Grade 12 Calculus and Vectors Exam Review Lesson (Unit 4) with solutions

This document was composed and formatted by Annie Bernatchez.

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Vectors Exam Review – Unit 4: Geometric Vectors

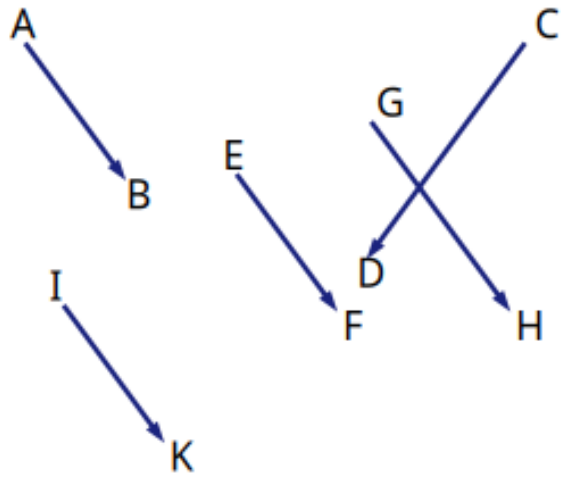
1) Write each true bearing as an equivalent quadrant bearing.

a) 130° $S50°E$	**b)** 330° $N30°W$

2) Write each quadrant bearing as an equivalent true bearing.

a) $S20°W$ 200°	**b)** $E47°N$ 043°

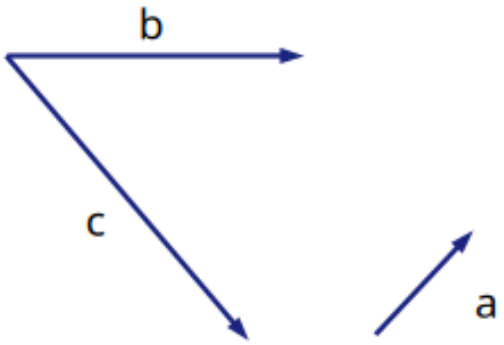
3) Consider these vectors. List the vectors in each set.



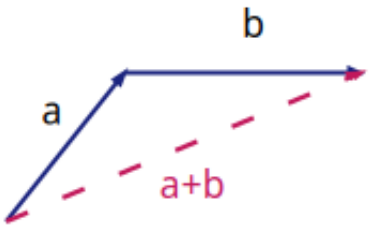
a) same direction as $\overrightarrow{AB}$ $\overrightarrow{EF}$, $\overrightarrow{GH}$, $\overrightarrow{IK}$	**b)** parallel to $\overrightarrow{AB}$ $\overrightarrow{CD}$, $\overrightarrow{EF}$, $\overrightarrow{GH}$, $\overrightarrow{IK}$
c) equivalent to $\overrightarrow{AB}$ $\overrightarrow{EF}$, $\overrightarrow{IK}$	**d)** opposite to $\overrightarrow{AB}$ $\overrightarrow{CD}$

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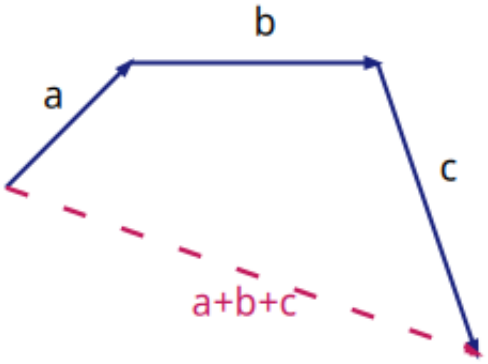
4) Given vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$, draw each of the following:



