Grade 12 Calculus and Vectors Exam Review Lesson (Units 1-3) with solutions

This document was composed and formatted by Annie Bernatchez.

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Calculus Midterm Exam Review

1) Differentiate each of the following with respect to x.

a) $y = 4$ $y' = 0$	**b)** $y = 3x$ $y' = 3$	**c)** $y = 5x^3 + 3x - 7$ $y' = 15x^2 + 3$
d) $y = \frac{3x^2}{2}$ $y' = 3x$	**e)** $y = \frac{3}{x^4}$ $y = 3x^{-4}$ $y' = \frac{-12}{x^5}$	**f)** $y = 4\sqrt[3]{x} - 8$ $y = 4x^{1/3} - 8$ $y' = \frac{4}{3x^{2/3}}$
g) $y = (2x + 3)(5x - 1)$ $y' = 2(5x - 1) + 5(2x + 3)$ $y' = 20x + 13$	**h)** $y = (4x^2 - x + 2)(x - 3)$ $y' = (8x - 1)(x - 3) + 1(4x^2 - x + 2)$ $y' = 12x^2 - 26x + 5$	**i)** $y = (2x + 1)(x^2 - 3)(3x - 2)$ $y' = (6x^2 + 2x - 6)(3x - 2) + 3(2x + 1)(x^2 - 3)$ $y' = 24x^3 - 3x^2 - 40x + 3$
j) $y = (3x - 2)^2$ $y' = 2(3x - 2)(3)$ $y' = 18x - 12$	**k)** $y = \sqrt{2x - 5}$ $y' = \frac{1}{2}(2x - 5)^{-1/2}(2)$ $y' = \frac{1}{\sqrt{2x - 5}}$	**l)** $y = \frac{1}{(4x^2 - 3x)^3}$ $y' = -3(4x^2 - 3x)^{-4}(8x - 3)$ $y' = \frac{-3(8x - 3)}{(4x^2 - 3x)^4}$
m) $y = (x^2 + 2x)^3(4x + 1)^2$ $y' = 3(x^2 + 2x)^2(2x + 2)(4x + 1)^2 + 2(4x + 1)(4)(x^2 + 2x)^3$ $y' = 2(x^2 + 2x)^2(4x + 1)(16x^2 + 23x + 3)$	**n)** $y = \frac{2x}{x - 5}$ $y' = \frac{2(x - 5) - 1(2x)}{(x - 5)^2}$ $y' = \frac{-10}{(x - 5)^2}$	**o)** $y = \frac{3x}{x^2 + 2x - 4}$ $y' = \frac{3(x^2 + 2x - 4) - (2x + 2)(3x)}{(x^2 + 2x - 4)^2}$ $y' = \frac{-3(x^2 + 4)}{(x^2 + 2x - 4)^2}$
p) $y = \frac{3x}{(x - 2)^2}$ $y' = \frac{3(x - 2)^2 - 2(x - 2)(3x)}{(x - 2)^4}$ $y' = \frac{-3(x + 2)}{(x - 2)^3}$	**q)** $y = \left[\frac{1}{(4x + x^2)^3}\right]^3$ $y' = -9(4x + x^2)^{-10}(4 + 2x)$ $y' = \frac{-18(2 + x)}{(4x + x^2)^{10}}$	**r)** $y = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{3}} + 6\sqrt[3]{x}$ $y' = -x^{-3/2} + \frac{1}{\sqrt{3}} + 2x^{-2/3}$ $y' = \frac{-1}{x^{3/2}} + \frac{2}{x^{2/3}} + \frac{1}{\sqrt{3}}$

2) Determine $f''(-1)$ for the function $f(x) = \frac{3}{(2x - 3)^2}$.

$f(x) = 3(2x - 3)^{-2}$

$f'(x) = -6(2x - 3)^{-3}(2) = -12(2x - 3)^{-3}$

$f''(x) = 36(2x - 3)^{-4}(2) = \frac{72}{(2x - 3)^4}$

$f''(-1) = \frac{72}{[2(-1) - 3]^4} = \frac{72}{625}$

3) Determine the equation of the tangent to the curve $y = (2x^2 - 3)(x + 1)^2$ at the point where $x = 1$.

Point: $y(1) = [2(1)^2 - 3](1 + 1)^2 = -4$, so the point is $(1, -4)$.

$y' = 4x(x + 1)^2 + 2(x + 1)(2x^2 - 3)$

$y'(1) = 4(1)(2)^2 + 2(2)[2(1)^2 - 3] = 16 + 4(-1) = 12$

Using $y = mx + b$: $-4 = 12(1) + b \Rightarrow b = -16$

$y = 12x - 16$

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4) For what values of x does the curve $y = -x^3 + 6x^2$ have a tangent with a slope of 12?

$y' = -3x^2 + 12x$

$12 = -3x^2 + 12x$

$0 = -3x^2 + 12x - 12 = -3(x-2)^2$

$x = 2$

5) A tangent to the parabola $y = 3x^2 - 7x + 5$ is perpendicular to $x + 5y - 10 = 0$. Determine the equation of the tangent.

The line $x + 5y - 10 = 0$ has slope $y = -\frac{1}{5}x + 2$, so $m = -\frac{1}{5}$; the perpendicular slope is $m_\perp = 5$.

$y' = 6x - 7 = 5 \Rightarrow x = 2$

$y(2) = 3(2)^2 - 7(2) + 5 = 3$, so the point is $(2, 3)$.

$3 = 5(2) + b \Rightarrow b = -7$

$y = 5x - 7$

6) Water is being drained from the bottom of a tank. The equation that gives the volume V of water, in liters, remaining in the tank after t minutes is $V(t) = 3000\left(1 - \frac{t}{10}\right)^2$, $0 \le t \le 10$.

a) Find the volume of water in the tank after 5 min.

$V(5) = 3000\left(1 - \frac{5}{10}\right)^2 = 750$ L

b) Find the rate at which the water is draining from the tank after **i)** 3 min **ii)** 8 min

$V'(t) = 6000\left(1 - \frac{t}{10}\right)\left(-\frac{1}{10}\right) = -600\left(1 - \frac{t}{10}\right)$

i) $V'(3) = -420$ L/min

ii) $V'(8) = -120$ L/min

7) For the function $f(x) = x^3 - x^2 - x + 1$, determine the points where the tangent line is horizontal.

$f'(x) = 3x^2 - 2x - 1$

$0 = 3x^2 - 2x - 1 = (x-1)(3x+1)$

$x = 1$ or $x = -\frac{1}{3}$

$f(1) = 0, \quad f\left(-\frac{1}{3}\right) = \frac{32}{27}$

The points are $(1, 0)$ and $\left(-\frac{1}{3}, \frac{32}{27}\right)$.

8) The amount of pollution in a certain lake is $P(t) = \left(t^{1/4} + 3\right)^3$, where t is measured in years and P is measured in parts per million (ppm). At what rate is the amount of pollution changing after 16 years?

$P'(t) = 3\left(t^{1/4} + 3\right)^2\left(\frac{1}{4}t^{-3/4}\right)$

$P'(16) = 3\left(16^{1/4} + 3\right)^2\left[\frac{1}{4}(16)^{-3/4}\right] = \frac{75}{32} \approx 2.34$ ppm/year

9) An object moves in a straight line, and its position, s, in metres after t seconds is $s(t) = 8 - 7t - t^2$.

a) Determine the velocity when $t = 5$.

$v(t) = s'(t) = -7 - 2t \Rightarrow v(5) = -7 - 2(5) = -17$ m/s

b) Determine the acceleration when $t = 5$.

