ units **c)** ≈ 39.4 units **4)a)** -5 **b)** -45 **5)a)** ≈ 70.88 **b)** $\approx -27\,833.94$ **c)** -71 **d)** -57 **6)** $[125, 216.5]$ **7)a)** $[12, -7]$ **b)** $\sqrt{13}$ **c)** 3 **d)** $\sqrt{29}$ **8)a)** $\theta \approx 156.2°$ **b)** $\theta \approx 57.3°$ **9)a)** $\approx 92.57\,\hat{v}$ **b)** $\frac{97}{170}[7, -11]$ **10)a)** 4.4 **b)** $\left[-\frac{66}{25}, -\frac{88}{25}\right]$ **11)a)** 438 J **b)** ≈ 826.59 J **12)** Yes; the right angle is at Q **13)** $\sqrt{813}$ **14)a)** $\approx 127.9°$ **b)** $\approx 91.3°$ **15)a)** $[-60, 90, 70]$ **b)** -8 **c)** -8 **d)** $[-28, -42, 30]$ **e)** $[-3, -21, 15]$ **f)** $\vec{0}$ **16)** $t = -2$ **17)** $\sqrt{6707}$ units^2 **18)** $\sqrt{1070}$ units^2 **19)** 9 units^2 **20)** 151 units^3 **21)** ≈ 288.4 N·m, into the material (bolt is being tightened) **22)** $\left[-\frac{8}{\sqrt{77}}, \frac{2}{\sqrt{77}}, -\frac{3}{\sqrt{77}}\right]$ **23)a)** $[36, 18, 30]$ **b)** $[23, 7, 3]$ **c)** $2\sqrt{2309}$ **24)** ≈ 586.5 J **25)** ≈ 107.1 J

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