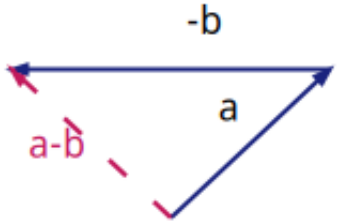
a) $\vec{a} + \vec{b}$



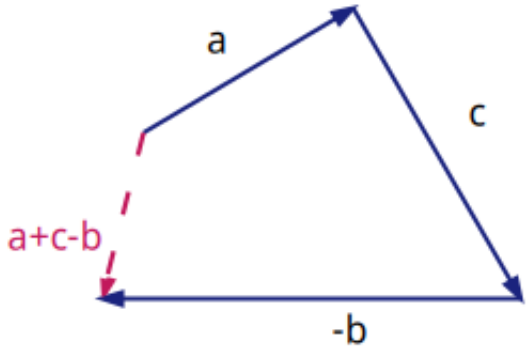
b) $\vec{a} + \vec{b} + \vec{c}$



c) $\vec{a} - \vec{b}$

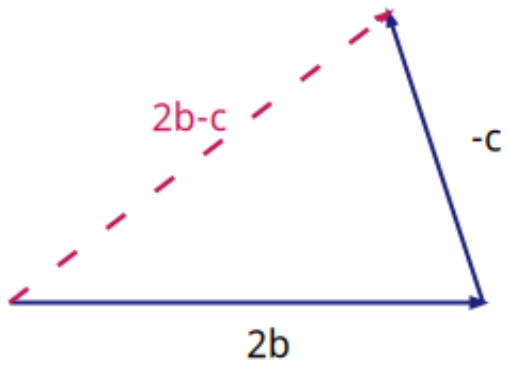


d) $\vec{a} + \vec{c} - \vec{b}$



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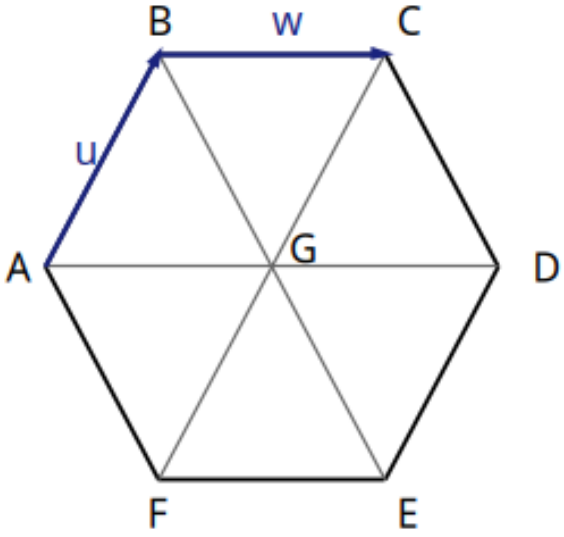
e) $2\vec{b} - \vec{c}$



f) $-2\vec{a}$



5) Consider the regular hexagon $ABCDEF$ with G at the center. $\overrightarrow{AB} = \vec{u}$ and $\overrightarrow{BC} = \vec{w}$. Write each of the following vectors in terms of $\vec{u}$ and $\vec{w}$. *Note: each triangle is equilateral.*



a) $\overrightarrow{FE}$

$\overrightarrow{FE} = \overrightarrow{BC} = \vec{w}$

b) $\overrightarrow{DE}$

$\overrightarrow{DE} = -\overrightarrow{AB} = -\vec{u}$

c) $\overrightarrow{DA}$

$\overrightarrow{DA} = -2\overrightarrow{BC} = -2\vec{w}$

d) $\overrightarrow{GE}$

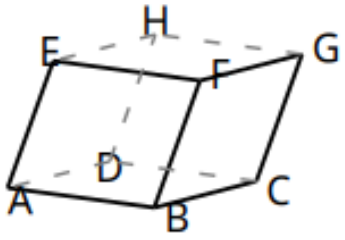
$\overrightarrow{GE} = \overrightarrow{GD} + \overrightarrow{DE}$
$\overrightarrow{GE} = \vec{w} - \vec{u}$

e) $\overrightarrow{AE}$

$\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE}$
$\overrightarrow{AE} = 2\vec{w} - \vec{u}$