$a(t) = v'(t) = -2 \Rightarrow a(5) = -2$ m/s^2

10) A store sells 250 pairs of Brand X running shoes each month when priced at $100 per pair. It has been determined that for every $5 decrease in price, an additional 20 pairs will be sold.

a) Determine the demand or price function.

Let x = number sold, n = number of $5 price decreases. $x = 250 + 20n \Rightarrow n = \frac{x - 250}{20}$

$p(x) = 100 - 5\left(\frac{x - 250}{20}\right) = 100 - \frac{x - 250}{4}$

$p(x) = -0.25x + 162.5$

b) Determine the revenue function.

$R(x) = x \cdot p(x) = x(-0.25x + 162.5)$

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$R(x) = -0.25x^2 + 162.5x$

c) Find the revenue when sales are 250 pairs and 290 pairs of shoes per month.

$R(250) = \$25\,000$ $R(290) = \$26\,100$

d) Determine the marginal revenue when sales are 290 pairs of shoes per month.

$R'(x) = -0.5x + 162.5 \Rightarrow R'(290) = \$17.50$ per pair sold

11) A theatre has found that if the price of a ticket is \$24 then 400 tickets will be sold. For every \$2 decrease in price, an additional 50 tickets will be sold.

a) Determine the demand or price function based on the number of price decreases.

Let x = number of \$2 price decreases: $p(x) = 24 - 2x$

b) Determine a revenue function based on the number of price decreases.

Tickets sold $= 400 + 50x$

$R(x) = (24 - 2x)(400 + 50x)$

$R(x) = 9600 + 400x - 100x^2$

c) Determine the marginal revenue for the revenue function developed in part a).

$R'(x) = 400 - 200x$

d) When is this marginal revenue function equal to zero? What is the total revenue at this time? How can the owners of the theatre use this information?

$400 - 200x = 0 \Rightarrow x = 2$, so the price is $24 - 2(2) = \$20$.

$R(2) = 9600 + 400(2) - 100(2)^2 = \$10\,000$

This is the price/revenue combination that maximizes total revenue — the owners should price tickets at \$20 to generate the greatest possible revenue.

e) If the theatre has a maximum capacity of 450, what price will maximize revenue?

Capacity constraint: $400 + 50x \le 450 \Rightarrow x \le 1$. Since $R(x)$ is a downward parabola with unconstrained maximum at $x = 2$, R is still increasing on $0 \le x \le 1$, so the constrained maximum occurs at $x = 1$.

Price $= 24 - 2(1) = \$22$

12) Given the revenue function $R(x) = 30x - 0.025x^2$ and the cost function $C(x) = 2x + 5$, where x is the number of items being sold, determine:

a) the revenue and marginal revenue when 500 items are sold

$R(500) = 30(500) - 0.025(500)^2 = \$8750$

$R'(x) = 30 - 0.05x$

$R'(500) = \$5$ per item sold

b) the cost and marginal cost when 500 items are produced

$C(500) = 2(500) + 5 = \$1005$ $C'(x) = 2 \Rightarrow C'(500) = \$2$ per item sold

c) the profit and marginal profit from sales of 500 items

$P(x) = R(x) - C(x) = 28x - 0.025x^2 - 5$

$P(500) = 28(500) - 0.025(500)^2 - 5 = \$7745$

$P'(x) = 28 - 0.05x \Rightarrow P'(500) = \$3$ per item sold

13) Determine the equation of all tangent lines to the curve $y = 2x - \frac{8}{x}$ that have a slope of 4.

$y' = 2 + 8x^{-2} = 4 \Rightarrow \frac{8}{x^2} = 2 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

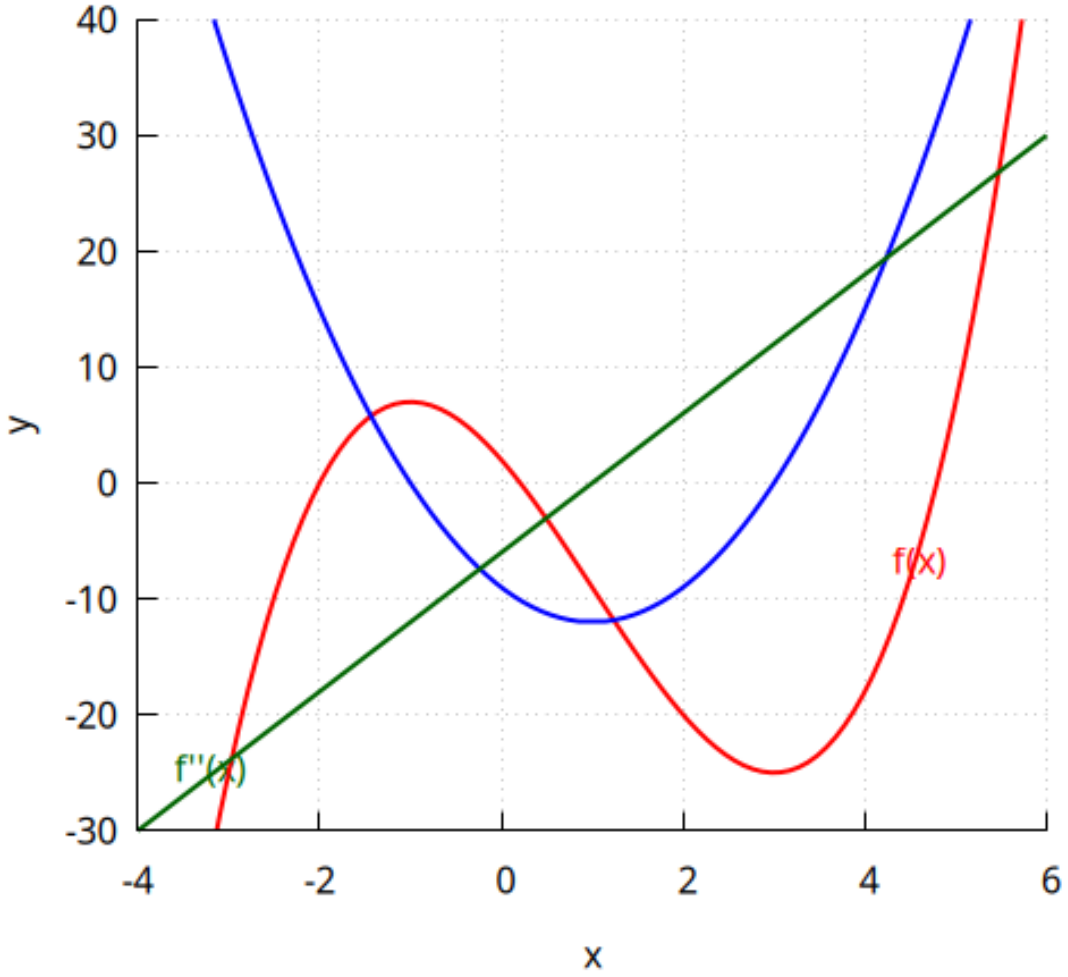
$y(2) = 0, \quad y(-2) = 0$

For $(2, 0)$: $0 = 4(2) + b \Rightarrow b = -8 \Rightarrow y = 4x - 8$

For $(-2, 0)$: $0 = 4(-2) + b \Rightarrow b = 8 \Rightarrow y = 4x + 8$

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14) Sketch a graph of the first and second derivative for the function $f(x)$ shown:



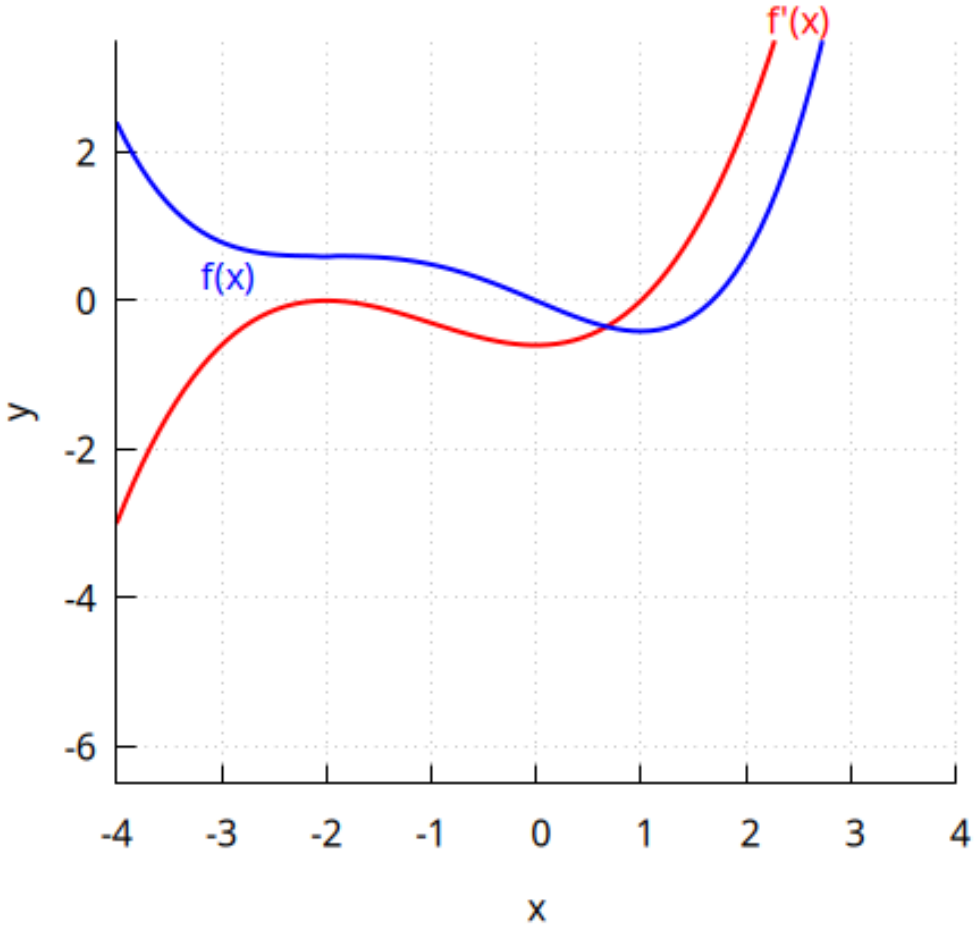
(A schematic cubic, $f(x) = x^3 - 3x^2 - 9x + 2$, is used here to illustrate the technique — any cubic matching the graph's qualitative shape works the same way.)

$f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

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15) Given the graph of $f'(x)$, state the intervals of increase and decrease for the function $f(x)$. Then sketch a possible graph of $f(x)$.



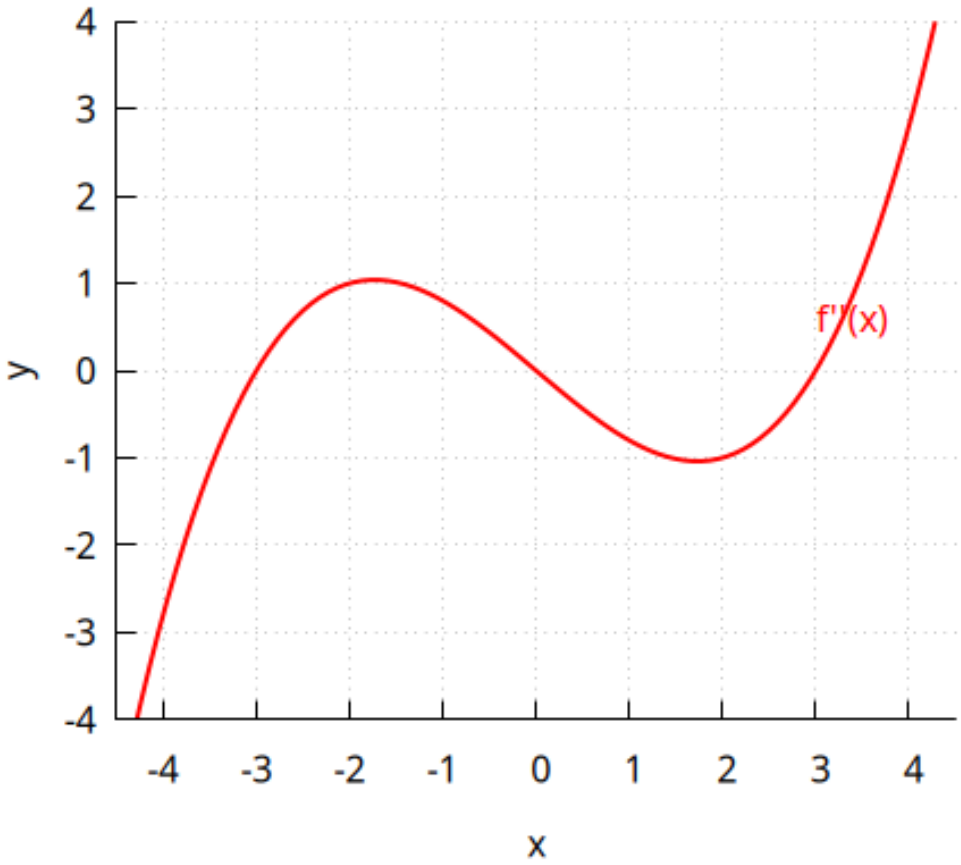
Increasing: $x > 1$

Decreasing: $x < -2, \quad -2 < x < 1$

($f'(x)$ touches but does not cross zero at $x = -2$, so the sign of f' does not change there.)

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16) Given the graph of $f''(x)$, state the intervals of concavity for the function $f(x)$.



Concave up: $-3 < x < 0,\ x > 3$

Concave down: $x < -3,\ 0 < x < 3$

17) Find the absolute maximum and minimum values of $f(x) = x^3 + 3x^2 + 1$ on the interval $-2 \le x \le 2$.

$f'(x) = 3x^2 + 6x = 3x(x+2) \Rightarrow x = 0,\ x = -2$

Test critical numbers and endpoints:

$f(-2) = 5,\quad f(0) = 1,\quad f(2) = 21$

Absolute minimum: $(0,\ 1)$ Absolute maximum: $(2,\ 21)$

18) Consider the function defined by $f(x) = -4x^3 + 6x^2 + 2$

a) Find the critical points and classify them using the second derivative test.

$f'(x) = -12x^2 + 12x = -12x(x-1) \Rightarrow x = 0,\ x = 1$

$f(0) = 2,\quad f(1) = 4$

Critical points: $(0,\ 2)$ and $(1,\ 4)$

$f''(x) = -24x + 12$

$f''(0) = 12 > 0$: $(0,\ 2)$ is a local min. $f''(1) = -12 < 0$: $(1,\ 4)$ is a local max.

b) Find the points of inflection and identify the intervals of concavity.

$0 = -24x + 12 \Rightarrow x = \frac{1}{2} \Rightarrow f(\frac{1}{2}) = 3$, possible POI at $(0.5,\ 3)$

Concave up: $x < \frac{1}{2}$ Concave down: $x > \frac{1}{2}$ POI at $(0.5,\ 3)$

19) Determine the local extrema for each function and classify them as local maxima or minima.

a) $y = 81 - x^4$

$y' = -4x^3 = 0 \Rightarrow x = 0,\quad y(0) = 81$. First derivative test: $y' > 0$ for $x < 0$, $y' < 0$ for $x > 0$.