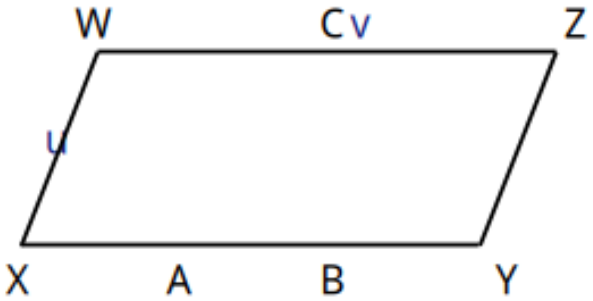
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6) The diagram shows a parallelepiped. Determine a single vector that is equivalent to each sum or difference.



a) $\overrightarrow{AB} - \overrightarrow{BF}$ $\overrightarrow{AB} - \overrightarrow{BF} = \overrightarrow{AB} - \overrightarrow{AE}$ $\overrightarrow{AB} - \overrightarrow{BF} = \overrightarrow{EB}$	**b)** $\overrightarrow{AB} + \overrightarrow{CG}$ $\overrightarrow{AB} + \overrightarrow{CG} = \overrightarrow{AB} + \overrightarrow{BF}$ $\overrightarrow{AB} + \overrightarrow{CG} = \overrightarrow{AF}$
c) $\overrightarrow{EF} + \overrightarrow{DH}$ $\overrightarrow{EF} + \overrightarrow{DH} = \overrightarrow{AB} + \overrightarrow{BF}$ $\overrightarrow{EF} + \overrightarrow{DH} = \overrightarrow{AF}$	**d)** $\overrightarrow{AB} + \overrightarrow{HD} + \overrightarrow{FG}$ $\overrightarrow{AB} + \overrightarrow{HD} + \overrightarrow{FG} = \overrightarrow{AB} - \overrightarrow{BF} + \overrightarrow{FG} = \overrightarrow{EB} + \overrightarrow{FG}$ $\overrightarrow{AB} + \overrightarrow{HD} + \overrightarrow{FG} = \overrightarrow{EB} + \overrightarrow{BC} = \overrightarrow{EC}$
e) $\overrightarrow{AB} - \overrightarrow{HD} + \overrightarrow{FG}$ $\overrightarrow{AB} - \overrightarrow{HD} + \overrightarrow{FG} = \overrightarrow{AB} + \overrightarrow{BF} + \overrightarrow{FG}$ $\overrightarrow{AB} - \overrightarrow{HD} + \overrightarrow{FG} = \overrightarrow{AF} + \overrightarrow{FG} = \overrightarrow{AG}$	**f)** $\overrightarrow{ED} + \overrightarrow{AB} - (\overrightarrow{HG} + \overrightarrow{FB})$ $\overrightarrow{ED} + \overrightarrow{AB} - (\overrightarrow{HG} + \overrightarrow{FB}) = \overrightarrow{ED} + \overrightarrow{DC} - (\overrightarrow{HG} + \overrightarrow{GC})$ $\overrightarrow{ED} + \overrightarrow{AB} - (\overrightarrow{HG} + \overrightarrow{FB}) = \overrightarrow{EC} - \overrightarrow{HC} = \overrightarrow{EC} - \overrightarrow{EB} = \overrightarrow{BC}$

7) In the diagram, C is the midpoint of WZ, and A and B are the points of trisection of XY. Express each vector in terms of a linear combination of $\vec{u} = \overrightarrow{XW}$ and $\vec{v} = \overrightarrow{WZ}$.



a) $\overrightarrow{AB}$ $\overrightarrow{AB} = \frac{1}{3}\vec{v}$	**b)** $\overrightarrow{AZ}$ $\overrightarrow{AZ} = \frac{2}{3}\vec{v} + \vec{u}$
c) $\overrightarrow{WB}$ $\overrightarrow{WB} = -\vec{u} + \frac{2}{3}\vec{v}$	**d)** $\overrightarrow{AC}$ $\overrightarrow{AC} = -\frac{1}{3}\vec{v} + \vec{u} + \frac{1}{2}\vec{v}$ $\overrightarrow{AC} = \frac{1}{6}\vec{v} + \vec{u}$