Local max at $(0,\ 81)$

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b) $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x + 9$

$f'(x) = 12x^3 + 24x^2 - 12x - 24 = 12(x - 1)(x + 1)(x + 2)$

Critical points at $x = 1, \ -1, \ -2$: $f(1) = -10, \ f(-1) = 22, \ f(-2) = 17$. First derivative test gives sign pattern $-, \ +, \ -, \ +$ across $-2, \ -1, \ 1$.

Local min: $(-2, \ 17)$ and $(1, \ -10)$ Local max: $(-1, \ 22)$

c) $f(x) = -3x^3 + 9x^2 - 8$

$f'(x) = -9x^2 + 18x = -9x(x - 2) \Rightarrow x = 0, \ 2$

$f(0) = -8, \quad f(2) = 4$

Second derivative test: $f''(x) = -18x + 18$. $f''(0) = 18 > 0$ (local min at $(0, \ -8)$); $f''(2) = -18 < 0$ (local max at $(2, \ 4)$).

20) Determine the intervals over which each function is increasing/decreasing.

a) $y = x^3 - 12x + 8$

$y' = 3x^2 - 12 = 3(x - 2)(x + 2) \Rightarrow x = \pm 2$

Increasing: $x < -2, \ x > 2$ Decreasing: $-2 < x < 2$

b) $f(x) = 3x^4 - 2x^3 - 9x^2$

$f'(x) = 12x^3 - 6x^2 - 18x = 6x(2x - 3)(x + 1)$

Critical numbers: $x = 0, \ 1.5, \ -1$. Sign pattern: $-, \ +, \ -, \ +$ across $-1, \ 0, \ 1.5$.

Increasing: $-1 < x < 0, \ x > 1.5$ Decreasing: $x < -1, \ 0 < x < 1.5$

21) Determine the points of inflection and intervals of concavity for each function.

a) $y = \frac{x^4}{3} - 8x^2$

$y'' = 4x^2 - 16 = 4(x - 2)(x + 2) \Rightarrow x = \pm 2$

$y(\pm 2) = -\frac{80}{3}$

Points of inflection: $\left(2, \ -\frac{80}{3}\right)$ and $\left(-2, \ -\frac{80}{3}\right)$

Concave up: $x < -2, \ x > 2$ Concave down: $-2 < x < 2$

b) $f(x) = x^4 - 8x^3 + 3x - 5$

$f''(x) = 12x^2 - 48x = 12x(x - 4) \Rightarrow x = 0, \ 4$

$f(0) = -5, \quad f(4) = -249$

Points of inflection: $(0, \ -5)$ and $(4, \ -249)$

Concave up: $x < 0, \ x > 4$ Concave down: $0 < x < 4$

22) Sketch each function using the algorithm for curve sketching.

a) $h(x) = x^3 + 6x^2 + 9x + 4$

No restrictions, no asymptotes. $h(x) = (x + 4)(x + 1)^2$, so x-ints: $(-4, \ 0), \ (-1, \ 0)$; y-int: $(0, \ 4)$.

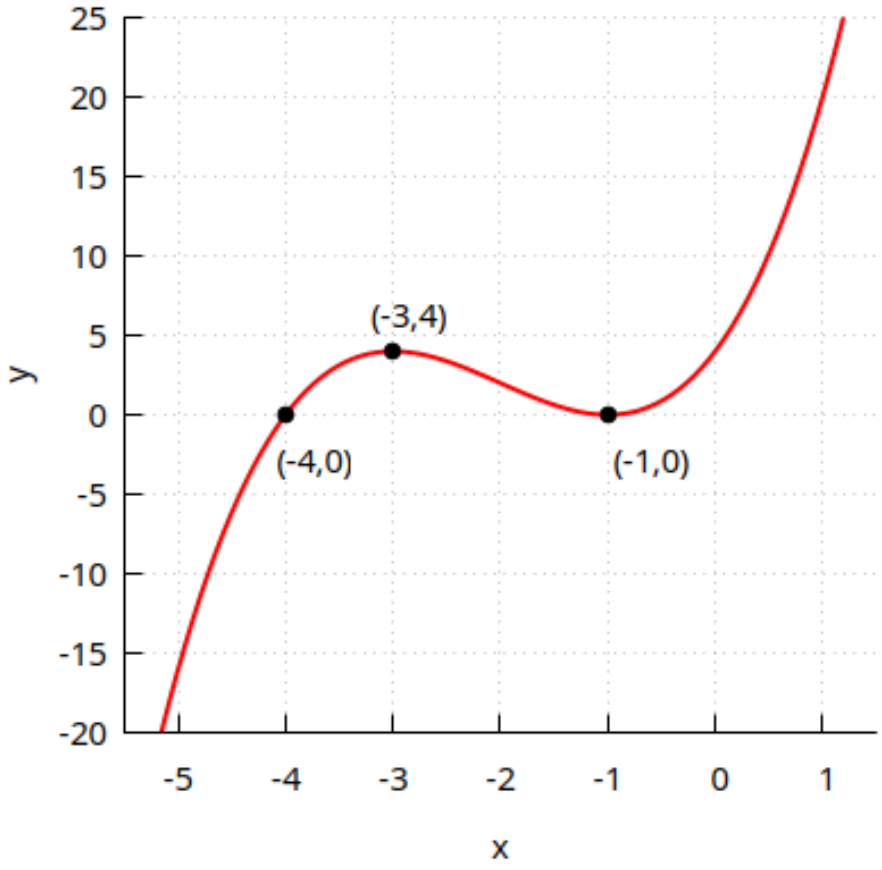
$h'(x) = 3x^2 + 12x + 9 = 3(x + 3)(x + 1) \Rightarrow x = -3, \ -1$

$h(-3) = 4, \ h(-1) = 0$. Critical points: $(-3, \ 4)$ (local max) and $(-1, \ 0)$ (local min).

$h''(x) = 6x + 12 = 0 \Rightarrow x = -2, \quad h(-2) = 2$

Possible POI: $(-2, \ 2)$. Decreasing/concave-down to the left, through a max, min, and inflection, increasing/concave-up to the right (see graph).

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b) $f(x) = -3x^4 - 2x^3 + 15x^2 - 12x + 2$

No restrictions, no asymptotes. x-ints near $(1,\ 0)$, $(-2.9,\ 0)$, $(0.23,\ 0)$; y-int: $(0,\ 2)$.