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8) Given that $|\vec{u}| = 12$ and $|\vec{v}| = 5$ and the angle between $\vec{u}$ and $\vec{v}$ is $30°$, determine:

a) the unit vector in the direction of $\vec{u} + \vec{v}$

The angle between $\vec{u}$ and $\vec{v}$ placed tail to tail is $30°$, so the interior angle of the triangle formed is $150°$:

$|\vec{u} + \vec{v}|^2 = 12^2 + 5^2 - 2(12)(5)\cos(150°)$

$|\vec{u} + \vec{v}|^2 = 169 - 120\left(-\frac{\sqrt{3}}{2}\right)$

$|\vec{u} + \vec{v}|^2 = 169 + 60\sqrt{3}$

$|\vec{u} + \vec{v}| = \sqrt{169 + 60\sqrt{3}}$

$\text{unit vector} = \frac{1}{|\vec{u} + \vec{v}|}(\vec{u} + \vec{v})$

$\text{unit vector} = \frac{1}{\sqrt{169 + 60\sqrt{3}}}(\vec{u} + \vec{v})$

b) $|3\vec{u} + 2\vec{v}|$

$|3\vec{u} + 2\vec{v}|^2 = 36^2 + 10^2 - 2(36)(10)\cos(150°)$

$|3\vec{u} + 2\vec{v}|^2 = 1396 - 720\left(-\frac{\sqrt{3}}{2}\right)$

$|3\vec{u} + 2\vec{v}|^2 = 1396 + 360\sqrt{3}$

$|3\vec{u} + 2\vec{v}| = \sqrt{1396 + 360\sqrt{3}}$

9) An airplane is flying with airspeed 400 km/h on a heading of $000°$. There is a 50 km/h wind blowing from the direction $090°$. Calculate the ground velocity of the plane.

The wind (from $090°$, i.e. blowing toward $270°$) is perpendicular to the plane's heading, so the ground velocity triangle is right-angled:

$|\vec{v}|^2 = 400^2 + 50^2$

$|\vec{v}| \approx 403.1$ km/h

$\theta = \tan^{-1}\left(\frac{50}{400}\right)$

$\theta \approx 7.1°$

The ground velocity is 403.1 km/h at $N7.1°W$.

10) An object weighing 400 N is hanging from two ropes attached to the ceiling. One rope makes an angle of $40°$ with the ceiling, the other makes an angle of $50°$ with the ceiling. Determine the tension in each rope.

The two ropes and the vertical 400 N weight form a force triangle with angles $40°$, $50°$, and $90°$:

$\sin(40°) = \frac{|\vec{T_1}|}{400}$

$|\vec{T_1}| \approx 257.1$ N (the rope at $40°$ to the ceiling)

$\cos(40°) = \frac{|\vec{T_2}|}{400}$

$|\vec{T_2}| \approx 306.4$ N (the rope at $50°$ to the ceiling)

11) A car is moving north at a speed of 25 m/s. A child in the back seat of the car throws a toy to the passenger in the front seat with a speed of 10 m/s in the direction forward $50°$ right (relative to the car). Calculate the speed and direction of the toy relative to the road.

The angle between the car's velocity (25 m/s) and the toy's velocity relative to the car (10 m/s, $50°$ off forward) is $50°$, so the angle inside the resultant triangle is $130°$:

$|\vec{r}|^2 = 25^2 + 10^2 - 2(25)(10)\cos(130°)$

$|\vec{r}| \approx 32.35$ m/s

$\cos\theta = \frac{10^2 - 25^2 - 32.35^2}{-2(25)(32.35)}$

$\theta \approx 13.7°$

The toy's speed is 32.35 m/s with a direction of $N13.7°E$ (forward $13.7°$ right, relative to the road).

12) A sign weighing 98 N is suspended from the middle of a 4 m long chain. The ends of the chain are attached to a ceiling at points 3 m apart. Determine the tensions in the chains.

Each half of the chain is 2 m long, and the two halves together with the 3 m ceiling span form an isosceles triangle:

$\cos\theta = \frac{2^2 - 3^2 - 2^2}{-2(3)(2)}$

$\theta \approx 41.41°$ (the base angle at each end of the chain)

By symmetry, the two chain tensions are equal. Using the force triangle (base angles $48.59°$, apex angles $41.41°$):

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$$\frac{|\vec{T_2}|}{\sin(48.59°)} = \frac{|\vec{T_1}|}{\sin(48.59°)} = \frac{98}{\sin(82.82°)}$$

$$|\vec{T_1}| = |\vec{T_2}| \approx 74.1 \text{ N}$$

13) A canoeist leaves a dock and paddles her canoe at an angle across a river. The current is flowing at 3 km/h. The resulting velocity of the boat is 5.4 km/h downstream, in a direction that forms a 15° angle with the adjacent shore. Determine the canoeist's velocity relative to the water.

The paddling vector, the current (3 km/h along the shore), and the resultant (5.4 km/h at 15° to the shore) form a triangle with a 15° angle between the current and the resultant:

$$|\vec{v}|^2 = 3^2 + 5.4^2 - 2(3)(5.4)\cos(15°)$$

$$|\vec{v}| \approx 2.62 \text{ km/h}$$

$$\cos\theta = \frac{3^2 - 5.4^2 - 2.62^2}{-2(5.4)(2.62)}$$

$\theta \approx 17.24°$ (the angle between the paddling vector and the resultant)

Measured from the shore, the paddling direction is $15° + 17.24° \approx 32.24°$. The velocity relative to the water is 2.62 km/h at an angle of 32.24° with the adjacent shore.

14) A pilot flies with a heading of 160° and an airspeed of 250 km/h. There is a steady wind of 30 km/h from the direction 030°. Calculate the ground velocity of the plane.

The wind blows toward 210°; the angle between the heading (160°) and the wind direction (210°) is 50°, giving an interior angle of 130° in the velocity triangle:

$$|\vec{v}|^2 = 250^2 + 30^2 - 2(250)(30)\cos(130°)$$

$$|\vec{v}| \approx 270.26 \text{ km/h}$$

$$\cos\theta = \frac{30^2 - 250^2 - 270.26^2}{-2(250)(270.26)}$$

$\theta \approx 4.88°$

The ground velocity is 270.26 km/h at a heading of $S15.12°E$.

15) A pilot flies on a heading of $N40°W$ with an airspeed of 240 km/h. Her actual ground velocity is 250 km/h at a bearing of $N42°W$. Determine the velocity of the wind.

The angle between the airspeed vector and the ground velocity vector is 2° (the difference between the two bearings):

$$|\vec{w}|^2 = 250^2 + 240^2 - 2(250)(240)\cos(2°)$$

$$|\vec{w}| \approx 13.16 \text{ km/h}$$

$$\cos\theta = \frac{250^2 - 240^2 - 13.16^2}{-2(240)(13.16)}$$

$\theta \approx 138.94°$

Since the ground-velocity bearing is $N42°W$ (i.e. 130° measured the same way as θ), the wind's bearing is offset from it by $\beta = 138.94° - 130° \approx 8.94°$. The wind has a velocity of 13.16 km/h at a heading of $N81.1°W$.