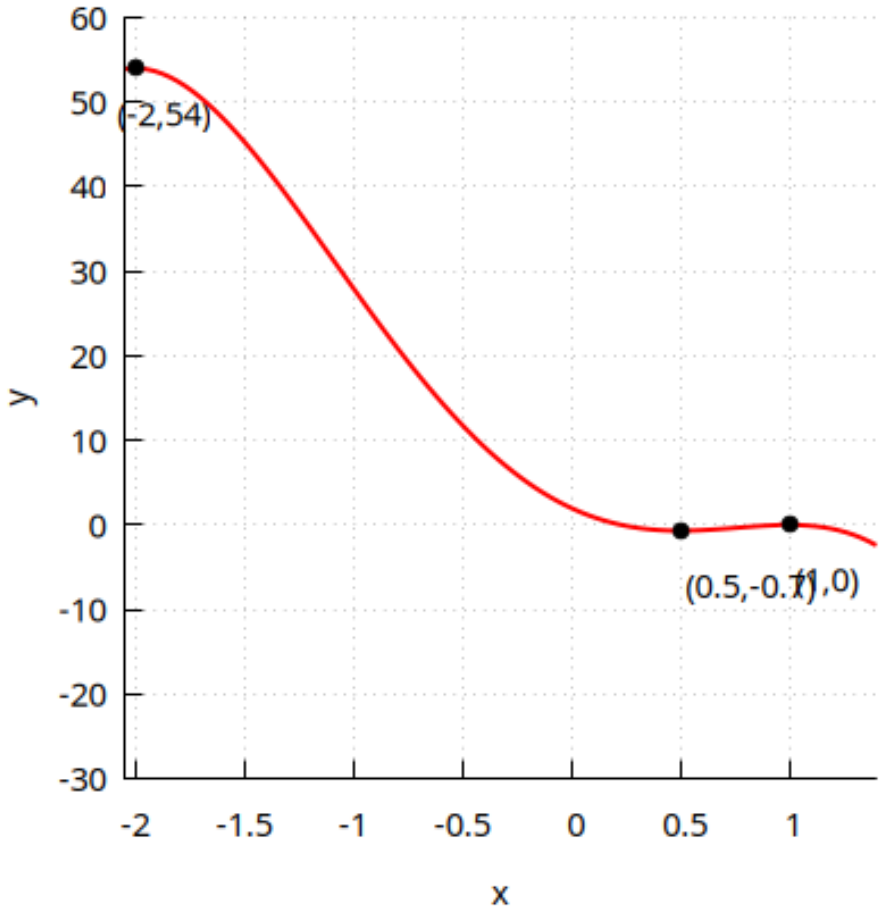
$f'(x) = -12x^3 - 6x^2 + 30x - 12 = -6(x - 1)(2x^2 + 4x - 2)$

Critical points: $(1,\ 0)$, $(0.5,\ -0.7)$, $(-2,\ 54)$.

$f''(x) = -36x^2 - 12x + 30$

Possible POI: $(0.76,\ -0.3)$ and $(-1.1,\ 31.6)$ (see graph).

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c) $f(x) = -x^4 - x^3 + 2x^2$

No restrictions, no asymptotes. $f(x) = -x^2(x+2)(x-1)$, so x-ints: $(0,\ 0),\ (-2,\ 0),\ (1,\ 0)$; y-int: $(0,\ 0)$.

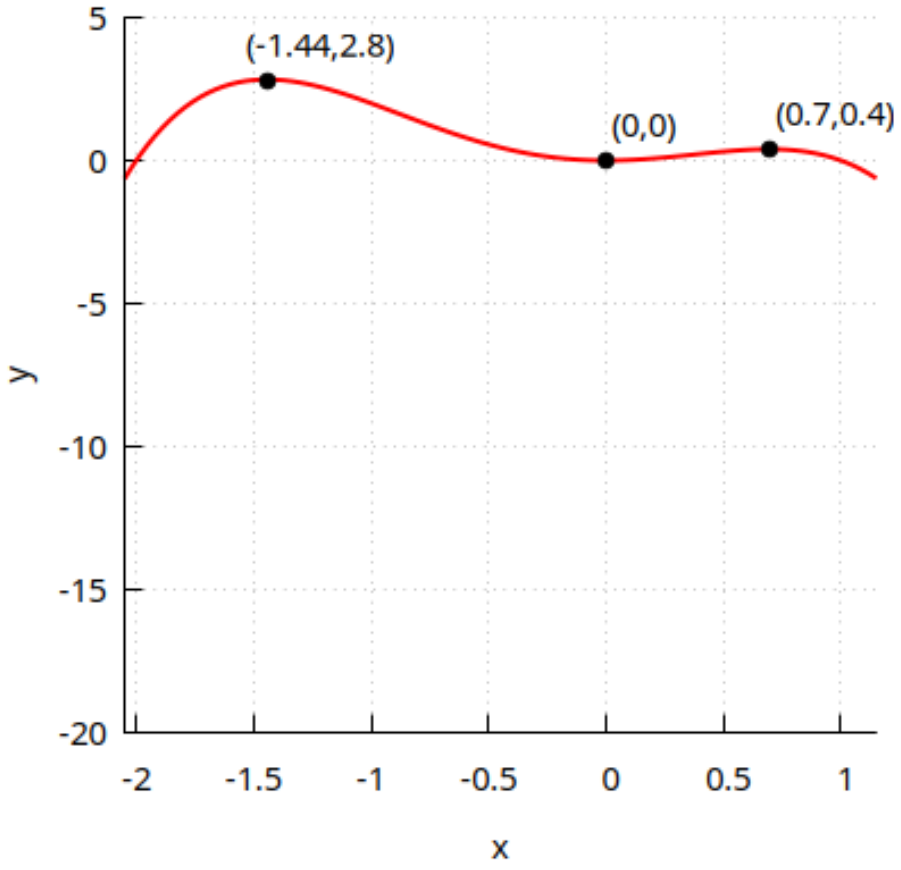
$f'(x) = -4x^3 - 3x^2 + 4x = -x(4x^2 + 3x - 4)$

Critical points: $(0,\ 0),\ (0.7,\ 0.4),\ (-1.44,\ 2.8)$.

$f''(x) = -12x^2 - 6x + 4$

Possible POI: $(0.38,\ 0.21)$ and $(-0.88,\ 1.63)$ (see graph).

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d) $y = \frac{5x}{x^2 - 1}$

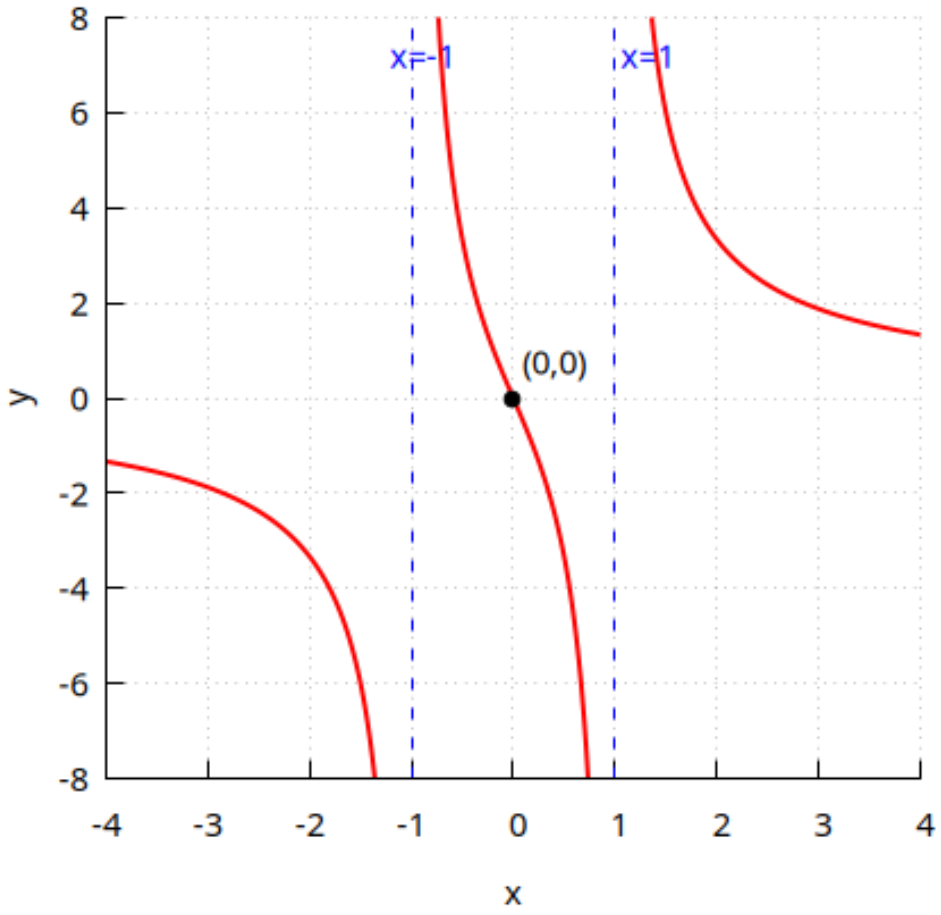
VA: $x = 1$ and $x = -1$ HA: $y = 0$

x-int and y-int both $(0, 0)$.

$y' = \frac{5(x^2 - 1) - 2x(5x)}{(x^2 - 1)^2} = \frac{-5(x^2 + 1)}{(x^2 - 1)^2}$

$x^2 + 1 \neq 0$, so there are no critical points. Possible POI at $(0, 0)$ (see graph).

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e) $y = \frac{1 - x^2}{x^2 + 1}$

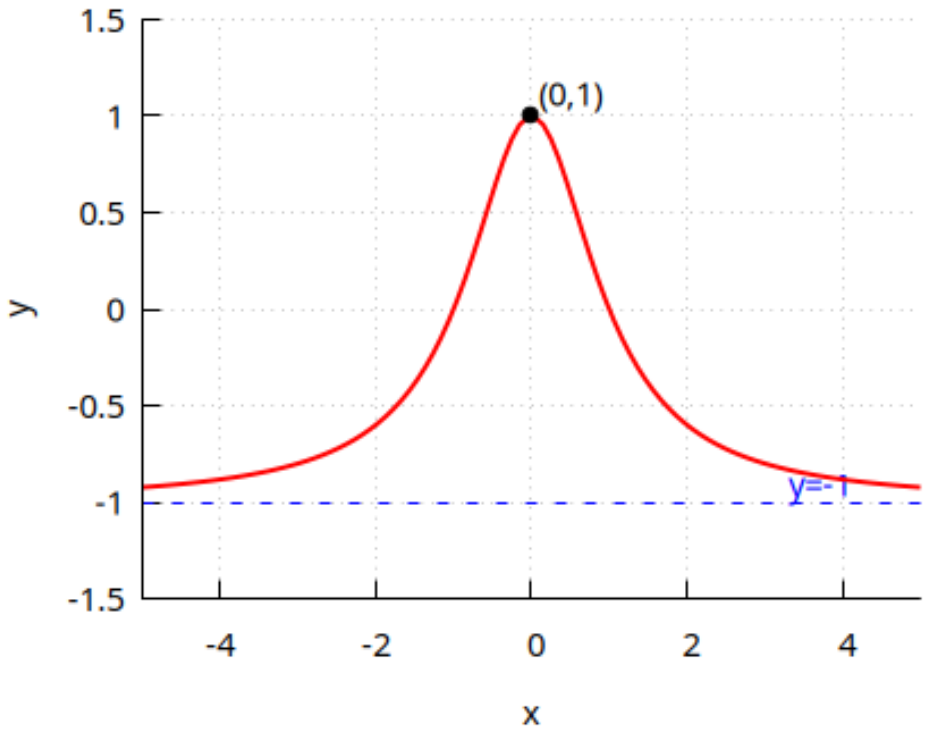
VA: none $(x^2 + 1 \neq 0)$ HA: $y = -1$

x-ints: $(1, 0)$ and $(-1, 0)$ y-int: $(0, 1)$

$y' = \frac{-4x}{(x^2 + 1)^2} = 0 \Rightarrow x = 0, \quad y(0) = 1$

Critical point: $(0, 1)$ (local max). Possible POI: $(-0.58, 0.5)$ and $(0.58, 0.5)$ (see graph).

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f) $f(x) = 3x^4 - 16x^3 + 18x^2$

No restrictions, no asymptotes. $f(x) = x^2(3x^2 - 16x + 18)$, x-ints: $(0,\ 0)$, $(3.72,\ 0)$, $(1.61,\ 0)$; y-int: $(0,\ 0)$.

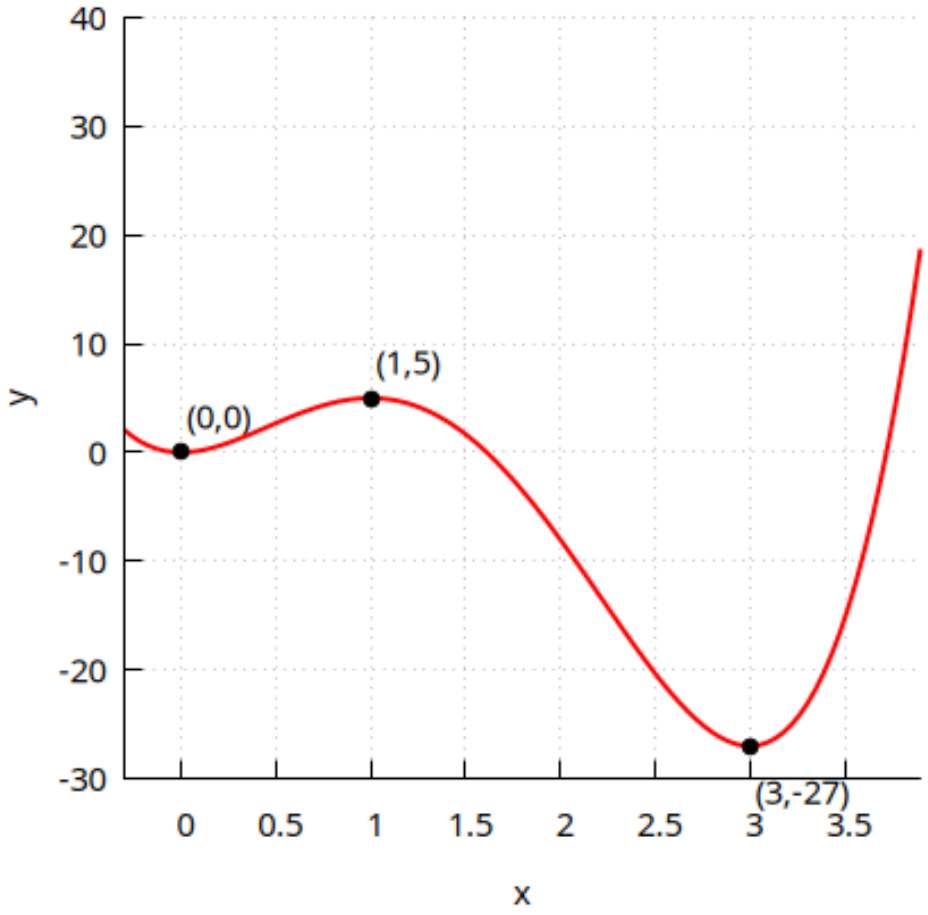
$f'(x) = 12x^3 - 48x^2 + 36x = 12x(x - 3)(x - 1)$

Critical points: $(0,\ 0)$, $(3,\ -27)$, $(1,\ 5)$.

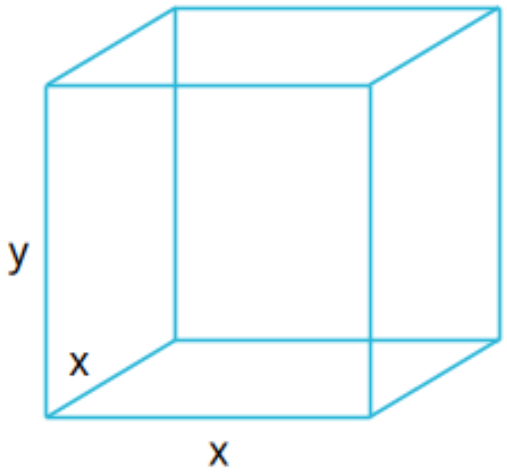
$f''(x) = 36x^2 - 96x + 36$

Possible POI: $(2.2,\ -13)$ and $(0.45,\ 2.3)$ (see graph).