16) Determine the vertical and horizontal components of each force.

a) 30 N at an inclination of 40° counterclockwise from the horizontal $\|\vec{F_x}\| = 30\cos(40°) \approx 22.98$ N $\|\vec{F_y}\| = 30\sin(40°) \approx 19.28$ N	**b)** 50 m/s, 50° clockwise from the horizontal $\|\vec{F_x}\| = 50\cos(50°) \approx 32.14$ m/s $\|\vec{F_y}\| = 50\sin(50°) \approx 38.30$ m/s

17) A 500 N crate is resting on a ramp that is inclined 8° counterclockwise from the horizontal. Resolve the weight into two rectangular components, one parallel to the ramp and the other perpendicular to the ramp.

Relative to the ramp, the 500 N weight makes an angle of 8° with the perpendicular to the ramp:

$|\vec{f}| = 500\sin(8°) \approx 69.59$ N (parallel to the ramp)

$|\vec{n}| = 500\cos(8°) \approx 495.13$ N (perpendicular to the ramp)

18) A person is pushing on the handle of a lawn mower with a force of 400 N acting 30° clockwise below the horizontal. What is the magnitude of the force pushing the lawn mower horizontally? Pushing down on the lawn

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mower?

$|\vec{F_x}| = 400\cos(30°) \approx 346.41$ N (horizontal)

$|\vec{F_y}| = 400\sin(30°) = 200$ N (downward)

19) A plane on takeoff has a velocity of 300 km/h at an angle of 10° up from the horizontal. Calculate the rate at which the plane is climbing and its horizontal speed, to one decimal place.

horizontal speed $= 300\cos(10°) \approx 295.4$ km/h

rate of climbing $= 300\sin(10°) \approx 52.1$ km/h

Answer Key

1)a) $S50°E$ **b)** $N30°W$ **2)a)** 200° **b)** 043° **3)a)** $\overrightarrow{EF}$, $\overrightarrow{GH}$, $\overrightarrow{IK}$ **b)** $\overrightarrow{CD}$, $\overrightarrow{EF}$, $\overrightarrow{GH}$, $\overrightarrow{IK}$ **c)** $\overrightarrow{EF}$, $\overrightarrow{IK}$ **d)** $\overrightarrow{CD}$ **4)** see diagrams above **5)a)** $\vec{w}$ **b)** $-\vec{u}$ **c)** $-2\vec{w}$ **d)** $\vec{w} - \vec{u}$ **e)** $2\vec{w} - \vec{u}$ **6)a)** $\overrightarrow{EB}$ or $\overrightarrow{HC}$ **b)** $\overrightarrow{AF}$ or $\overrightarrow{DG}$ **c)** $\overrightarrow{AF}$ or $\overrightarrow{DG}$ **d)** $\overrightarrow{EC}$ **e)** $\overrightarrow{AG}$ **f)** $\overrightarrow{BC}$ **7)a)** $\frac{1}{3}\vec{v}$ **b)** $\frac{2}{3}\vec{v} + \vec{u}$ **c)** $-\vec{u} + \frac{2}{3}\vec{v}$ **d)** $\frac{1}{6}\vec{v} + \vec{u}$ **8)a)** $\frac{1}{\sqrt{169 + 60\sqrt{3}}}(\vec{u} + \vec{v})$ **b)** $\sqrt{1396 + 360\sqrt{3}}$ **9)** 403.1 km/h $N7.1°W$ **10)** 40° rope: 257.1 N; 50° rope: 306.4 N **11)** 32.35 m/s, $N13.7°E$ **12)** 74.1 N **13)** 2.62 km/h at 32.24° with the adjacent shore **14)** 270.26 km/h $S15.12°E$ **15)** 13.16 km/h $N81.1°W$ **16)a)** $F_h \approx 22.98$ N, $F_v \approx 19.28$ N **b)** $F_h \approx 32.14$ m/s, $F_v \approx 38.30$ m/s **17)** $|\vec{n}| \approx 495.13$ N; $|\vec{f}| \approx 69.59$ N **18)** horizontal force: 346.41 N; downward force: 200 N **19)** horizontal speed ≈ 295.4 km/h; rate of climbing ≈ 52.1 km/h

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