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23) Iesha is making a jewelry box for her mother in technology class. The top and bottom of the box will be square and made out of a wood that costs \$0.002/cm², and the sides of the box will be made from a cheaper wood that costs \$0.001/cm². Find the dimensions of the box that will minimize the cost if she wants the box to have a volume of 4800 cm³.



$V = x^2y = 4800 \Rightarrow y = \frac{4800}{x^2}$. Surface area: top+bottom $= 2x^2$, sides $= 4xy$.

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$C(x) = 0.002(2x^2) + 0.001\left(\frac{19\,200}{x}\right) = 0.004x^2 + \frac{19.2}{x}$

$C'(x) = 0.008x - 19.2x^{-2} = 0 \Rightarrow x^3 = \frac{19.2}{0.008} = 2400 \Rightarrow x \approx 13.4$ cm

$y = \frac{4800}{13.4^2} \approx 26.7$ cm

$C''(x) = 0.008 + 38.4x^{-3} > 0$ at $x = 13.4$, confirming a minimum.

The dimensions that minimize the cost are 13.4 cm by 13.4 cm by 26.7 cm.

24) A manufacturing company is responsible for designing a 500 mL can for diced tomatoes.

a) Determine the dimensions of the can that will minimize the surface area of the can.

$V = \pi r^2 h = 500 \Rightarrow h = \frac{500}{\pi r^2}$

$SA(r) = 2\pi r^2 + 2\pi r\left(\frac{500}{\pi r^2}\right) = 2\pi r^2 + \frac{1000}{r}$

$SA'(r) = 4\pi r - 1000r^{-2} = 0 \Rightarrow r^3 = \frac{1000}{4\pi} \Rightarrow r \approx 4.3$ cm

$h = \frac{500}{\pi(4.3)^2} \approx 8.6$ cm

b) The lid and base of the can are made out of a stronger metal that costs twice as much as the sides. What dimensions will minimize the cost of the producing the can? How does your answer compare to your answer in part a)?

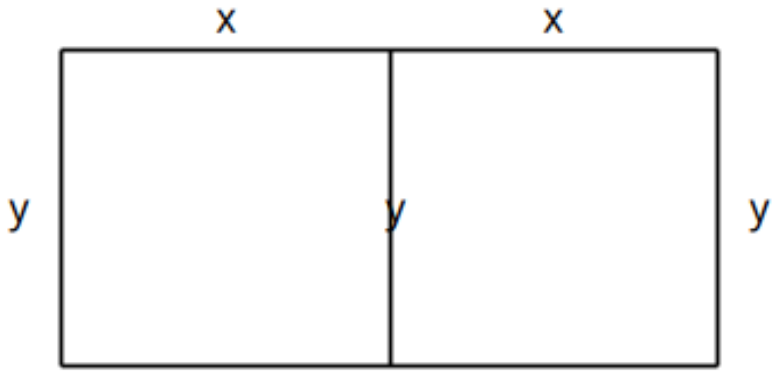
$C(r) = 2(2\pi r^2) + \frac{1000}{r} = 4\pi r^2 + \frac{1000}{r}$

$C'(r) = 8\pi r - 1000r^{-2} = 0 \Rightarrow r^3 = \frac{1000}{8\pi} \Rightarrow r \approx 3.4$ cm

$h = \frac{500}{\pi(3.4)^2} \approx 13.8$ cm

Weighting the top/bottom more heavily makes the optimal can narrower and taller than in part a) (less of the expensive top/bottom material is used relative to the cheaper sides).

25) A farmer has 800 m of fencing to enclose two rectangular pens. The pens should have equal areas. What dimensions will maximize the enclosed area?



$P = 4x + 3y = 800 \Rightarrow y = \frac{800 - 4x}{3}$. Total area $A = 2xy$.

$A(x) = 2x\left(\frac{800 - 4x}{3}\right) = \frac{1600x}{3} - \frac{4x^2}{3}$

$A'(x) = \frac{1600}{3} - \frac{16x}{3} = 0 \Rightarrow x = 100$ m

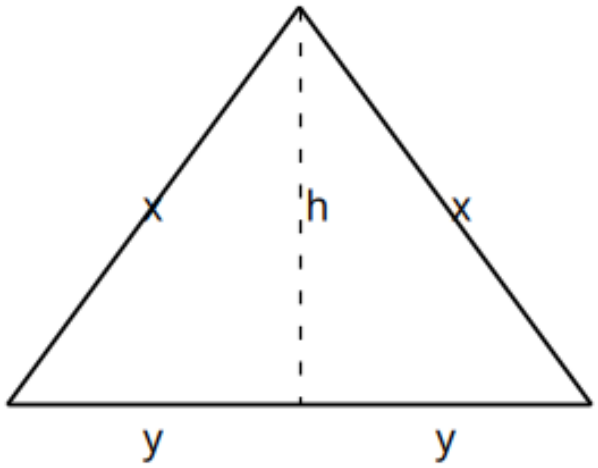
$y = \frac{800 - 4(100)}{3} \approx 133.3$ m

The rectangle formed by the two pens is 200 m by 133.3 m.

26) A piece of wire, 200 cm long, is to be bent into an isosceles triangle. What should the dimensions of the triangle

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be in order to maximize its area?



$P = 2x + 2y = 200 \Rightarrow x = 100 - y$. Height $h = \sqrt{x^2 - y^2} = 10\sqrt{100 - 2y}$ (using $x = 100 - y$).

$A(y) = \frac{1}{2}(2y)\left(10\sqrt{100 - 2y}\right) = 10y\sqrt{100 - 2y}$

$A'(y) = 10\sqrt{100 - 2y}\left[1 - \frac{y}{100 - 2y}\right] = 0 \Rightarrow y = 50$ or $y = \frac{100}{3}$; $y = 50$ is outside the domain (degenerate triangle), so the first-derivative test (limit of domain at $y = 50$) identifies a local max at $y = \frac{100}{3}$ cm.

$x = 100 - \frac{100}{3} = \frac{200}{3}$ cm

Dimensions of the triangle are $\frac{200}{3}$ by $\frac{200}{3}$ by $\frac{200}{3}$ cm — the equilateral triangle maximizes the area.

27) Differentiate each of the following with respect to x.

a) $y = 2x + 3e^x$ $y' = 2 + 3e^x$	**b)** $y = 10^x$ $y' = 10^x \ln(10)$	**c)** $y = 4^{3x^2}$ $y' = 4^{3x^2}[\ln(4)](6x)$ $y' = 6x(4)^{3x^2}\ln(4)$
d) $y = (x^4)2^x$ $y' = 4x^3(2^x) + 2^x[\ln(2)](x^4)$ $y' = x^3 2^x[4 + x\ln(2)]$	**e)** $y = 3\sin x + \cos(2x)$ $y' = 3\cos x - 2\sin(2x)$	**f)** $y = \tan(3x)$ $y' = [\sec^2(3x)](3)$ $y' = 3\sec^2(3x)$
g) $y = [\sin(2x)]e^{3x}$ $y' = (2\cos(2x))e^{3x} + 3e^{3x}[\sin(2x)]$ $y' = e^{3x}[2\cos(2x) + 3\sin(2x)]$	**h)** $y = x^2\sin x$ $y' = 2x\sin x + x^2\cos x$ $y' = x(2\sin x + x\cos x)$	**i)** $y = 5(2)^x$ $y' = 5[\ln(2)](2)^x$
j) $y = -2(10)^{3x}$ $y' = -2[\ln(10)](10)^{3x}(3)$ $y' = -6[\ln(10)](10)^{3x}$	**k)** $y = \sin^3(x^2)$ $y' = 3[\sin(x^2)]^2[\cos(x^2)](2x)$ $y' = 6x\cos(x^2)\sin^2(x^2)$	**l)** $y = \tan\sqrt{1-x}$ $y' = \sec^2\left(\sqrt{1-x}\right)\left(\frac{-1}{2\sqrt{1-x}}\right)$ $y' = \frac{-\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$
m) $y = [\sin(2x) + 1]^4$ $y' = 4[\sin(2x) + 1]^3[\cos(2x)](2)$ $y' = 8\cos(2x)[\sin(2x) + 1]^3$	**n)** $y = \sin[\cos(x^2)]$ $y' = \cos[\cos(x^2)] \cdot [-\sin(x^2)](2x)$ $y' = -2x\sin(x^2)\cos[\cos(x^2)]$	**o)** $y = 2\sin x - 3\cos(5x)$ $y' = 2\cos x + 15\sin(5x)$
p) $x^2y + y^2x = 1$ $2xy + x^2y' + 2xyy' + y^2 = 0$ $y' = \frac{-(y^2 + 2xy)}{2xy + x^2}$	**q)** $x = \sin(y)$ $1 = \cos(y)\,y'$ $y' = \frac{1}{\cos y}$	**r)** $y^5 + xy = 3$ $5y^4y' + y + xy' = 0$ $y' = \frac{-y}{5y^4 + x}$

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s) $\sin y + \sin x = 1$ $(\cos y)y' + \cos x = 0$ $y' = \frac{-\cos x}{\cos y}$	**t)** $yx^2 + e^y = x$ $y'x^2 + 2xy + e^y y' = 1$ $y' = \frac{1 - 2xy}{e^y + x^2}$	**u)** $y = \ln(x^3 + 3x)$ $y' = \frac{3x^2 + 3}{x^3 + 3x}$
v) $y = 4\log_5 x$ $y' = \frac{4}{x \ln 5}$	**w)** $y = \log_2(\cos x)$ $y' = \frac{-\sin x}{\cos x \ln 2}$	**x)** $y = \log(4x^2)$ $y' = \frac{2}{x \ln 10}$
y) $y = x^2 e^{\sin x}$ $y' = 2xe^{\sin x} + x^2 e^{\sin x}\cos x$ $y' = xe^{\sin x}(x\cos x + 2)$		

28) Find the slope of the line tangent to the curve $y = \sin(\cos x)$ at $x = \frac{\pi}{2}$.

$y' = \cos(\cos x) \cdot (-\sin x)$

$y'\left(\frac{\pi}{2}\right) = \cos(0) \cdot (-1) = -1$

$m = -1$

29) Find the equation of the line tangent to the curve $y = 2x^2 + \sin(4x)$ at $x = \frac{\pi}{3}$.

$y' = 4x + 4\cos(4x) \Rightarrow y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + 4\cos\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} - 2$

$y\left(\frac{\pi}{3}\right) = 2\left(\frac{\pi}{3}\right)^2 + \sin\left(\frac{4\pi}{3}\right) = \frac{2\pi^2}{9} - \frac{\sqrt{3}}{2}$

$y = \frac{4\pi - 6}{3}x + \frac{12\pi - 4\pi^2 - 9\sqrt{3}}{18}$

30) The function for a current is $I(t) = 9\cos(120\pi t)$ where I is the current, in amperes, and t is the time in seconds.

a) Find the current when $t = 6$ s.

$I(6) = 9\cos(720\pi) = 9\cos(0) = 9$ A (720π is a whole number of revolutions)

b) The electromotive force, $E(t)$, in a circuit, is given in volts by finding the first derivative of the current function $I(t)$. Find the function for the electromotive force $E(t)$.

$E(t) = I'(t) = -1080\pi\sin(120\pi t)$

c) Determine the electromotive force when $t = 3$.

$E(3) = -1080\pi\sin(360\pi) = 0$ V

d) Find the maximum and minimum electromotive forces and the times that they occur.

$E(t)$ has amplitude $1080\pi \approx 3392.9$.

Minimum -3392.9 V when $\sin(120\pi t) = 1$, i.e. $t = \frac{1 + 4k}{240}$, $k \in \mathbb{Z}$, $k \geq 0$

Maximum 3392.9 V when $\sin(120\pi t) = -1$, i.e. $t = \frac{3 + 4k}{240}$, $k \in \mathbb{Z}$, $k \geq 0$

31) The movement of an engine piston can be modelled by $h(t) = 4\sin t$ where h is the height of the piston, in centimetres, above the neutral position and t is time, in seconds.

a) Determine the velocity of the piston when $t = 5$.

$v(t) = h'(t) = 4\cos t \Rightarrow v(5) = 4\cos(5) \approx 1.135$ cm/s

b) Determine the acceleration of the piston when $t = 5$.

$a(t) = v'(t) = -4\sin t \Rightarrow a(5) = -4\sin(5) \approx 3.836$ cm/s^2

32) A virus is spreading according to the function $p(t) = 125(2)^{t/4}$ where $p(t)$ is the number of people infected after t days.

a) How many people had the virus initially?

$p(0) = 125(2)^0 = 125$ people

b) How many people will be infected after 4 weeks?

4 weeks $= 28$ days: $p(28) = 125(2)^7 = 125(128) = 16\,000$ people

c) How fast will the virus be spreading after 4 weeks?

$p'(t) = \frac{125\ln(2)}{4}(2)^{t/4}$

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$p'(28) = \frac{125\ln(2)}{4}(128) = 4000\ln(2) \approx 2773$ people/day

d) How long will it take until 22 500 people are infected?

$125(2)^{t/4} = 22\,500 \Rightarrow (2)^{t/4} = 180 \Rightarrow t = \frac{4\ln(180)}{\ln(2)} \approx 30$ days

33) Find the equation of the line tangent to the curve $y = \left(\frac{1}{2}\right)^x$ at $(-3, 8)$.

$y' = \left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right) \Rightarrow y'(-3) = 8\ln\left(\frac{1}{2}\right)$

$8 = 8\ln\left(\frac{1}{2}\right)(-3) + b$

$y = 8\ln\left(\frac{1}{2}\right)x + 8 + 24\ln\left(\frac{1}{2}\right)$

34) Determine the equation of the tangent to the curve $y = x\sin x$ at the point where $x = \frac{\pi}{2}$.

$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}; \quad y' = \sin x + x\cos x \Rightarrow y'\left(\frac{\pi}{2}\right) = 1 + 0 = 1$

$\frac{\pi}{2} = 1\left(\frac{\pi}{2}\right) + b \Rightarrow b = 0$

$y = x$

35) Determine all the maximum and minimum values of $y = xe^x + 3e^x$.

$y = e^x(x+3) \Rightarrow y' = e^x(x+3) + e^x = e^x(x+4) = 0 \Rightarrow x = -4$

$y(-4) = e^{-4}(-4+3) = -e^{-4}$

$y' < 0$ for $x < -4$ and $y' > 0$ for $x > -4$, so this is the only critical point.

Local minimum at $(-4, -e^{-4})$. No local maximum.

36) Determine the equation of the tangent to the curve defined by $y = 2e^{3x}$ that is parallel to the line defined by $-6x + y = 2$.

The line has slope 6. $y' = 6e^{3x} = 6 \Rightarrow e^{3x} = 1 \Rightarrow x = 0, \quad y(0) = 2$

$2 = 6(0) + b \Rightarrow b = 2$

$y = 6x + 2$

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